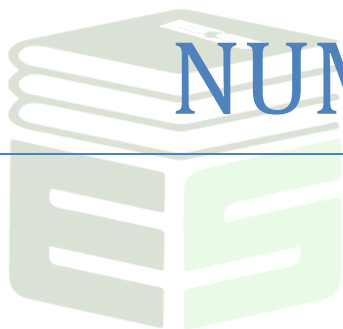




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NUMBER SYSTEMS

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Chapter – 1

A Source of Your Way to Success



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$$(i) \sqrt{ab} = \sqrt{a}\sqrt{b}$$

$$(ii) \sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

$$(iii) (\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b}) = a - b$$

$$(iv) (a + \sqrt{b})(a - \sqrt{b}) = a^2 - b$$

$$(v) (\sqrt{a} + \sqrt{b})^2 = a + 2\sqrt{ab} + b$$

- To rationalise the denominator of $\frac{1}{\sqrt{a} + b}$, we multiply this

by $\frac{\sqrt{a} - b}{\sqrt{a} - b}$, where a and b are integers and $\sqrt{a} + b \neq 0$.

- Let $a > 0$ be a real number and p and q be rational numbers. Then

$$(a) a^p \cdot a^q = a^{p+q}$$

$$(b) (a^p)^q = a^{pq}$$

$$(c) \frac{a^p}{a^q} = a^{p-q}$$

$$(d) a^p b^p = (ab)^p$$

EXERCISE 1.1

1. Is zero a rational number? Can you write it in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$?

Sol. Yes, zero is a rational number.

Zero can be written in any of the following forms :

$$\frac{0}{1}, \frac{0}{-1}, \frac{0}{2}, \frac{0}{-2} \text{ and so on.}$$

Thus, 0 can be written as $\frac{p}{q}$, where $p = 0$ and q is any non-zero integer.

Hence, 0 is a rational number.

2. Find six rational numbers between 3 and 4.

Sol. We know that between two rational numbers x and y , such that $x < y$, there is a rational number $\frac{x+y}{2}$. That is, $x < \frac{x+y}{2} < y$

A rational number between 3 and 4 is $\frac{1}{2}(3+4)$ i.e., $\frac{7}{2}$

$$\therefore 3 < \frac{7}{2} < 4$$

Now, a rational number between 3 and $\frac{7}{2}$ is

$$\frac{1}{2}\left(3 + \frac{7}{2}\right) = \frac{1}{2} \times \frac{6+7}{2} = \frac{13}{4}$$

A rational number between $\frac{7}{2}$ and 4 is

$$\frac{1}{2}\left(\frac{7}{2} + 4\right) = \frac{1}{2} \times \frac{7+8}{2} = \frac{15}{4}$$

$$\therefore 3 < \frac{13}{4} < \frac{7}{2} < \frac{15}{4} < 4$$

Further, a rational number between 3 and $\frac{13}{4}$ is

$$\frac{1}{2}\left(3 + \frac{13}{4}\right) = \frac{1}{2} \times \frac{12+13}{4} = \frac{25}{8}$$

A rational number between $\frac{15}{4}$ and 4 is

$$\frac{1}{2} \left(\frac{15}{4} + 4 \right) = \frac{1}{2} \times \frac{15 + 16}{4} = \frac{31}{8}$$

A rational number between $\frac{31}{8}$ and 4 is

$$\frac{1}{2} \left(\frac{31}{8} + 4 \right) = \frac{1}{2} \times \frac{31 + 32}{8} = \frac{63}{16}$$

$$\therefore 3 < \frac{25}{8} < \frac{13}{4} < \frac{7}{2} < \frac{15}{4} < \frac{31}{8} < \frac{63}{16} < 4$$

Hence, six rational numbers between 3 and 4 are :

$$\frac{25}{8}, \frac{13}{4}, \frac{7}{2}, \frac{15}{4}, \frac{31}{8} \text{ and } \frac{63}{16}$$

ALTERNATIVE METHOD

Since we want 6 rational numbers between 3 and 4, so

$$\text{we write } 3 = \frac{3 \times 7}{1 \times 7} = \frac{21}{7} \text{ and } 4 = \frac{4 \times 7}{1 \times 7} = \frac{28}{7}.$$

We know that $21 < 22 < 23 < 24 < 25 < 26 < 27 < 28$

$$\Rightarrow \frac{21}{7} < \frac{22}{7} < \frac{23}{7} < \frac{24}{7} < \frac{25}{7} < \frac{26}{7} < \frac{27}{7} < \frac{28}{7}.$$

Hence, six rational numbers between $3 = \frac{21}{7}$ and $4 = \frac{28}{7}$ are

$$\frac{22}{7}, \frac{23}{7}, \frac{24}{7}, \frac{25}{7}, \frac{26}{7} \text{ and } \frac{27}{7}.$$

3. Find five rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$.

Sol. Since we want 5 rational numbers between $\frac{3}{5}$ and

$\frac{4}{5}$, so we write

$$\frac{3}{5} = \frac{3 \times 6}{5 \times 6} = \frac{18}{30}$$

and $\frac{4}{5} = \frac{4 \times 6}{5 \times 6} = \frac{24}{30}$

We know that $18 < 19 < 20 < 21 < 22 < 23 < 24$

$$\Rightarrow \frac{18}{30} < \frac{19}{30} < \frac{20}{30} < \frac{21}{30} < \frac{22}{30} < \frac{23}{30} < \frac{24}{30}$$

Hence, 5 rational numbers between $\frac{3}{5} = \frac{18}{30}$ and $\frac{4}{5} = \frac{24}{30}$

are : $\frac{19}{30}, \frac{20}{30}, \frac{21}{30}, \frac{22}{30}, \frac{23}{30}$ and $\frac{24}{30}$.

4. State whether the following statements are true or false. Give reasons for your answers.

(i) Every natural number is a whole number.

(ii) Every integer is a whole number.

(iii) Every rational number is a whole number.

Sol. (i) True : Every natural number lies in the collection of whole numbers.

(ii) False : -3 is not a whole number.

(iii) False : $\frac{3}{5}$ is not a whole number.

EXERCISE 1.2

1. State whether the following statements are true or false. Justify your answers.

(i) Every irrational number is a real number.

(ii) Every point on the number line is of the form $\sqrt{m}$, where m is a natural number.

(iii) Every real number is an irrational number.

Sol. (i) True as a real number is either rational or irrational.

(ii) False as numbers of other types also lie on the number line.

(iii) False as rational numbers are also real numbers.

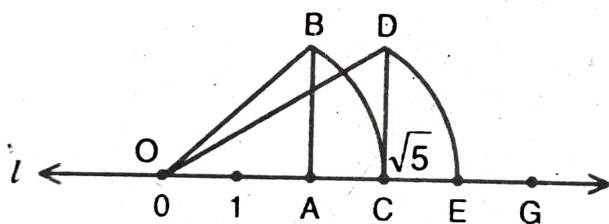
2. Are the square roots of all positive integers irrational? If not, give an example of the square root of a number that is a rational number.

Sol. No, as $\sqrt{4} = 2$ is a natural number.

3. Show how $\sqrt{5}$ can be represented on the number line.

Sol. We shall now show how to represent $\sqrt{5}$ on the number line.

We first represent $\sqrt{5}$ on the number line l . We construct a right-angled $\triangle OAB$, right-angled at A such that $OA = 2$ and $AB = 1$ unit (see figure).

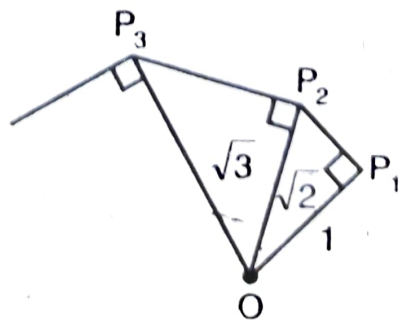


$$\text{Then, } OB = \sqrt{OA^2 + AB^2} = \sqrt{4 + 1} = \sqrt{5}$$

Now, we cut off a length $OC = OB = \sqrt{5}$ on the number line.

Then the point C represents the irrational number $\sqrt{5}$.

4. **Classroom activity**
(Constructing the 'square root spiral') : Take a large sheet of paper and construct the 'square root spiral' in the following fashion. Start with a point O and draw a line segment OP_1 of unit length. Draw a line segment P_1P_2 perpendicular to OP_1 of



unit length (see figure). Now draw a line segment P_2P_3 perpendicular to OP_2 . Then draw a line segment P_3P_4 perpendicular to OP_3 . Continuing in the manner, you can get the line segment $P_{n-1}P_n$ by drawing a line segment of unit length perpendicular to OP_{n-1} . In this manner, you will have created the points $P_2, P_3, \dots, P_n, \dots$ and joined them to create a beautiful spiral depicting $\sqrt{2}, \sqrt{3}, \sqrt{4}, \dots$.

Sol. Classroom activity – Do as directed.

EXERCISE 1.3

1. Write the following in decimal form and say what kind of decimal expansion each has :

(i) $\frac{36}{100}$

(ii) $\frac{1}{11}$

(iii) $4\frac{1}{8}$

(iv) $\frac{3}{13}$

(v) $\frac{2}{11}$

(vi) $\frac{329}{400}$

Sol. (i) $\frac{36}{100} = 0.36$, terminating.

(ii) By long division, we have

$$\begin{array}{r}
 0.090909 \\
 11 \overline{) 1.000000} \\
 \underline{99} \\
 100 \\
 \underline{99} \\
 100 \\
 \underline{99} \\
 1
 \end{array}$$

$\therefore \frac{1}{11} = 0.090909... = 0.\overline{09}$, non-terminating and repeating.

(iii) $4\frac{1}{8} = \frac{4 \times 8 + 1}{8} = \frac{33}{8}$

By long division, we have

$$\begin{array}{r} 4.125 \\ 8 \overline{) 33.000} \\ \underline{32} \\ 10 \\ \underline{8} \\ 20 \\ \underline{16} \\ 40 \\ \underline{40} \\ 0 \end{array}$$

$\therefore \frac{33}{8} = 4.125$, terminating.

(iv) By long division, we have

$$\begin{array}{r} 0.23076923... \\ 13 \overline{) 3.0000000} \\ \underline{26} \\ 40 \\ \underline{39} \\ 100 \\ \underline{91} \\ 90 \\ \underline{78} \\ 120 \\ \underline{117} \\ 30 \\ \underline{26} \\ 40 \\ \underline{39} \\ 1 \end{array}$$

$\therefore \frac{3}{13} = 0.23076923... = 0.\overline{230769}$, non-terminating and repeating.

(v) By long division, we have

$$\begin{array}{r}
 0.181818... \\
 11 \overline{) 2.00000} \\
 \underline{11} \\
 90 \\
 \underline{88} \\
 20 \\
 \underline{11} \\
 90 \\
 \underline{88} \\
 20 \\
 \underline{11} \\
 90 \\
 \underline{88} \\
 2 \\
 \underline{}
 \end{array}$$

$\therefore \frac{2}{11} = 0.181818... = 0.\overline{18}$, non-terminating and repeating.

(vi) By long division, we have

$$\begin{array}{r}
 0.8225 \\
 400 \overline{) 329.0000} \\
 \underline{3200} \\
 900 \\
 \underline{800} \\
 1000 \\
 \underline{800} \\
 2000 \\
 \underline{2000} \\
 0 \\
 \underline{}
 \end{array}$$

$$\therefore \frac{329}{400} = 0.8225, \text{ terminating.}$$

2. You know that $\frac{1}{7} = 0.\overline{142857}$. Can you predict what the decimal expansions of $\frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}$ are, without actually doing the long division? If so, how?

Sol. Yes. All of the above will have repeating decimals which are permutations of 1, 4, 2, 8, 5, 7.

For example, here

$$\begin{array}{r}
 0.1428571\ldots \\
 7 \overline{) 1.0000000} \\
 \underline{7} \\
 30 \\
 \underline{28} \\
 20 \leftarrow \\
 \underline{14} \\
 60 \\
 \underline{56} \\
 40 \\
 \underline{35} \\
 50 \\
 \underline{49} \\
 10 \\
 \underline{7} \\
 30
 \end{array}$$

To find $\frac{2}{7}$, locate when the remainder becomes 2 and respective quotient (here it is 2), then write the new quotient beginning from there (the arrows drawn in the above division) using the repeating digits 1, 4, 2, 8, 5, 7.

$$\therefore \frac{2}{7} = 0.\overline{285714}$$

$$\text{Similarly, } \frac{3}{7} = 0.\overline{428571}, \frac{4}{7} = 0.\overline{571428},$$

$$\frac{5}{7} = 0.\overline{714285} \text{ and } \frac{6}{7} = 0.\overline{857142}.$$

3. Express the following in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

(i) $0.\overline{6}$

(ii) $0.4\overline{7}$

(iii) $0.00\overline{1}$

Sol. (i) Let $x = 0.\overline{6}$

Then, $x = 0.666\ldots$...(1)

Here, we have only one repeating digit. So, we multiply both sides of (1) by 10 to get

$$10x = 6.66\ldots$$
 ...(2)

Subtracting (1) from (2), we get

$$10x - x = (6.66\ldots) - (0.66\ldots)$$

$$\Rightarrow 9x = 6 \Rightarrow x = \frac{2}{3}$$

Hence, $0.\overline{6} = \frac{2}{3}$

(ii) Let $x = 0.4\overline{7}$

Clearly, there is just one digit on the right side of the decimal point which is without bar. So, we multiply both sides by x so that only the repeating decimal is left on the right side of the decimal point.

$$\therefore 10x = 4.\overline{7}$$

$$\Rightarrow 10x = 4 + 0.\overline{7}^*$$

$$\Rightarrow 10x = 4 + \frac{7}{9}$$

***Rule :** (i) Write the number without decimal point as numerator.

(ii) Write as many 9's as there are different repeating digits for the denominator.

For example : $0.\overline{7} = \frac{7}{9}$, $0.\overline{35} = \frac{35}{99}$ etc.

$$\Rightarrow 10x = \frac{4 \times 9 + 7}{9} \Rightarrow 10x = \frac{43}{9}$$

$$\Rightarrow x = \frac{43}{90}$$

Hence $0.4\overline{7} = \frac{43}{90}$

ALITER

Let $x = 0.4\overline{7} = 0.4777...$

$\therefore 10x = 4.777... \quad \dots(1)$

and $100x = 47.777... \quad \dots(2)$

Subtracting (1) from (2), we get

$$100x - 10x = (47.777...) - (4.777...)$$

$$\Rightarrow 90x = 43 \Rightarrow x = \frac{43}{90}$$

Hence, $0.4\overline{7} = \frac{43}{90}$

(iii) Let $x = 0.\overline{001}$

$$\Rightarrow x = 0.001001001... \quad \dots(1)$$

Here, we have three repeating digits after the decimal point. So, we multiply (1) by $10^3 = 1000$, we get

$$1000x = 1.001001... \quad \dots(2)$$

Subtracting (1) from (2), we get

$$1000x - x = (1.001001...) - (0.001001...)$$

$$\Rightarrow 999x = 1 \Rightarrow x = \frac{1}{999}$$

Hence, $0.\overline{001} = \frac{1}{999}$

4. Express $0.99999\dots$ in the form $\frac{p}{q}$. Are you surprised by your answer ? With your teacher and classmates discuss why the answer makes sense.

Sol. Let $x = 0.9999\dots$... (1)

Here, we have only one repeating digit. So, we multiply both sides of (1) by 10 to get

$$10x = 9.999\dots \quad \dots (2)$$

Subtracting (1) from (2), we get

$$\begin{aligned} 10x - x &= (9.999\dots) - (0.999\dots) \\ \Rightarrow 9x &= 9 \quad \Rightarrow x = 1 \end{aligned}$$

Hence, $0.9999\dots = 1$

Since $0.9999\dots$ goes on forever. So, there is no gap between 1 and $0.9999\dots$ and hence they are equal.

5. What can the maximum number of digits be in the repeating block of digits in the decimal expansion $\frac{1}{17}$? Perform the division to check your answer.

Sol. We have

$$\begin{array}{r} 0.588235294117647\dots \\ 17 \overline{) 1.0000000000000000} \\ \underline{85} \\ 150 \\ \underline{136} \\ 140 \\ \underline{136} \\ 40 \\ \underline{34} \\ 60 \\ \underline{51} \end{array}$$

$$\begin{array}{r}
 90 \\
 85 \\
 \hline
 50 \\
 34 \\
 \hline
 160 \\
 153 \\
 \hline
 70 \\
 68 \\
 \hline
 20 \\
 17 \\
 \hline
 30 \\
 17 \\
 \hline
 130 \\
 119 \\
 \hline
 110 \\
 102 \\
 \hline
 80 \\
 68 \\
 \hline
 120 \\
 119 \\
 \hline
 1
 \end{array}$$

Thus, $\frac{1}{17} = 0.\overline{0588235294117647}$

$\therefore$ The maximum number of digits in the quotient while computing $\frac{1}{17}$ are 15.

6. Look at several examples of rational numbers in the form $\frac{p}{q}$ ($q \neq 0$), where p and q are integers with no common factors other than 1 and having terminating decimal representations (expansions). Can you guess what property q must satisfy ?

Sol. Consider several rational numbers in the form $\frac{p}{q}$ ($q \neq 0$), where p and q are integers with common factors other than 1 and having terminating decimal representation.

Let the various such rational numbers be $\frac{1}{2}, \frac{1}{4}, \frac{7}{8}, \frac{37}{25},$

$\frac{8}{125}, \frac{17}{20}, \frac{31}{16}$ etc.

In all cases, we think of the natural number which when multiplied by their respective denominators gives 10 or a power of 10.

$$\frac{1}{2} = \frac{1 \times 5}{2 \times 5} = \frac{5}{10} = 0.5 \quad [\because 2 \times 5 = 10]$$

$$\frac{1}{4} = \frac{1 \times 25}{4 \times 25} = \frac{25}{100} = 0.25 \quad [\because 4 \times 25 = 100]$$

$$\frac{7}{8} = \frac{7 \times 125}{8 \times 125} = \frac{875}{1000} = 0.875 \quad [\because 8 \times 125 = 1000]$$

$$\frac{37}{25} = \frac{37 \times 4}{25 \times 4} = \frac{148}{100} = 1.48 \quad [\because 25 \times 4 = 100]$$

$$\frac{8}{125} = \frac{8 \times 8}{125 \times 8} = \frac{64}{1000} = 0.064 \quad [\because 125 \times 8 = 1000]$$

$$\frac{17}{20} = \frac{17 \times 5}{20 \times 5} = \frac{85}{100} = 0.85 \quad [\because 20 \times 5 = 100]$$

$$\frac{31}{16} = \frac{31 \times 625}{16 \times 625} = \frac{19375}{10000} = 1.9375 [\because 16 \times 625 = 10000]$$

We have seen that those rational numbers whose denominators when multiplied by a suitable integer produce a power of 10 are expressible in the finite decimal form. But this can always be done only when the denominator of the given rational number has either 2 or 5 or both of them as the only prime factors. Thus, we obtain the following property :

If the denominator of a rational number in standard form has no prime factors other than 2 or 5, then and only then it can be represented as a terminating decimal.

7. Write three numbers whose decimal expansions are non-terminating non-recurring.

Sol. Three numbers whose decimal representations are non-terminating and non-repeating are

$\sqrt{2}$, $\sqrt{3}$ and $\sqrt{5}$ or we can say 0.100100010001..., 0.20200200020002... and .003000300003...

8. Find three different irrational numbers between the rational numbers $\frac{5}{7}$ and $\frac{9}{11}$.

Sol. We have

$$\begin{array}{r}
 0.714285... \\
 7 \overline{) 5.000000} \\
 \underline{49} \\
 10 \\
 \underline{7} \\
 30 \\
 \underline{28} \\
 20 \\
 \underline{14} \\
 60 \\
 \underline{56} \\
 40 \\
 \underline{35} \\
 5
 \end{array}$$

$$\begin{array}{r}
 0.81... \\
 11 \overline{) 9.00} \\
 \underline{88} \\
 20 \\
 \underline{11} \\
 9
 \end{array}$$

$$\therefore \frac{5}{7} = 0.\overline{714285} \text{ and } \frac{9}{11} = 0.\overline{81}$$

Thus, three different irrational numbers between $\frac{5}{7}$ and

$\frac{9}{11}$ are

0.75075007500075000075..., 0.767076700767000... and 0.80800800080000...

9. Classify the following numbers as rational or irrational :

(i) $\sqrt{23}$

(ii) $\sqrt{225}$

(iii) 0.3796

(iv) 7.478478...

(v) 1.101001000100001...

Sol. (i) $\sqrt{23}$ is an irrational number as 23 is not a perfect square.

$$\begin{aligned} \text{(ii)} \quad \sqrt{225} &= \sqrt{3 \times 3 \times 5 \times 5} \\ &= 3 \times 5 \\ &= 15 \end{aligned}$$

3	225
3	75
5	25
5	5
	1

Thus, 15 is a rational number.

(iii) 0.3796 is a rational number as it is terminating decimal.

(iv) 7.478478... is non-terminating but repeating, so, it is a rational number.

(v) 1.101001000100001... is non-terminating and non-repeating so, it is an irrational number.

EXERCISE 1.4

1. Visualise 3.765 on the number line, using successive magnification.

Sol. We know that 3.765 lies between 3 and 4, that is, in the interval [3, 4] to have a rough idea where it is. Now, we divide the interval [3, 4] into 10 equal parts and look at [3.7, 3.8] through a magnifying glass and realize that 3.765 lies between 3.7 and 3.8 (see figure (i)). Now, we imagine that each of the new intervals [3.1, 3.2], [3.2, 3.3], ..., [3.9, 4] has been sub divided into 10 equal parts. As before, we can now visualize through the magnify glass that 3.765 lies in the interval [3.76, 3.77] (see figure (ii)).

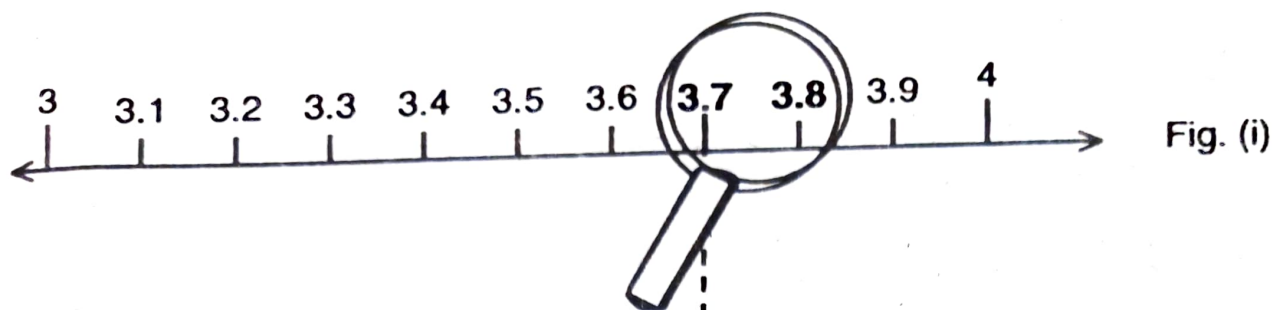


Fig. (i)

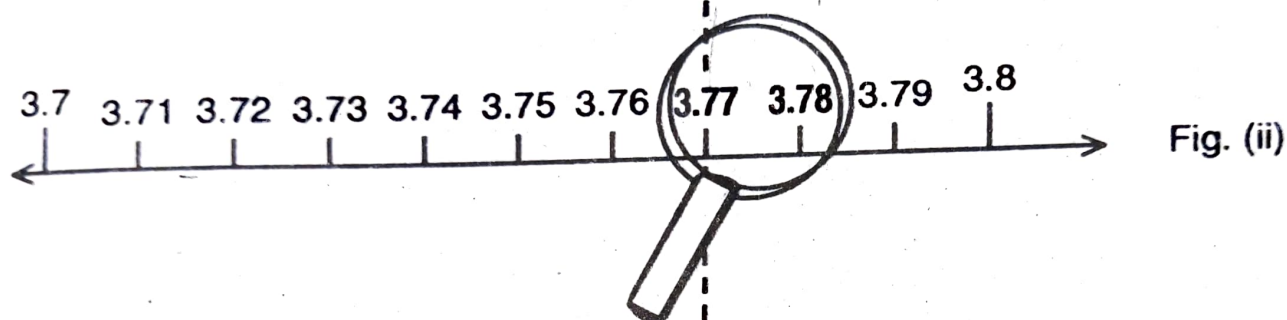


Fig. (ii)

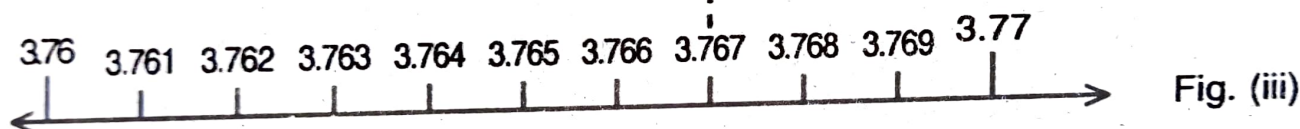


Fig. (iii)

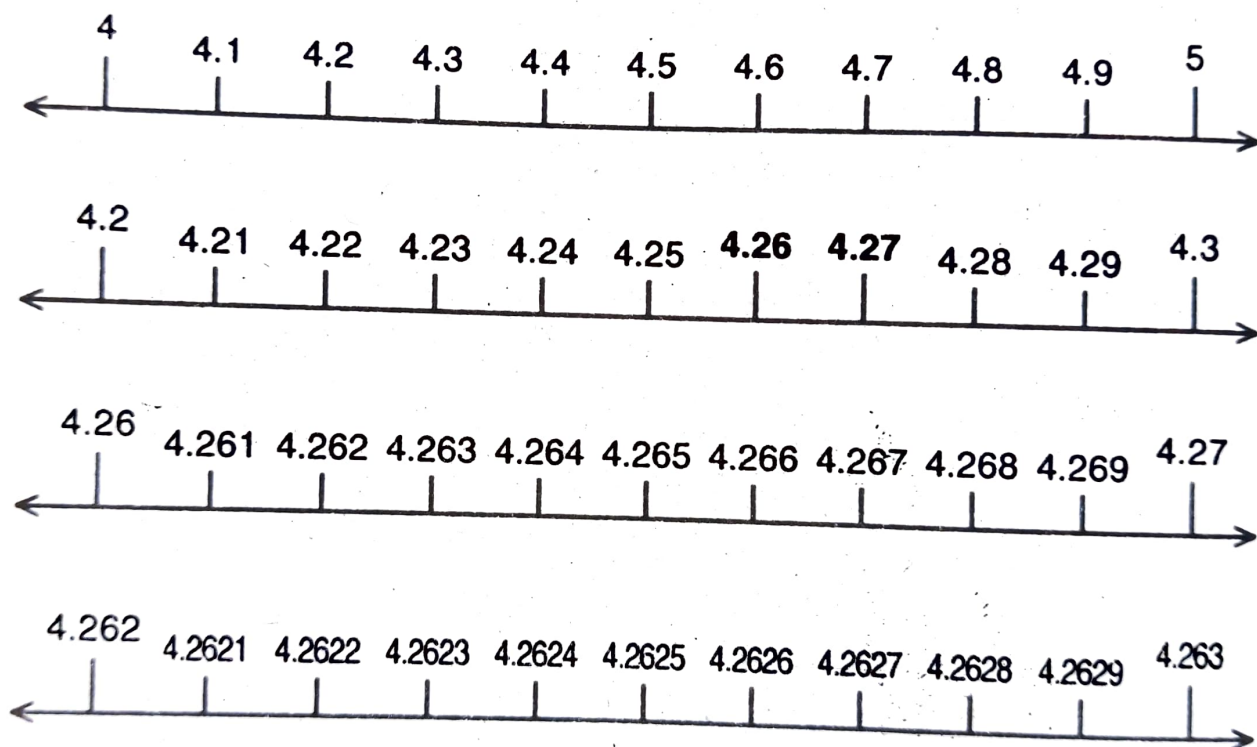
So, we have seen that it is possible by sufficient successive magnifications to visualize the position (or representation) of a real number with a terminating decimal expansion on the number line. Let us now try and visualize the position (or representation) of a real number with a non-terminating recurring decimal expansion on the number line. We can look at appropriate intervals through a magnifying glass and by successive magnifications visualize the position of the number on the number line.

2. Visualise $4.\overline{26}$ on the number line, upto 4 decimal places.

Sol. We proceed by successive magnifications, and successively decrease the lengths of the intervals in which $4.\overline{26}$ is located. $4.\overline{26}$ is located in the interval $[4, 5]$ of length 1. We further locate $4.\overline{26}$ in the interval $[4.2, 4.3]$ of length 0.1. To get more accurate visualization of the representation,

we divide even this interval into 10 equal parts and use a magnifying glass to visualize that $4.\overline{26}$ lies in the interval $[4.26, 4.27]$ of length 0.01. To visualize $4.\overline{26}$ in an interval of length 0.001, we again divide each of the new intervals into 10 equal parts and visualize the representation of $4.\overline{26}$ in the interval $[4.262, 4.263]$ of length 0.001. Notice that $4.\overline{26}$ is located closer to 4.263 than to 4.262.

Note : We can proceed endlessly in this manner and simultaneously imagining the decrease in the length of the interval in which $4.\overline{26}$ is located.



EXERCISE 1.5

1. Classify the following numbers as rational or irrational :

(i) $2 - \sqrt{5}$

(ii) $(3 + \sqrt{23}) - \sqrt{23}$

(iii) $\frac{2\sqrt{7}}{7\sqrt{7}}$

(iv) $\frac{1}{\sqrt{2}}$

(v) 2π

Sol. (i) $2 - \sqrt{5}$ is an irrational number being a difference between a rational and an irrational.

(ii) $(3 + \sqrt{23}) - \sqrt{23} = 3 + \sqrt{23} - \sqrt{23} = 3$, which is a rational number.

(iii) $\frac{2\sqrt{7}}{7\sqrt{7}} = \frac{2}{7}$, which is a rational number.

(iv) $\frac{1}{\sqrt{2}}$ is irrational being the quotient of a rational and an irrational.

(v) 2π is irrational being the product of rational and irrational.

2. Simplify each of the following expressions :

(i) $(3 + \sqrt{3})(2 + \sqrt{2})$

(ii) $(3 + \sqrt{3})(3 - \sqrt{3})$

(iii) $(\sqrt{5} + \sqrt{2})^2$

(iv) $(\sqrt{5} - \sqrt{2})(\sqrt{5} + \sqrt{2})$

Sol. (i) $(3 + \sqrt{3})(2 + \sqrt{2}) = 3 \times 2 + 3\sqrt{2} + 2\sqrt{3} + \sqrt{2} \times \sqrt{3}$
 $= 6 + 3\sqrt{2} + 2\sqrt{3} + \sqrt{6}$

(ii) $(3 + \sqrt{3})(3 - \sqrt{3}) = (3)^2 - (\sqrt{3})^2$
 $= 9 - 3 = 6$

(iii) $(\sqrt{5} + \sqrt{2})^2 = (\sqrt{5})^2 + 2\sqrt{5}\sqrt{2} + (\sqrt{2})^2$
 $= 5 + 2\sqrt{10} + 2 = 7 + 2\sqrt{10}$

(iv) $(\sqrt{5} - \sqrt{2})(\sqrt{5} + \sqrt{2}) = (\sqrt{5})^2 - (\sqrt{2})^2$
 $= 5 - 2 = 3$

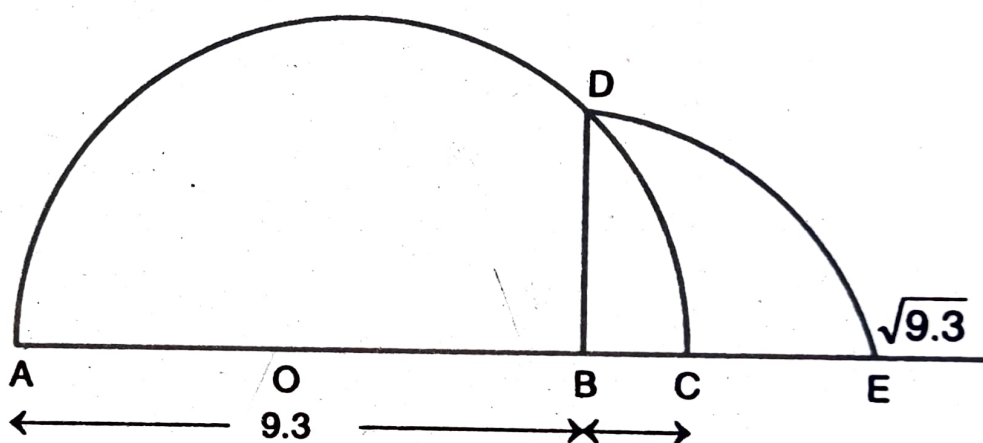
3. Recall, π is defined as the ratio of the circumference (say c) of a circle to its diameter (say d).

That is $\pi = \frac{c}{d}$. This seems to contradict the fact that π is irrational. How will you resolve this contradiction ?

Sol. There is no contradiction as either c or d irrational and hence π is an irrational number.

4. Represent $\sqrt{9.3}$ on the number line.

Sol. Mark the distance 9.3 units from a fixed point A on a given line to obtain a point B such that $AB = 9.3$ units. From B, mark a distance of 1 unit and mark the new point as C. Find the mid-point of AC and mark that point as O. Draw a semi-circle with centre O and radius OC. Draw a line perpendicular to AC passing through B and intersecting the semi circle at D.



Then $BD = \sqrt{9.3}$. To represent $\sqrt{9.3}$ on the number line. Let us treat the line BC as the number line, with B as zero, C as 1, and so on. Draw an arc with centre B and radius BD, which intersects the number line in E. Then, E represent $\sqrt{9.3}$.

5. Rationalise the denominators of the following :

(i) $\frac{1}{\sqrt{7}}$

(ii) $\frac{1}{\sqrt{7} - \sqrt{6}}$

(iii) $\frac{1}{\sqrt{5} + \sqrt{2}}$

(iv) $\frac{1}{\sqrt{7} - 2}$

Sol. (i) $\frac{1}{\sqrt{7}} = \frac{1 \times \sqrt{7}}{\sqrt{7} \times \sqrt{7}} = \frac{\sqrt{7}}{7}$

(ii) $\frac{1}{\sqrt{7} - \sqrt{6}} = \frac{1}{\sqrt{7} - \sqrt{6}} \times \frac{\sqrt{7} + \sqrt{6}}{\sqrt{7} + \sqrt{6}} = \frac{\sqrt{7} + \sqrt{6}}{(\sqrt{7})^2 - (\sqrt{6})^2}$

$$= \frac{\sqrt{7} + \sqrt{6}}{7 - 6} = \sqrt{7} + \sqrt{6}$$

$$(iii) \quad \frac{1}{\sqrt{5} + \sqrt{2}} = \frac{1}{\sqrt{5} + \sqrt{2}} + \frac{\sqrt{5} - \sqrt{2}}{\sqrt{5} - \sqrt{2}} = \frac{\sqrt{5} - \sqrt{2}}{(\sqrt{5})^2 - (\sqrt{2})^2}$$

$$= \frac{\sqrt{5} - \sqrt{2}}{5 - 2} = \frac{\sqrt{5} - \sqrt{2}}{3}$$

$$(iv) \quad \frac{1}{\sqrt{7} - 2} = \frac{1}{\sqrt{7} - 2} \times \frac{\sqrt{7} + 2}{\sqrt{7} + 2} = \frac{\sqrt{7} + 2}{(\sqrt{7})^2 - (2)^2}$$

$$= \frac{\sqrt{7} + 2}{7 - 4} = \frac{\sqrt{7} + 2}{3}$$

EXERCISE 1.6

1. Find : (i) $64^{\frac{1}{2}}$ (ii) $32^{\frac{1}{5}}$ (iii) $125^{\frac{1}{3}}$

$$\text{Sol. (i)} \quad 64^{\frac{1}{2}} = (8 \times 8)^{\frac{1}{2}} = (8^2)^{\frac{1}{2}}$$

$$= 8^{2 \times \frac{1}{2}} = 8^1 = 8$$

$$(ii) \quad 32^{\frac{1}{5}} = (2 \times 2 \times 2 \times 2 \times 2)^{\frac{1}{5}} = (2^5)^{\frac{1}{5}}$$

$$= 2^{5 \times \frac{1}{5}} = 2^1 = 2$$

$$(iii) \quad 125^{\frac{1}{3}} = (5 \times 5 \times 5)^{\frac{1}{3}} = (5^3)^{\frac{1}{3}}$$

$$= 5^{3 \times \frac{1}{3}} = 5^1 = 5$$

2. Find : (i) $9^{\frac{3}{2}}$ (ii) $32^{\frac{2}{5}}$

(iii) $16^{\frac{3}{4}}$ (iv) $125^{\frac{-1}{3}}$

$$\text{Sol. (i)} \quad 9^{\frac{3}{2}} = (3 \times 3)^{\frac{3}{2}} = (3^2)^{\frac{3}{2}} = 3^{2 \times \frac{3}{2}}$$

$$= 3^3 = 3 \times 3 \times 3 = 27$$

$$(ii) \quad 32^{\frac{2}{5}} = (2 \times 2 \times 2 \times 2 \times 2)^{\frac{2}{5}} = (2^5)^{\frac{2}{5}}$$

$$= 2^{5 \times \frac{2}{5}} = 2^2 = 2 \times 2 = 4$$

$$(iii) \quad 16^{\frac{3}{4}} = (2 \times 2 \times 2 \times 2)^{\frac{3}{4}} = (2^4)^{\frac{3}{4}}$$

$$= 2^{4 \times \frac{3}{4}} = 2^3 = 2 \times 2 \times 2 = 8$$

$$(iv) \quad 125^{-\frac{1}{3}} = \frac{1}{125^{\frac{1}{3}}} = \frac{1}{(5 \times 5 \times 5)^{\frac{1}{3}}} = \frac{1}{5}$$

3. Simplify : (i) $2^{\frac{2}{3}} \cdot 2^{\frac{1}{5}}$

$$(ii) \left(\frac{1}{3^3} \right)^7$$

$$(iii) \frac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}}$$

$$(iv) 7^{\frac{1}{2}} \cdot 8^{\frac{1}{2}}$$

$$\text{Sol. (i)} \quad 2^{\frac{2}{3}} \cdot 2^{\frac{1}{5}} = 2^{\frac{2}{3} + \frac{1}{5}} = 2^{\frac{10+3}{15}} = 2^{\frac{13}{15}}$$

$$(ii) \quad \left(\frac{1}{3^3} \right)^7 = \frac{(1)^7}{3^{3 \times 7}} = \frac{1}{3^{21}}$$

$$(iii) \quad \frac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}} = 11^{\frac{1}{2} - \frac{1}{4}} = 11^{\frac{1}{4}}$$

$$(iv) \quad 7^{\frac{1}{2}} \cdot 8^{\frac{1}{2}} = (7 \times 8)^{\frac{1}{2}} = 56^{\frac{1}{2}}$$

$$= (4 \times 14)^{\frac{1}{2}} = (56)^{\frac{1}{2}}$$