Chapter :- 1(b)

Systems of Units :-

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⇒ Need of Measurement :-

Physics as we know, is a branch of science which deals with the study of nature and natural phenomimena. for a precise description of an such phenomimina, measurement of the quantities involved is essential.

All the quantities in terms of which law of physics are described and whose measurment is necessary are called physical quantities.

⇒ Systems of Units :- four type of system of Unit :-

a) f.p.s :- (British Engineering System of Units)

f :- Foot (unit of length)
p :- pound (Unit of mass)
s :- second (unit of time)

b) C.g.s :- (Gaussian System) (German Mathematician Cari Friedrich in 1832)

c :- Centimetre (Unit of length)
g :- gram (unit of Mass)
s :- Second (Unit of time)

(c) m.k.s :- (Italian physicist discovered m.k.s (Giovanni Giorgi) 1935.

m :- meter (unit of length)
k :- kilograms (unit of mass)
s :- second (unit of time)

(d) S.I :- International system of Unit.

(This system Introduced in 1960, by the General Conference of weight and measures.

This system of units sessentially a modification of m.k.s system and is ∴ Called Rationalised m.k.s units.

The S.I. is based on the following seven fundamental Units and two supplymentaly Units :-

Fundamental Units.

Basic Physical Quantity	Fundamental Units	Symbol used.
Mass	Kilogram	Kg
length	metre	m
Time	second	s
Temperature	kelvin	k
Electric current	ampere	A

Luminous intensity	Candela	cd.
Quantity of matter	mole	mol.

Supplementary of units

Supplementary physical quantity	Supplementary unit	Symbol
plane angle	radian	rad
Solid angle	steradian	sr.

Some Important practical units.

1) AU (Astronomical unit) :- It is the average distance of the Centre of the sun from the Centre of the earth.

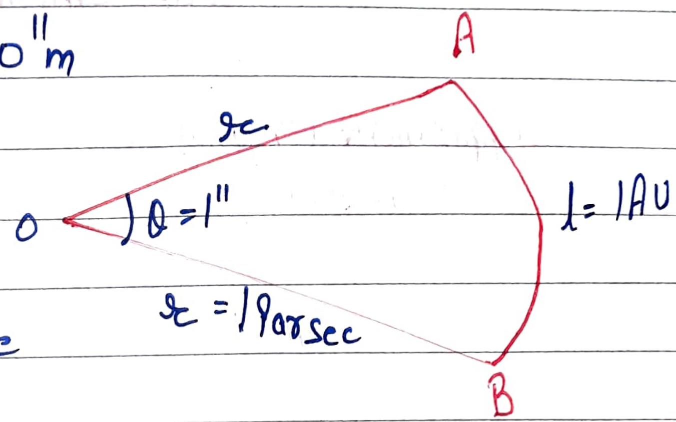
$$1\,AU = 1.496 \times 10^{11} m \cong 1.5 \times 10^{11} m.$$

2) Light Year :- It is an Important Unit of long distance. One light year is the distance travelled by light in vaccum in one year.

$$1\,ly = (365 \times 24 \times 60 \times 60)\text{ metre} \times 3 \times 10^{8}$$
$$= 9.46 \times 10^{15} m.$$

3) Par sec :- It is yet another unit of long distance and represents a parallactic second. one Par sec is the distance at which an arc 1 AU long subtends an angle of 1".

$l = 1AU = 1.5 \times 10^{11} m$



$\theta = 1 sec$

$= \frac{1}{60}$ min

$= \frac{1}{60 \times 60}$ degree

$= \frac{1}{60 \times 60} \times \frac{\pi}{180}$ radian

Here $\theta = \frac{arc}{Radius} = \frac{AB}{OB}$

$\theta = \frac{l}{r} \Rightarrow r = \frac{l}{\theta}$

$= \frac{1.5 \times 10^{11} m}{\frac{\pi}{60 \times 60 \times 180}}$

$= \frac{1.5 \times 10^{11} \times 60 \times 60 \times 180}{\pi}$

$1 \text{ Par sec} = 3.1 \times 10^{16} m.$

Relation between AU, ly and Par sec.

Here $1\,AU = 1.5 \times 10^{11}\,m$

$1\,ly = 9.46 \times 10^{15}\,m$

and $1\,par\,sec = 3.1 \times 10^{16}\,m$

$$\frac{1\,ly}{1\,AU} = \frac{9.46 \times 10^{15}}{1.5 \times 10^{11}}$$

$$= \frac{9.46}{1.56} \times 10^{4}$$

$$= 6.3 \times 10^{4}$$

$$\boxed{1\,ly = 6.3 \times 10^{4}\,AU}$$

$$\frac{1\,par\,sec}{1\,ly} = \frac{3.1 \times 10^{16}}{9.46 \times 10^{15}} = 3.26$$

$$1\,par\,sec = 3.26\,ly.$$

(Imp. Note)

Solar day :- It is the time taken by the earth to Complet one Rotation about its axis w.r.t. the sun.

Sedrial day :- It is the time taken by the earth to Complete one rotation about its axis w.r.t. a fixed star.

Solar year :- It is the time taken by the earth to Complete one revolution around the sun

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in its orbit.

1 solar year = 365.25 of average solar days

= 366.25 sedrial days.

Lunar month :- It is the time taken by moon to complete one revolution around the earth in its orbit.

1 lunar month = 27.3 days.

Shake :- It is smallest practical unit of time

1 shake :- 10^{-8} s.

formulae used :-

1 AU = 1.496×10^{11} m

1 ly = 9.46×10^{15} m

1 parsec = 3.1×10^{16} m

1 A° = 10^{-10} m

1 μm = 10^{-6} m

1 mm = 10^{-3} m

—/—/—

Numericals :-

Example: Calculate the number of light years in one metre.

Sol:- We know $1\ ly = 9.46 \times 10^{15}\ m$

$$\therefore 1\ m = \frac{1}{9.46 \times 10^{15}}\ ly = \frac{1}{9.46} \times 10^{-15}\ ly$$

$$1m = 1.057 \times 10^{-16}\ ly$$

Example 2:- The mass of an electron is 9.11×10^{-31} kg. How many electrons would make 1kg.

Sol:- Here, mass of an electron $= 9.11 \times 10^{-31}$ kg

total mass $= 1$ kg

$$\therefore \text{no. of electrons} = \frac{\text{total mass}}{\text{mass of each electron}}$$

$$= \frac{1}{9.11 \times 10^{-31}} = 1.1 \times 10^{30}$$

Chapter :- 1(c)

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Measurement of Length, Mass and Time

⇒ Method to Calculate Distance :-

(a) Direct Method to Calculate Distances :-

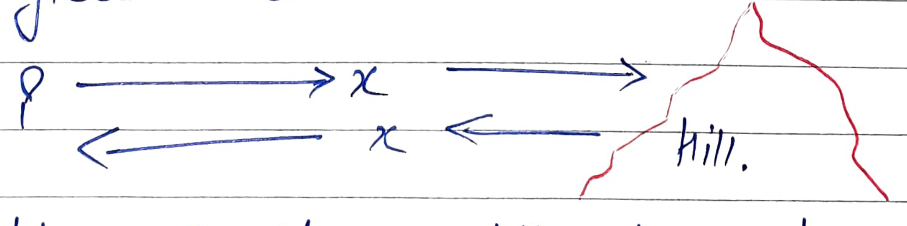
The direct Method is the Common method that employs a Chain, tape or any other instrument to measure the linear distance.

(b) Indirect Method to Calculate Distances :-

The Indirect Method Can be determined indirectly on the base of Known Sides and angles, rather than being measured directly.

eg :- 1) Echo Method or Reflection Method :-

An echo is the phenomenon of repetition of Sound on reflection from an obstacle. That is why echo method is also Called Reflection method.



Let us Consider a hill at a distance (x) from the observer at Point (P). Let a Shot be fired in air at (P).

The sound of fire travell distance x and then Reflect back to the observer.

i.e.

$$\text{distance} = \text{velocity} \times \text{time}$$
$$x + x = v \times t$$
$$2x = v \times t$$
$$\boxed{x = \frac{v \times t}{2}}$$

Here v = velocity of sound. (336 m/s)

2) ~~Acoustic~~ LASER Method :-

L (Light) A (Amplification) by S (Stimulation) E (Emission) of R (Radiation).

This method is used to Calculate long distances
eg:- Calculate the distance of Moon from earth.

The laser beam transmitted from earth is received back on earth after Reflection from the moon.

$$\text{Distance} = \text{Velocity} \times \text{time}$$
$$x + x = c \times t$$
$$2x = c \times t$$
$$\boxed{x = \frac{c \times t}{2}}$$

Here c = velocity of light.

RADAR METHOD:-

RA(Radio) D(Detection) and R(Ranging)

It is a Powerful equipment which is used for detection of an object like aeroplane and measuring distance using Radio waves.

The Radio waves are transmitted in space all around from the RADAR station. When they are intercepted by an object like aeroplane, they get reflected and are received back at the the RADAR station.

$$x + x = c \times t$$

$$2x = c \times t$$

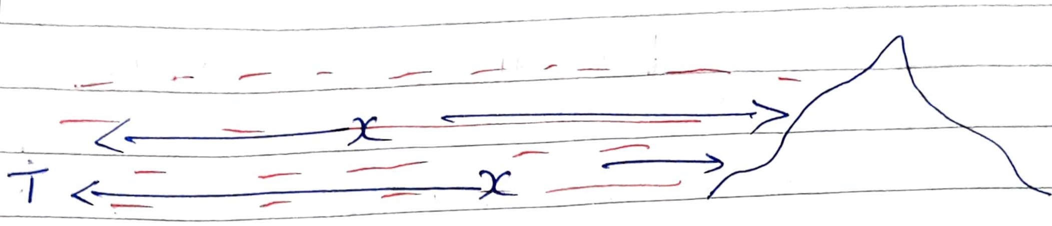
$$\boxed{x = \frac{c \times t}{2}}$$

SONAR Method:- SO(Sound) N (Navigation) A and R (Ranging)

This Method is used to detecting an object under water and measuring its distance using ultrasonics.

The ultrasonics waves from a transmitted from under water from a

Transmitter (T). When they intercepted by an object like rock, they get reflected.

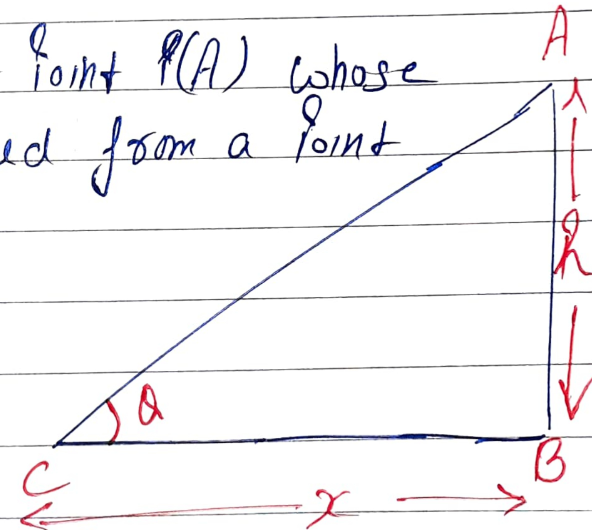


$$x + x = v + t$$
$$2x = v + t$$
$$\boxed{x = \frac{v+t}{2}}$$

Where (v) is velocity of ultrasonic waves is sea water.

Triangulation Method :- This is the geometrical Method to measuring the height of a distant object.

Let us consider a object at point P(A) whose height is h is to be measured from a point (C) at a distance (x).



Here $\angle ACB = \theta$

In ΔACB

$$\tan\theta = \frac{P}{B} = \frac{AB}{BC} = \frac{h}{x}$$

$$\boxed{h = x\tan\theta}$$

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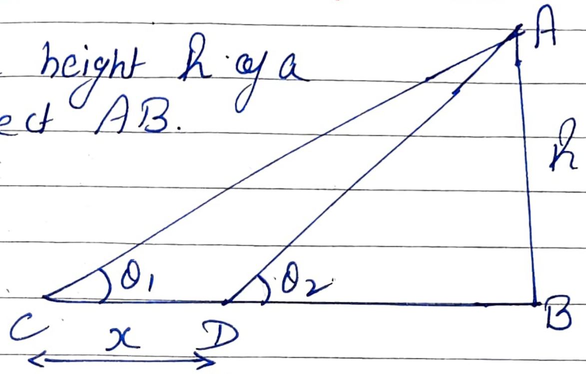
If distance (x) of object from the observation point is known, we can calculate its height.

In case object is another side of the River

To measure the height h of a inaccessible object AB.



The sextent is then moved horizontal to any other position D where ($CD = x$)

In ΔABC, $\cot\theta_1 = \dfrac{CB}{AB}$ —— (I)

In ΔABD, $\cot\theta_2 = \dfrac{DB}{AB}$ —— (II)

Subtract (II) from (I)

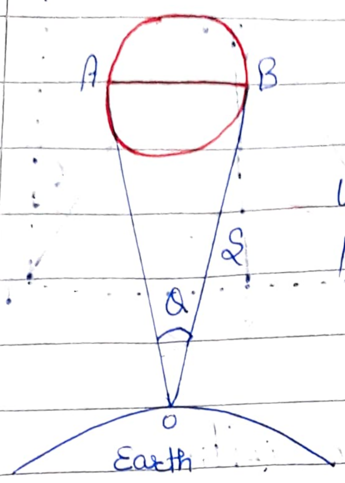
$$\cot\theta_1 - \cot\theta_2 = \frac{CB}{AB} - \frac{DB}{AB}$$

$$= \frac{CB - DB}{AB}$$

$$\cot\theta_1 - \cot\theta_2 = \frac{CD}{AB} = \frac{x}{h}$$

$$\boxed{h = \frac{x}{\cot\theta_1 - \cot\theta_2}}$$

⇒ Size of an ASTRONOMICAL OBJECT.



Let us consider the astronomical object like moon can be measured using an astronomical telescope. Here O is the observation point on earth.

Here

$$\angle BOA = \theta$$

Let (S) be the average distance of moon from the earth.

Now

Here AB = length of circular arc of Radius (S)

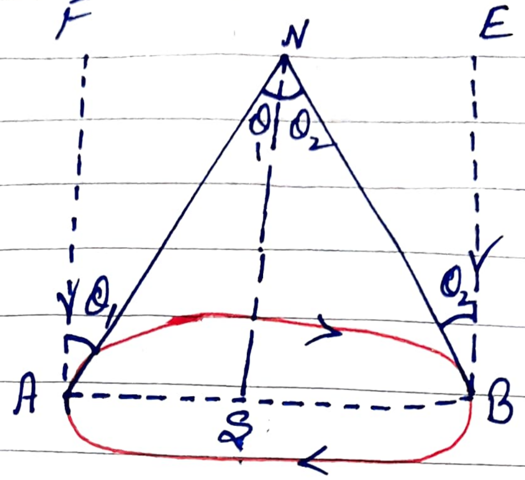
Here angle $= \dfrac{\text{arc}}{\text{Radius}}$

$$\theta = \frac{AB}{S}$$

$$\boxed{AB = \theta S}$$

⇒ **PARALLEX Method** :- Parallex is the name given to change in the position of a object with Respect to the background, when object is seen from two different positions. The distance between the two positions is called the basis.

The parallex method has been used for measuring distances of stars which are less than 100 light years away.



Here AB = diameter of earth orbit.

~~N is the~~

S is the position of ~~star~~ sun.

Here $\angle FAN = \theta_1$ and $\angle EBN = \theta_2$

$\angle ANS = \theta_1$ and ~~$\angle ABS$~~ $\angle BNS = \theta_2$

Cleary $\angle ANB = \angle ANS + \angle BNS$

$= \theta_1 + \theta_2$

This is the angle which the nearby star (N) formed on the orbital diameter of earth

$$\text{angle} = \frac{\text{arc}}{\text{Radius}}$$

$$\theta_1 + \theta_2 = \frac{AB}{AN}$$

$$AN = \frac{AB}{\theta_1 + \theta_2}$$

__/__/__

Here $AB = 2AU$

$= 2 \times 1.5 \times 10^{11} m = 3 \times 10^{11} m$

ie $\boxed{AN = \frac{3 \times 10^{11} m}{\theta_1 + \theta_2}}$

Problem:-

The Paraller of a heavenly body measured from two points diametrically opposite on equator of earth is 2.0 minute. If Radius of earth is 6400 km, calculate distance of heavenly body.

Sol: Here $\theta = 2$ min $\Rightarrow \theta = \frac{2''}{60} = \frac{2}{60} \times \frac{\pi}{180}$ Radian

$l =$ diameter of earth $= 2 \times 6400$ km

$= 1.28 \times 10^4$ km

$= 1.28 \times 10^4 \times 1000$ m

$= 1.28 \times 10^7$ m

$r = ?$

$\therefore l = r\theta$

$r = l/\theta = \frac{1.28 \times 10^7}{\frac{2}{60} \times \frac{\pi}{180}} = \frac{1.28 \times 10^7 \times 180 \times 30}{\pi}$

$= 2{,}201.27 \times 10^7$ m

$= 2.201 \times 10^{10}$ m

Q difference between Mass and weight ?

Ans:

	Mass	weight
1)	Mass is an essential property of material body.	weight is not an essential property of material body.
2)	Mass of a body is the quantity of matter in the body.	weight of a body is the force with which it is attracted toward the Centre of earth.
3)	Mass of the body is Constant	weight of a body varies from place to place.
4)	Mass of material body is never zero	Weight of a body is zero at the Centre of earth.
5	It is a scalar quantity	It is a vector quantity.
6)	S.I. Unit kg	S.I. Unit is Newton.

Chapter: 1(d)

Dimensional analysis

⇒ **Dimensions of a Physical Quantity :-** It is define the dimensions of a physical quantity as the powers to which the fundamental units of mass, length and time have have to be raised to represent a derived unit of the quantity.

eg :- Volume = length x Breadth x height

$$= L \times B \times h$$
$$= [L] \times [L] \times [L]$$
$$= [L^3]$$
$$= [M^0 L^3 T^0]$$

⇒ **Diminsional Equation and Diminsional formula :-**

$[M^0 L^1 T^{-1}]$ is the diminsional formula of velocity.

$[V] = [M^0 L^1 T^{-1}]$ is the dimensional equation of velocity.

The diminsional formula of a physical quantity is an expression which tells us :

(1) the fundamental units on which the quantity depends

(ii) the nature of the dependence.

$\Rightarrow [M^0 L T^{-1}]$

When a physical quantity is equated to its dimensional formula, what we obtain is the dimensional equation of the physical quantity.

i.e $[F] = [MLT^{-2}]$

⇒ Uses of Dimensional Equations :-

following are the three uses of dimensional equations :-

1) Conversion of one system of units into another.
ii) Checking the accuracy of various formulae.
iii) Derivation of formulae.

⇒ Conversion of one system of units into Another:-

This is based on the fact that magnitude of a physical quantity remains the same, whatever be the system of its measurement i.e

$$Q = n_1 u_1 = n_2 u_2 \quad \text{——} \quad (1)$$

Here U_1 and U_2 are two unit of measurement of the quantity Q. n_1 and n_2 are their respective numerical values.

let M_1, L_1, T_1 be the fundamental units of one system and M_2, L_2 and T_2 be the fundamental units of Mass, length and time in the other system of unit.

Here $U_1 = [M_1^a L_1^b T_1^c]$

and $U_2 = [M_2^a L_2^b T_2^c]$

Now using equation ①

$$n_2 = \frac{n_1 U_1}{U_2}$$

$$= \frac{n_1 [M_1^a L_1^b T_1^c]}{[M_2^a L_2^b T_2^c]}$$

$$\boxed{n_2 = n_1 \left(\frac{M_1}{M_2}\right)^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c} \quad \text{②}$$

(*i*) acceleration $= \dfrac{\text{change in velocity}}{\text{time taken}}$

$= \dfrac{L/T}{T} = LT^{-2} = [M^0\ L^1\ T^{-2}]$

(*ii*) As force = mass × acceleration

$\therefore \quad F = [M] \times [L^1 T^{-2}]$

$= [M^1\ L^1\ T^{-2}]$, and so on

The dimensional formulae of some of the important mechanical quantities are derived below and are listed in Table 1(d).1. The SI units of all these quantities are also given in the table.

TABLE 1(d).1. Dimensional formulae and SI units of Physical Quantities

S.No.	Physical quantity	Relation with other quantities	Dimensional formula	SI unit
1.	*Area*	length × breadth	$L \times L = L^2 = [M^0 L^2 T^0]$	m^2
2.	*Volume*	length × breadth × height	$L \times L \times L = L^3 = [M^0\ L^3\ T^0]$	m^3
3.	*Density*	$\dfrac{\text{mass}}{\text{volume}}$	$\dfrac{M}{L^3} = [M^1 L^{-3} T^0]$,	kg m^{-3}
4.	*Specific gravity*	$\dfrac{\text{density of body}}{\text{density of water at 4°C}}$	$\dfrac{M/L^3}{M/L^3} = 1 = [M^0\ L^0 T^0]$ →no dimensions	No units
5.	*Speed or velocity*	$\dfrac{\text{displacement}}{\text{time}}$	$\dfrac{L}{T} = LT^{-1} = [M^0 L^1 T^{-1}]$	ms^{-1}
6.	*Linear momentum*	mass × velocity	$M[LT^{-1}] = [M^1 L^1 T^{-1}]$	kg ms^{-1}
7.	*Acceleration*	$\dfrac{\text{change in velocity}}{\text{time taken}}$	$\dfrac{L/T}{T} = LT^{-2} = [M^0\ L^1 T^{-2}]$	ms^{-2}
8.	*Acceleration due to gravity (g)*	$\dfrac{\text{change in velocity}}{\text{time taken}}$	$\dfrac{L/T}{T} = LT^{-2} = [M^0 L^1 T^{-2}]$	ms^{-2}
9.	*Force*	mass × acc.	$M[LT^{-2}] = [M^1 L^1 T^{-2}]$	N (newton)
10.	*Impulse*	force × time	$[MLT^{-2}] \times T = [M^1 L^1 T^{-1}]$	N*s*
11.	*Pressure*	force/area	$\dfrac{MLT^{-2}}{L^2} = [M^1 L^{-1} T^{-2}]$	Nm^{-2}
12.	*Universal constant of gravitation* (G)	From Newton's law of gravitation. $F = \dfrac{Gm_1 m_2}{r^2}$ or $G = \dfrac{Fr^2}{m_1 m_2}$, where F is force between masses m_1, m_2 at a distance r	$G = \dfrac{(MLT^{-2})\ L^2}{MM} = [M^{-1}\ L^3\ T^{-2}]$	$Nm^2\ kg^{-2}$
13.	*Work*	force × distance	$MLT^{-2} \times L = [M^1 L^2 T^{-2}]$	J (joule)
14.	*Energy* (including Potential energy, Kinetic energy, heat energy, light energy etc.)	work	$[M^1\ L^2 T^{-2}]$	J (joule)
15.	*Moment of force*	force × distance	$MLT^{-2} \times L = [M^1 L^2 T^{-2}]$	N-*m*
16.	*Power*	$\dfrac{\text{work}}{\text{time}}$	$\dfrac{ML^2\ T^{-2}}{T} = [M^1 L^2 T^{-3}]$	W (watt)

S.No	Physical quantity	Relation with other quantities	Dimensional formula	SI unit
17.	*Surface tension*	$\frac{\text{force}}{\text{length}}$	$\frac{MLT^{-2}}{L} = [M^1 L^0 T^{-2}]$	Nm^{-1}
18.	*Surface energy*	Pot. energy/area	$[M^1 L^0 T^{-2}]$	Jm^{-2}
19.	*Force constant*	$\frac{\text{force}}{\text{displacement}}$	$\frac{MLT^{-2}}{L} = [M^1 L^0 T^{-2}]$	Nm^{-1}
20.	*Thrust*	force	$[M^1 L^1 T^{-2}]$	N (newton)
21.	*Tension*	force	$[M^1 L^1 T^{-2}]$	N (newton)
22.	*Stress*	$\frac{\text{force}}{\text{area}}$	$\frac{MLT^{-2}}{L^2} = [M^1 L^{-1} T^{-2}]$	$N m^{-2}$
23.	*Strain*	$\frac{\text{change in configuration}}{\text{original configuration}}$	$\frac{L}{L} = 1 = [M^0 L^0 T^0]$	No units
24.	*Coefficient of elasticity*	$\frac{\text{stress}}{\text{strain}}$	$\frac{M^1 L^{-1} T^{-2}}{1} = [M^1 L^{-1} T^{-2}]$	Nm^{-2}
25.	*Radius of gyration (K)*	distance	$L = [M^0 L^1 T^0]$	m
26.	*Moment of inertia (I)*	mass (distance)^2	$ML^2 = [M^1 L^2 T^0]$	kg m^2
27.	*Angle (θ)*	length (l)/radius (r)	$\frac{L}{L} = 1 = [M^0 L^0 T^0]$	radian
28.	*Angular velocity (ω)*	$\frac{\text{angle } (\theta)}{\text{time } (t)}$	$\frac{1}{T} = T^{-1} = [M^0 L^0 T^{-1}]$	rad s^{-1}
29.	*Angular acc. (α)*	$\frac{\text{change in angular vel.}}{\text{time taken}}$	$\frac{1/T}{T} = T^{-2} = [M^0 L^0 T^{-2}]$	rad s^{-2}
30.	*Angular momentum*	$I\omega$	$(ML^2)(T^{-1}) = [M^1 L^2 T^{-1}]$	kg m^2 s^{-1}
31.	*Torque*	$I\alpha$	$(ML^2)(T^{-2}) = [M^1 L^2 T^{-2}]$	N-m
32.	*Wavelength (λ)*	length of one wave i.e. distance	$L = [M^0 L^1 T^0]$	m
33.	*Frequency (ν)*	number of vibrations/sec	$1/T = T^{-1} = [M^0 L^0 T^{-1}]$	s^{-1} or Hz (hertz)
34.	*Velocity of light in vacuum (c)*	$\frac{\text{distance travelled}}{\text{time taken}}$	$\frac{L}{T} = [M^0 L^1 T^{-1}]$	ms^{-1}
35.	*Velocity gradient*	$\frac{\text{velocity}}{\text{distance}}$	$\frac{LT^{-1}}{L} = T^{-1} = [M^0 L^0 T^{-1}]$	s^{-1}
36.	*Rate of flow*	$\frac{\text{volume}}{\text{time}}$	$\frac{L^3}{T} = L^3 T^{-1} = [M^0 L^3 T^{-1}]$	$m^3 s^{-1}$
37.	*Planck's constant (h)*	$\frac{\text{energy (E)}}{\text{frequency } (\nu)}$	$\frac{ML^2 T^{-2}}{T^{-1}} = [M^1 L^2 T^{-1}]$	J-s
38.	*Mass of unit length (m)*	$\frac{\text{mass}}{\text{length}}$	$\frac{M}{L} = [M^1 L^{-1} T^0]$	kg m^{-1}
39.	*Distance travelled in nth second*	$\frac{\text{distance}}{\text{time}}$	$\frac{L}{T} = [M^0 L^1 T^{-1}]$	ms^{-1}
40.	*Avogadro's number (N)*	Number of atoms/molecules in one gram atom/mole	$[M^0 L^0 T^0]$	mole^{-1}

Numericals :-

⇒ Convert 1 Newton into dyne.

Sol: Newton is the unit of force

dimension formula of force is $[MLT^{-2}]$

Here $a = 1,\ b = 1,\ c = -2$

Newton	dyne
SI	C.g.s
$M_1 = 1\,kg$	$M_2 = 1\,g$
$L_1 = 1\,m$	$L_2 = 1\,cm$
$T_1 = 1\,s$	$T_2 = 1\,s$

Now using formula

$$n_1 U_1 = n_2 U_2$$

$$n_2 = \frac{n_1 U_1}{U_2}$$

$$n_2 = n_1 \left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c$$

$$= 1 \left[\frac{1\,kg}{1\,g}\right]^1 \left[\frac{1\,m}{1\,cm}\right]^1 \left[\frac{1\,\cancel{s}}{1\,\cancel{s}}\right]^{-2}$$

$$n_2 = \left[\frac{1000\,\cancel{g}}{1\,\cancel{g}}\right]\left[\frac{100\,\cancel{cm}}{1\,\cancel{cm}}\right]$$

$$n_2 = 100000$$

$$n_1 = 10^5$$

Here 1 Newton = 10^5 dyne.

2) Convert an energy of one joule into ergs.

Sol: Joule is unit of energy

ie dimension formula of energy $[ML^2T^{-2}]$

Here $a = 1$, $b = 2$, $c = -2$

Now using formula

SI	C.g.s
$M_1 = 1kg$	$M_2 = 1g$
$L_1 = 1m$	$L_2 = 1cm$
$T_1 = 1s$	$T = 1s$
$n_1 = 1$	$n_2 = ?$

Now

$$n_1 U_1 = n_2 U_2$$

$$n_2 = n_1 \frac{U_1}{U_2}$$

$$= n_1 \left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c$$

$$= n_1 \left[\frac{1kg}{1g}\right]^1 \left[\frac{1m}{1cm}\right]^2 \left[\frac{1s}{1s}\right]^{-2}$$

$$n_2 = 1\left[\frac{1000\cancel{g}}{1\cancel{g}}\right]\left[\frac{100\cancel{cm}}{\cancel{1cm}}\right]^2$$

$n_2 = 10^7$
1 Joule = 10^7 earg.

⇒ Convert 100 dyne to Newton.

The dyne is cgs and Newton is S.I. Unit and the unit of force.

Here Force = $[MLT^{-2}]$
Here $a=1$, $b=1$, $c=-2$

C.g.s	S.I.
$M_1 = 1g$	$M_2 = 1kg$
$L_1 = 1cm$	$L_2 = 1m$
$T_1 = 1s$	$T_2 = 1s$
$n_1 = 100$	$n_2 = ?$

Now using formula

$$n_1 U_1 = n_2 U_2$$

$$n_2 = n_1 \frac{U_1}{U_2}$$

$$n_2 = n_1 \left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c$$

$$= n_1 \left[\frac{1g}{1kg}\right]^a \left[\frac{1cm}{1m}\right]^b \left[\frac{1s}{1s}\right]^c$$

$$= 100 \left[\frac{1g}{1000g}\right]^1 \left[\frac{1cm}{100cm}\right]^1 \left[\frac{1s}{1s}\right]^{-2}$$

$$n_2 = 100 \times \frac{1}{1000} \times \frac{1}{100}$$

$n_2 = 10^{-3}$

Here 100 dyne = 10^{-3} Newton.

⇒ Convert 10 erg to Joule.

erg and Joule is the unit of energy.

dimensional formula of energy = $[ML^2T^{-2}]$

Here $a = 1$, $b = 2$, $c = -2$

C.g.s	S.I
$M_1 = 1g$	$M_2 = 1Kg$
$T_1 = 1s$	$T_2 = 1s$
$L_1 = 1cm$	$L_2 = 1m$
$n_1 = 10$	$n_2 = ?$

using formula

$$n_1 u_1 = n_2 u_2$$

$$n_2 = n_1 \left[\frac{u_1}{u_2}\right]$$

$$= n_1 \left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c$$

$$= 10 \left[\frac{1g}{1kg}\right]^1 \left[\frac{1cm}{1m}\right]^2 \left[\frac{1\cancel{s}}{1\cancel{s}}\right]^{-2}$$

$$n_2 = 10 \left[\frac{1\cancel{g}}{1000\cancel{g}}\right] \left[\frac{1\cancel{cm}}{100\cancel{cm}}\right]^2$$

$$n_2 = 10\left[\frac{1}{1000}\right]\left[\frac{1}{10000}\right]$$

$$n_2 = 10 \times 10^{-7}$$

$$n_2 = 10^{-6}$$

Here $\underline{10 \text{ erg}} = \underline{10^{-6} \text{ Joule}}$

⇒ Find the value of a force of 100 N on a system based upon the metre, the kilogram and the minute as the fundamental units.

Sol: The Newton is the unit of force

Force (dimensional formula) = $[MLT^{-2}]$

Here $a = 1$, $b = 1$, $c = -2$

SI to New system

$M_1 = 1\text{kg}$ $\quad\quad$ $M_2 = 1\text{kg}$

$L_1 = 1\text{m}$ $\quad\quad$ $L_2 = 1\text{m}$

$T_1 = 1\text{s}$ $\quad\quad$ $T_2 = 1\text{min}$

$n_1 = 100$ $\quad\quad$ $n_2 = ?$

$$n_2 = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^{-2}$$

$$= 100\left[\frac{1\text{Kg}}{1\text{ kg}}\right]^1\left[\frac{1\text{m}}{1\text{m}}\right]^1\left[\frac{1\text{s}}{1\text{min}}\right]^{-2}$$

$$n_2 = 100\left[1\right]\left[1\right]\left[\frac{1s}{60}\right]^{-2}$$

$$n_2 = 100 \times (60)^2$$

$$= 3600 \times 100$$

$n_2 = 36 \times 10^4$ new units of force.

Checking the accuracy of formulae:-

⇒ Checking the correctness of the relation $v^2 - u^2 = 2as$, where the symbols have their usual meaning.

Sol: The given equation.

$$v^2 - u^2 = 2as \quad \text{— ①}$$

Here $v = [M^0 L T^{-1}]$

$u = [M^0 L T^{-1}]$

$a = [M^0 L T^{-2}]$

$s = [M^0 L T^0]$

Using in equation ①

$$[M^0 L T^{-1}]^2 - [M^0 L T^{-1}]^2 = [M^0 L T^{-2}][M^0 L T^0]$$

$$[M^0 L^2 T^{-2}] - [M^0 L^2 T^{-2}] = [M^0 L^2 T^{-2}]$$

L.H.S = R.H.S

formula is correct.

⇒ Check the accuracy of the Relation $\nu = \frac{1}{2l}\sqrt{\frac{T}{m}}$ where (ν) is the frequency, l is length, T is tension and (m) is mass of unit length of the string.

Sol

The given Relation is

$$\nu = \frac{1}{2l}\sqrt{\frac{T}{m}} \quad \text{—} \; ①$$

Here $\nu = [M^0 L^0 T^{-1}]$

$l = [M^0 L T^0]$

$T = [M^1 L^1 T^{-2}]$

$m = [M L^{-1} T^0]$

Using in equation ①

$$[M^0 L^0 T^{-1}] = \frac{1}{2[M^0 L T^0]}\sqrt{\frac{[M^1 L^1 T^{-2}]}{[M L^{-1} T^0]}}$$

$$[M^0 L^0 T^{-1}] = \frac{1}{2}\sqrt{\frac{[M^0 L^{-2} T^0][M^1 L^1 T^{-2}]}{[M L^{-1} T^0]}}$$

$$[M^0 L^0 T^{-1}] = \frac{1}{2}\left[M^{1-1} L^{-2+1+1} T^{-2}\right]^{1/2}$$

$$[M^0 L^0 T^{-1}] = \frac{1}{2}[M^0 L^0 T^{-2}]^{1/2}$$

$$[M^0 L^0 T^{-1}] = \frac{1}{2}[M^0 L^0 T^{-1}]$$

Here L.H.S = R.H.S

∴ formula is Correct.

Derivation of formulae :

Q Derive an expression for time period (t) of a Simple pendulum, which may depend upon mass of bob (m), length of pendulum (l) and acceleration due to gravity (g).

Sol:- Let $t \propto m^a l^b g^c$

Where a, b, c are the dimensions.

$$t = k\, m^a l^b g^c \quad \text{—} (1)$$

Where k = Constant

m = mass = $[M]$

l = length = $[L]$

g = acc. due to gravity = $[LT^{-2}]$

t = time = $[T]$

using in equation (1)

$$[M^0 L^0 T] = k\,[M]^a\,[L]^b\,[LT^{-2}]^c$$

$$[M^0 L^0 T] = k\,[M^a L^{b+c} T^{-2c}]$$

Comparing powers (using principle of homogeneity)

$$a = 0$$

$$b + c = 0$$

$$-2c = 1$$

$$c = -\frac{1}{2}$$

Here $-\frac{1}{2} + b = 0 \Rightarrow \boxed{b = \frac{1}{2}}$

$$t = K\, m^0\, l^{1/2}\, g^{-1/2}$$

$$\boxed{t = K\sqrt{\frac{l}{g}}}$$

⇒ The frequency of vibration (ν) of a string may depend upon length (l) of the string, tension (T) in the string and mass per unit length (m) of the string. Using the method of dimensions, derive the formula for ν.

Sol Let $\nu \propto l^a T^b m^c$

$$\nu = K\, l^a T^b m^c \quad \text{——} \ (1)$$

Here l = length of the string
T = tension in the string
m = mass per unit length of the string.
ν = frequency

Here

$\nu = [T^{-1}]$, $T = [MLT^{-2}]$
$l = [L]$, $m = [ML^{-1}]$

using in equation (1)

$$[T^{-1}] = k\,[L]^a [MLT^{-2}]^b [ML^{-1}]^c$$

$$[T^{-1}] = k\,[M^{b+c}\, L^{a+b-c}\, T^{-2b}]$$

Applying the principle of homogenity of dimensions.

$b + c = 0$ $\Rightarrow \frac{1}{2} + c = 0$

$a + b - c = 0$ $c = -\frac{1}{2}$

$-2b = -1$

$\boxed{b = \frac{1}{2}}$ $\Rightarrow a + \frac{1}{2} + \frac{1}{2} = 0$

$\Rightarrow \boxed{a = -1}$

using the value of a, b and c in (1)

$$\nu = k\, l^{-1}\, T^{\frac{1}{2}}\, m^{-\frac{1}{2}}$$

$$\boxed{\nu = \frac{k}{l}\sqrt{\frac{T}{m}}}$$

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Chapter - 1(e)

Errors of Measurement :-

⇒ ~~Error of~~

⇒ **Significant figures :-** Significant figures in the measured value of a physical quantity tell the number of digits in which we have confidence. Larger the number of significant figures obtained in the measurement, greater is the accuracy of the measurement and reverse is also true.

⇒ **Common Rules for counting significant figures :-**

Rule 1 :- All the non-zero digits are significant.
eg :- $x = 13897$.

Rule 2 : All the zeros occurring between two non-zero digits are significant.
eg :- 1007 , 1000908.

Rule 3 :- In a number less than one, all zeros to the right of decimal point and to the left of a non-zero digit are not significant.

eg :- $x = 0.00094$,
Significant figure :- 2.

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Rule 4:- All zeros on the right of the last non-zero digit in the decimal part are significant.

eg:- $x = 0.89000$, $x = 0.0092000$
significant :- five and five

Rule 5:- All zero on the Right of non-zero digit are NOT significant.

eg:- $x = 1000$, 378000.
significand figure = one and three.

Rule 6:- All zeros on the right of the last non zero digits become significant, when they come from a measurement.

Rounding off:- while Rounding off measurements, we use the following rules

Rule 1:- If the digit to be dropped is less than 5, then the preceding digit left unchanged.

eg:- $x = 7.82$ round off 7.8
$x = 3.94$ round off 3.9

Rule 2:- If the digit to be dropped is more than 5, then the preceding digit is ~~left unchanged~~ raised by one
eg:- $x = 7.87$ is round of to 7.9

Rule 3:- If the digit to be dropped is 5 followed by digits other than zero then the preceding digit is raised by one.

eg:- $x = 16.351$ is round off to 16.4

Rule 4:- If the digit to be dropped is 5 or followed by zero, then the preceding digit is left unchanged, if it is even.

eg:- $x = 3.250$ become 3.2.

Error of Measurement :-

The difference in the true value and the measured value of a quantity is called error of measurement.

Type of Errors :-

The errors of measurement can be divided into the following three types.

(I) Systematic Errors
(II) Random Errors
(III) Gross Errors.

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(a) Systematic Errors:- These errors are whose Causes are known and there errors Can be minimised.
eg:-

(1) Instrumental error may be due imperfection of design and erroneous manufacture of the instruments.

2) Personal error may be due to inexperience of the observer
eg:- lack of proper Setting of the apparatus.

3) Error due to imperfection arise on account of ignoring certain facts.
eg:- error in weighing arising out of buoyancy is usually ignored.

4) Error due to external causes arise due to Changes in temperature, pressure, humidty etc.

(b) Random Errors:- These errors may arise due to a large Variety of factors. The Causes of such errors are, therefore not known precisely.

eg:- when same person perform same experiment get different observate

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Gross Errors :- These error arise on account of shear carelessness of the observer.

eg :- (i) Reading an instrument without setting it properly.

(ii) Recording the observation wrongly.

⇒ **Absolute Error, Relative Error and Percentage Error :-**

a) **Absolute Error :-** Absolute error in the measurement of a physical quantity is the magnitude of the difference between the true value and the measured value of the quantity.

Let us consider physical quantity be measured n times.
Let the measured values be $a_1, a_2, ---, a_n$
The arithmetic mean of these values

$$a_m = \frac{a_1 + a_2 + --- + a_n}{n} \quad \text{——— (1)}$$

or $$a_n = \frac{1}{n}\sum_{i=1}^{n} a_i \quad \text{——— (2)}$$

usually, a_m is taken as the true value of the quantity

By defination,

$$\Delta a_1 = a_m - a_1$$
$$\Delta a_2 = a_m - a_2$$
$$------$$
$$\Delta a_n = a_m - a_n$$

absolute errors may be (+ve) in certain cases and negative in certain other cases

(b) Mean absolute error :- It is the arithmetic mean of the magnitudes of absolute errors in all the measurements of the quantity. It is Reperesented by $\overline{\Delta a}$.

$$\overline{\Delta a} = \frac{|\Delta a_1| + |\Delta a_2| + \dots + |\Delta a_n|}{n}$$

$$\overline{\Delta a} = 1/n \times \sum_{i=1}^{i=n} |\Delta a_i| \quad \text{—— } 3$$

Hence the final result of measurement may be

$$\boxed{a = a_m \pm \overline{\Delta a}}$$

=> The value lies between $(a_m + \overline{\Delta a})$ and $(a_m - \overline{\Delta a})$

Relative error or fractional error :-

The Relative is defined as the Ratio of mean absolute error to the mean value of the quantity measured.

Relative error or fractional error

$$= \frac{\text{mean absolute error}}{\text{mean value}} = \frac{\overline{\Delta a}}{a_m}$$

when the Relative error is expressed in percentage

$$\boxed{\text{Percentage error} = \frac{\overline{\Delta a}}{a_m} \times 100\%}$$

⇒ The refractive Index of water is found to have the value 1.29, 1.33, 1.34, 1.35, 1.32, 1.36, 1.30 and 1.33. Calculate the mean value, absolute error, the relative error and the percentage error.

Sol:- given value are (mean of given values)

$$a_m = \frac{1.29 + 1.33 + 1.34 + 1.35 + 1.32 + 1.36 + 1.30 + 1.33}{8}$$

$a_m = 1.327 = 1.33$

absolute errors in measurement are:-

$$\Delta a_1 = 1.33 - 1.29 = 0.04$$
$$\Delta a_2 = 1.33 - 1.33 = 0.00$$
$$\Delta a_3 = 1.33 - 1.34 = -0.01$$
$$\Delta a_4 = 1.33 - 1.24 = -0.02$$
$$\Delta a_5 = 1.33 - 1.32 = +0.01$$
$$\Delta a_6 = 1.33 - 1.36 = -0.03$$
$$\Delta a_7 = 1.33 - 1.30 = +0.03$$
$$\Delta a_8 = 1.33 - 1.33 = 0.00$$

Mean absolute error, $\overline{\Delta a} = \dfrac{\sum_{i=1}^{i=n} |(\Delta a)_i|}{n}$

$$= \frac{0.04 + 0.00 + 0.01 + 0.02 + 0.01 + 0.03 + 0.03 + 0.00}{8}$$

$$\overline{\Delta a} = \frac{0.14}{8} = 0.0175 = 0.02$$

Relative error $= \pm \dfrac{\overline{\Delta a}}{a} = \dfrac{\pm 0.02}{1.33}$

$$= \pm 0.015$$
$$= \pm 0.02$$

% age error $= \pm 0.015 \times 100 = \pm 1.5\%$

_/_/_

Propagation or Combination of Errors :-

(a) Error in Summation :-

Suppose $x = a+b$ —— ①

let Δa = absolute error in measurement of a

Δb = absolute error in measurement of b

Δx = absolute error in measurement of x

∴ from ①

$$(x \pm \Delta x) = (a \pm \Delta a) + (b \pm \Delta b)$$

$$x \pm \Delta x = (a+b) \pm (\Delta a + \Delta b)$$

$$\cancel{x} \pm \Delta x = \cancel{x} \pm (\Delta a + \Delta b)$$

$$\Delta x = \pm \Delta a \pm \Delta b$$

Here The four possible value $(\Delta a + \Delta b)$ $(\Delta a - \Delta b)$, $(-\Delta a + \Delta b)$, $(-\Delta a - \Delta b)$.

$$\Delta x = \pm (\Delta a + \Delta b).$$

Hence maximum absolute error in sum of the two quantities is equal to sum of the absolute errors in the individual quantities.

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(b) Error in difference :-

let $x = a - b$ —①

let Δa, Δb, and Δx absolute error in the measurement of a, b and x.

from ①

$$(x \pm \Delta x) = (a \pm \Delta a) - (b \pm \Delta b)$$

$$(x \pm \Delta x) = (a - b) \pm (\Delta a - \Delta b)$$

$$(\cancel{x} \pm \Delta x) = \cancel{x} \pm (\Delta a - \Delta b)$$

$$\pm \Delta x = \pm \Delta a \mp \Delta b$$

possible four value are

$$(\Delta a \mp \Delta b), (-\Delta a - \Delta b), (-\Delta a + \Delta b), (\Delta a - \Delta b)$$

$$\Delta x = \pm (\Delta a + \Delta b).$$

maximum absolute error in difference of two quantities is equal to sum of the absolute errors in the individual quantities.

Numerical: In an experiment, two capacities measured are $(1.3 \pm 0.1)\ \mu F$ and $(2.4 \pm 0.2)\ \mu F$. Calculate the total capacity in ||rl ~~plate~~ with %age error.

Sol Here $C_1 = (1.3 \pm 0.1)\ \mu F$

and $C_2 = (2.4 \pm 0.2)\ \mu F$

||rl plates $C_p = C_1 + C_2$ (In ||rl Combination)

$$C_p = (1.3 + 2.4)$$

$$= 3.7\ \mu F$$

$$\Delta C_p = \pm (\Delta C_1 + \Delta C_2)$$

$$= \pm (0.1 + 0.2) = \pm 0.3$$

_/_/_

% age error $= \pm \frac{0.3}{3.7} \times 100$

$= \pm 0.081 \times 100$

$= \pm 8.8\%$

Hence $C_p = (3.7 \pm 0.3)\,\mu F$

$= 3.7\,\mu F \pm 8.1\%$

Numerical:- The length of two cylinders are measured to be $l_1 = (5.62 \pm 0.01)$ cm and $l_2 = (4.34 \pm 0.02)$ cm. Calculate difference in lengths with error limits.

Sol:- Here $l_1 = (5.62 \pm 0.01)$ cm
and $l_2 = (4.34 \pm 0.02)$ cm

$l' = l_1 - l_2 = 5.62 - 4.34 = 1.28$ cm

$\Delta l' = \pm(\Delta l_1 + \Delta l_2)$

$= \pm(0.01 + 0.02) = \pm 0.03$

% age error $= \frac{\pm 0.03}{1.28} \times 100$

$= \pm 2.34\%$

Hence, difference in length:

$= (1.28 \pm 0.03)$ cm

$= 1.28$ cm $\pm 2.34\%$

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(c) Error in product:

let $x = a \times b$ —— ①

let Δa, Δb and Δx absolute error in the measurement of a, b and x.

from ①

$$(x \pm \Delta x) = (a \pm \Delta a) \times (b \pm \Delta b)$$

$$x\left(1 \pm \frac{\Delta x}{x}\right) = a\left(1 \pm \frac{\Delta a}{a}\right) b\left(1 \pm \frac{\Delta b}{b}\right)$$

$$x\left(1 \pm \frac{\Delta x}{x}\right) = a \times b \left(1 \pm \frac{\Delta a}{a}\right)\left(1 \pm \frac{\Delta b}{b}\right)$$

$$\cancel{x}\left(1 \pm \frac{\Delta x}{x}\right) = \cancel{x}\left(1 \pm \frac{\Delta a}{a}\right)\left(1 \pm \frac{\Delta b}{b}\right)$$

$$\left(1 \pm \frac{\Delta x}{x}\right) = \left(1 \pm \frac{\Delta b}{b} \pm \frac{\Delta a}{a} \pm \frac{\Delta a \Delta b}{ab}\right)$$

Here $\left(\frac{\Delta a}{a}\right)$ and $\left(\frac{\Delta b}{b}\right)$ both are small, their product is still smaller and can be neglected

$$\left(1 \pm \frac{\Delta x}{x}\right) = \cancel{1} \pm \frac{\Delta b}{b} \pm \frac{\Delta a}{a}$$

$$\pm \frac{\Delta x}{x} = \pm\left(\frac{\Delta b}{b} + \frac{\Delta a}{a}\right)$$

possible values $\left(\frac{\Delta b}{b} + \frac{\Delta a}{a}\right)$, $\left(\frac{\Delta b}{b} - \frac{\Delta a}{a}\right)$

$\left(-\frac{\Delta b}{b} + \frac{\Delta a}{a}\right)$, $\left(-\frac{\Delta b}{b} - \frac{\Delta a}{a}\right)$.

Hence maximum fractional error or Relative error in product of quantities is equal to sum of the ~~[illegible]~~ fracti-onal or Relative errors in the individual quantities.

d) Error in division:-

Let $x = a/b$ — ①

Suppose Δa, Δb and Δx absolute error in the measurement of a, b and x.
from ①

$$x \pm \Delta x = \frac{a \pm \Delta a}{b \pm \Delta b}$$

$$x\left(1 \pm \frac{\Delta x}{x}\right) = \frac{a}{b} \frac{\left(1 \pm \frac{\Delta a}{a}\right)}{\left(1 \pm \frac{\Delta b}{b}\right)}$$

$$\cancel{x}\left(1 \pm \frac{\Delta x}{x}\right) = \cancel{x} \frac{\left(1 \pm \frac{\Delta a}{a}\right)}{\left(1 \pm \frac{\Delta b}{b}\right)}$$

$$\therefore \left(1 \pm \frac{\Delta x}{x}\right) = \left(1 \pm \frac{\Delta a}{a}\right)\left(1 \pm \frac{\Delta b}{b}\right)^{-1}$$

Using binomial theorem.

$$\left(1\pm\frac{\Delta b}{b}\right)^{-1} = 1\mp\frac{\Delta b}{b}\pm\frac{\Delta b^2}{b^2}+ \ldots$$

neglecting square terms.

$$\left(1\pm\frac{\Delta x}{x}\right) = \left(1\pm\frac{\Delta a}{a}\right)\left(1\mp\frac{\Delta b}{b}\right)$$

$$\cancel{1}\pm\frac{\Delta x}{x} = \left(\cancel{1}\mp\frac{\Delta b}{b}\pm\frac{\Delta a}{a}\mp\left(\frac{\Delta a}{a}\right)\left(\frac{\Delta b}{b}\right)\right)$$

$$\frac{\Delta x}{x} = \mp\frac{\Delta b}{b}\pm\frac{\Delta a}{a}$$

neglecting $\left(\frac{\Delta a}{a}\right)\left(\frac{\Delta b}{b}\right)$ because they are very small value.

$$\frac{\Delta x}{x} = \mp\frac{\Delta b}{b}\pm\frac{\Delta a}{a}$$

four possible values.

$$\frac{\Delta x}{x}\ \left(\frac{\Delta a}{a}+\frac{\Delta b}{b}\right), \left(\frac{\Delta a}{a}-\frac{\Delta b}{b}\right)$$

$$\left(-\frac{\Delta a}{a}+\frac{\Delta b}{b}\right), \left(-\frac{\Delta a}{a}-\frac{\Delta b}{b}\right)$$

$$\frac{\Delta x}{x} = \pm\left(\frac{\Delta a}{a}+\frac{\Delta b}{b}\right)$$

Hence the maximum value of fractional or relative error in division of quantities is equal to sum of the fractional relative errors in the individual quantities.

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Numerical:- The length and breadth of a rectangular lamina are measured to be (2.3 ± 0.2) cm and (1.6 ± 0.1) cm. Calculate area of the lamina with error limits.

Sol:- Here $l = (2.3 \pm 0.2)$ cm

$b = (1.6 \pm 0.1)$ cm

$$A = l \times b = 2.3 \times 1.6 = 3.68 \text{ cm}^2$$

$$\frac{\Delta A}{A} = \pm\left(\frac{\Delta l}{l} + \frac{\Delta b}{b}\right)$$

$$= \pm\left(\frac{0.2}{2.3} + \frac{0.1}{1.6}\right)$$

$$= \pm\frac{0.55}{3.68}$$

$$\Delta A = \pm\frac{0.55}{3.68} \times A = \pm\frac{0.55}{\cancel{3.68}} \times \cancel{3.68}$$

$$\Delta A = \pm 0.55$$

$$A = (3.68 \pm 0.55) \text{ cm}^2.$$