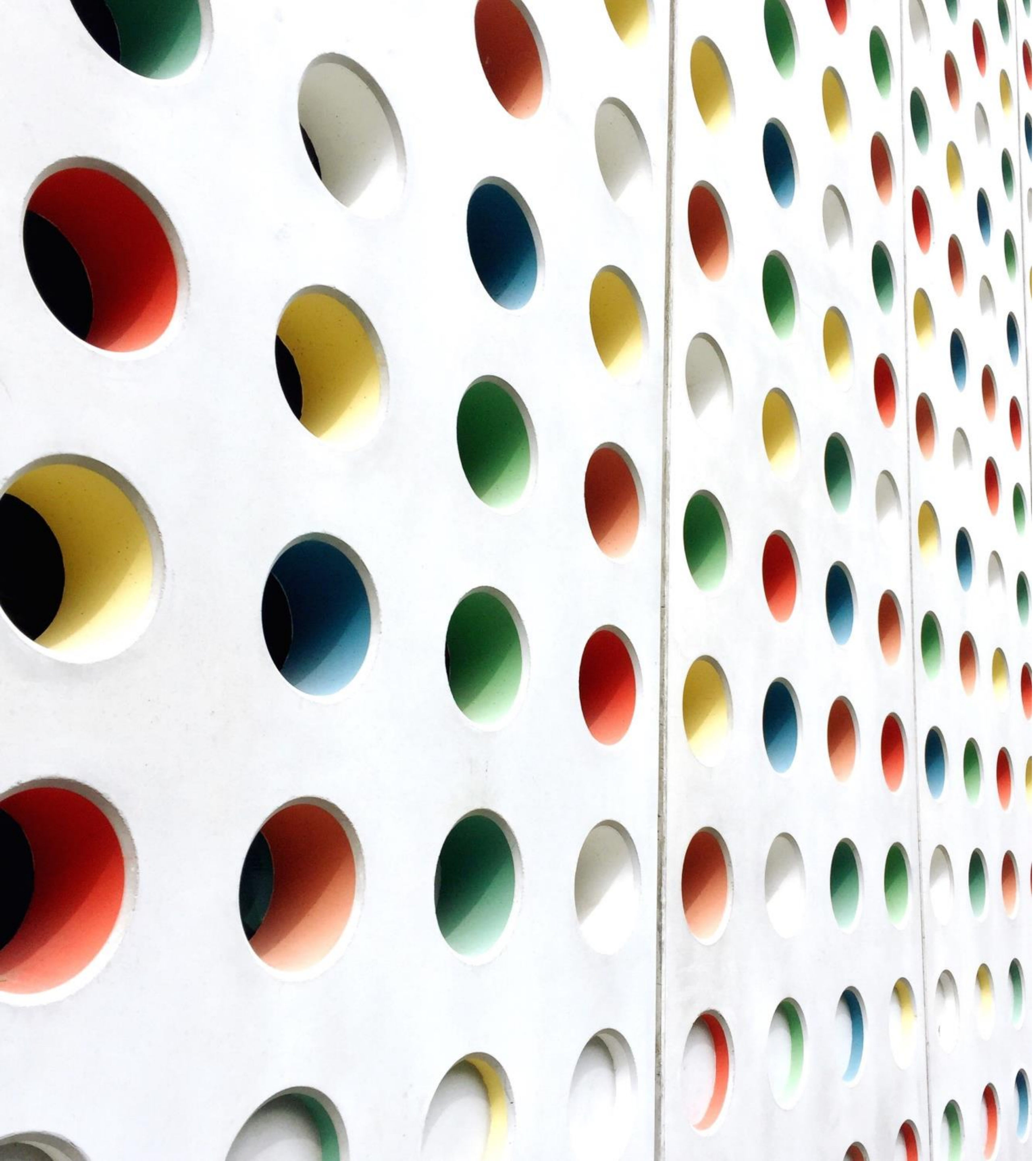

Real Numbers (Euclid's Division Lemma) (Chapter: 1)

CLASS: 10^{TH}

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(Euclid's Division Lemma) :

Given positive integers a and b, there exist unique integers q and r satisfying a relation

Remainder

$$a = bq + r, (0 \le r < b).$$

dividend

quotient

Division

$\boxed{0 \le r < b}$

$$3\overline{)\,10\,}\,3$$
$$9$$
$$1$$

$$3\overline{)\,9\,}\,3$$
$$9$$
$$0$$

$$b\overline{)\,a\,}\,q$$
$$x$$
$$r$$

-ve

Important step for Euclid's division:

$c > d$

To obtain the HCF of two positive integers, say c and d, with c > d, follow the steps below:

- *Apply Euclid's division lemma, to c and d. So, we find whole numbers, q and r such that*

  $c = dq + r, 0 \le r < d.$ $0 \le r < d$

- *If r = 0, d is the HCF of c and d. If r ≠ 0, apply the division lemma to d and r.*
- *: Continue the process till the remainder is zero. The divisor at this stage will be the required HCF.*

This algorithm works because HCF (c, d) = HCF (d, r) where the symbol HCF (c, d) denotes the HCF of c and d, etc.

$10 = 3 \times 3 + 1$

3

HCF

Example: Use Euclid's algorithm to find the HCF of s 455 and 42.

$455 > 42$

$455 = 42 \times 10 + 35$

$42 = 35 \times 1 + 7$

$35 = 7 \times 5 + 0$

HCF

$$\begin{array}{r} 7\overline{)455}\,65 \\ 42 \\ \hline 35 \\ 35 \\ \hline \times \end{array} \qquad \begin{array}{r} 2\overline{)42}\,6 \\ 42 \\ \hline \times \end{array}$$

$$\begin{array}{r} 42\overline{)455}\,10 \\ 420 \\ \hline 35\overline{)42}\,1 \\ 35 \\ \hline 7\overline{)35}\,5 \\ 35 \\ \hline 0 \end{array}$$

Use Euclid's division algorithm to find the HCF of :135 and 225

$a = bq + r$

$225 > 135$

$225 = 135 \times 1 + 90$

$135 = 90 \times 1 + 45$

$90 = 45 \times 2 + 0$

$\rightarrow$ H.C.F $= 45$

$$
\begin{array}{r|l}
135 & 225 \; (1 \\
 & 135 \\ \hline
90 & 135 \; (1 \\
 & 90 \\ \hline
45 & 90 \; (2 \\
 & 90 \\ \hline
 & 0
\end{array}
$$

$r \rightarrow 0$

Example: Use Euclid's algorithm to find the HCF of 4052 and 12576.

$12576 > 4052$

$$12576 = 4052 \times 3 + 420$$
$$4052 = 420 \times 9 + 272$$
$$420 = 272 \times 1 + 148$$
$$272 = 148 \times 1 + 124$$
$$148 = 124 \times 1 + 24$$
$$124 = 24 \times 5 + 4$$
$$24 = 4 \times 6 + 0$$

$$\text{H.C.F} = 4$$

4052) 12576 (3
12156
420) 4052 (9
3780
272) 420 (1
272
148

Show that any positive odd integer is of the form 6q + 1, or 6q + 3, or 6q + 5, where q is some integer.

$a = bq + r$ and $0 \le r < b$

we know that

$b = 6$ (division) $(0 \le r < 6)$

$r = 0, 1, 2, 3, 4, 5$ $(a = 6q + r)$

for $r = 0$

$a = 6q$ (6 divide 2) (even)

for $r = 1$

$a = 6q + 1$ (6 divisible 2, 1 is $\ne$ 2) (odd)

for $r = 2$

$a = 6q + 2$ $(6 \div 2)(2 \div 2)$

even

for $r = 3$

$a = 6q + 3$ $(6 \div 2)(3 \not\div 2)$

(odd)

for $r = 4$

$a = 6q + 4$ $(6 \div 2)(4 \div 2)$

(even)

for $r = 5$

$a = 6q + 5$ $(6 \div 2)(5 \not\div 2)$

odd

An army contingent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?

Total no. of members = 616

No. of members in army band = 32

HCF (616, 32)

616 > 32

$$616 = 32 \times 19 + 8$$

$$32 = 8 \times 4 + 0$$

$$\begin{array}{r} 32 \\ \times 9 \\ \hline 288 \end{array}$$

$$32\overline{)616}(19$$
$$\underline{32}$$
$$296$$
$$\underline{288}$$
$$8)32(4$$
$$\underline{32}$$
$$0$$

HCF = 8

maximum no. of columns = 8

$3m,\ 3m+1,\ 3m+1$

Use Euclid's division lemma to show that the square of any positive integer is either of the form 3m or 3m + 1 for some integer m.

$a = bq + r \quad (0 \le r < b)$

let a = positive Integer

and $b = 3$

ie $r = 0, 1, 2$

generation

$a = 3q + r$

$\therefore a = 3q,\ 3q+1,\ 3q+2$

$a^2 = (3q)^2,\ (3q+1)^2,\ (3q+2)^2$

$= 9q^2,\ 9q^2+1+6q,\ 9q^2+4+12q$

$= 3(3q^2),\ 9q^2+6q+1,\ 9q^2+12q+3+1$

$= 3(3q^2),\ 3(3q^2+2q)+1,\ 3(3q^2+4q+1)+1$

$= 3k_1,\ 3k_2+1,\ 3k_3+1$

where k_1, k_2, k_3 = Integer

$k_1 = k_2 = k_3 = m$ Integer

Use Euclid's division lemma to show that the cube of any positive integer is of the form 9m, 9m + 1 or 9m + 8

let a = Positive Integer

$a = bq + r \quad (0 \le r < b)$

$b = 3 \quad (r = 0, 1, 2)$

$a = 3q + r$

$a = 3q, \ 3q+1, \ 3q+2$

cubing

$a^3 = (3q)^3, \ (3q+1)^3, \ (3q+2)^3$

$a = 27q^3 = 9(3q^3) = \boxed{9(m)} \quad (m = 3q^3)$

$a = (3q+1)^3 = (3q)^3 + 1^3 + 3(3q)^2 + 3(3q)$

$= 27q^3 + 1 + 27q^2 + 9q$

$= (27q^3 + 27q^2 + 9q) + 1$

$= 9(3q^3 + 3q^2 + q) + 1$

$\boxed{a = 9m+1} \quad (3q^3 + 3q^2 + q = m$

$a = (3q+2)^3 = 27q^3 + 8 + 27q^2 + 9q$

$= 9(3q^3 + 3q^2 + q) + 8$

$\boxed{a = 9m+8}$