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POLYNOMIALS

Chapter – 2

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CLASS: 9TH

MATHEMATICS

EXERCISE 2.1

1. Which of the following expressions are polynomials in one variable and which are not ? State reasons for your answer.

(i) $4x^2 - 3x + 7$ **(ii)** $y^2 + \sqrt{2}$

(iii) $3\sqrt{t} + t\sqrt{2}$ **(iv)** $y + \frac{2}{y}$

(v) $x^{10} + y^3 + t^{50}$.

Sol. (i) In $4x^2 - 3x + 7$, all the indices of x are whole numbers so it is a polynomial in one variable x.

(ii) In $y^2 + \sqrt{2}$, the index of y is a whole number so it is a polynomial in one variable y.

(iii) $3\sqrt{t} + t\sqrt{2} = 3t^{\frac{1}{2}} + \sqrt{2}t$, here the exponent of the first term is $\frac{1}{2}$, which is not a whole number. Therefore, it is not a polynomial.

(iv) $y + \frac{2}{y} = y + 2y^{-1}$, here the exponent of the second term is -1, which is not a whole number and so it is not a polynomial.

(v) $x^{10} + y^3 + t^{50}$ is not a polynomial in one variable as three variables x, y, t occur in it.

2. Write the coefficients of x^2 in each of the following :

(i) $2 + x^2 + x$ **(ii)** $2 - x^2 + x^3$

(iii) $\frac{\pi}{2}x^2 + x$ **(iv)** $\sqrt{2}x - 1$

Sol. Coefficient of x^2 :

(*i*) in $2 + x^2 + x$ is 1

(*ii*) in $2 - x^2 + x^3$ is -1

(*iii*) in $\frac{\pi}{2}x^2 + x$ is $\frac{\pi}{2}$

(*iv*) in $\sqrt{2}x - 1$ is 0.

3. Give one example each of a binomial of degree 35, and of a monomial of degree 100.

Sol. Binomial of degree 35 may be taken as $x^{35} + 4x$

Monomial of degree 100 may be taken as $5x^{100}$.

4. Write the degree of each of the following polynomials :

(*i*) $5x^3 + 4x^2 + 7x$

(*ii*) $4 - y^2$

(*iii*) $5t - \sqrt{7}$

(*iv*) 3

Sol. (*i*) The highest power term is $5x^3$ and the exponent is 3. So, the degree is 3.

(*ii*) The highest power term is $-y^2$ and the exponent is 2. So, the degree is 2.

(*iii*) The highest power term is $5t$ and the exponent is 1. So, the degree is 1.

(*iv*) The only term here is 3 which can be written as $3x^0$ and so the exponent is 0. Therefore, the degree is 0.

5. Classify the following as linear, quadratic and cubic polynomials :

(*i*) $x^2 + x$

(*ii*) $x - x^3$

(*iii*) $y + y^2 + 4$

(*iv*) $1 + x$

(*v*) $3t$

(*vi*) r^2

(*vii*) $7x^3$

Sol. (*i*) The highest degree of $x^2 + x$ is 2, so it is a quadratic.

(*ii*) The highest degree of $x - x^3$ is 3, so it is a cubic.

(*iii*) The highest degree of $y + y^2 + 4$ is 2, so it is a quadratic.

(*iv*) The highest degree of x in $1 + x$ in 1. So it is a linear polynomial.

(*v*) The highest degree of t in $3t$ is 1. So it is a linear polynomial.

(*vi*) The highest degree of r in r^2 is 2. So, it is a quadratic polynomial.

(*vii*) The highest degree of x in $7x^3$ is 3. So, it is cubic polynomial.

EXERCISE 2.2

1. Find the value of the polynomial $5x - 4x^2 + 3$ at

(*i*) $x = 0$ **(*ii*) $x = -1$** **(*iii*) $x = 2$**

Sol. Let $p(x) = 5x - 4x^2 + 3$

(*i*) At $x = 0$: $p(0) = 5(0) - 4(0)^2 + 3$

$= 0 - 0 + 3 = 3$

(*ii*) At $x = -1$: $p(-1) = 5(-1) - 4(-1)^2 + 3$

$= -5 - 4 + 3 = -6.$

(*iii*) At $x = 2$: $p(2) = 5(2) - 4(2)^2 + 3 = 10 - 16 + 3$

$= 13 - 16 = -3.$

2. Find $p(0)$, $p(1)$ and $p(2)$ for each of the following polynomials :

(*i*) $p(y) = y^2 - y + 1$ **(*ii*) $p(t) = 2 + t + 2t^2 - t^3$**

(*iii*) $p(x) = x^3$ **(*iv*) $p(x) = (x - 1)(x + 1)$**

Sol. (*i*) We have, $p(y) = y^2 - y + 1$

$p(0) = (0)^2 - 0 + 1 = 0 - 0 + 1 = 1,$

$p(1) = (1)^2 - 1 + 1 = 1 - 1 + 1 = 1,$

and $p(2) = (2)^2 - 2 + 1 = 4 - 2 + 1 = 3$

(*ii*) We have, $p(t) = 2 + t + 2t^2 - t^3$

$\therefore$ $p(0) = 2 + 0 + 2(0)^2 - (0)^3$

$= 2 + 0 + 0 - 0 = 2,$

$p(1) = 2 + 1 + 2(1)^2 - (1)^3$

$= 2 + 1 + 2 - 3 = 5 - 3 = 2,$

and $p(2) = 2 + 2 + 2(2)^2 - (2)^3$

$= 2 + 2 + 8 - 8 = 4$

(*iii*) We have, $p(x) = x^3$

$\therefore$ $p(0) = (0)^3 = 0,$

$p(1) = (1)^3 = 1,$

and $p(2) = (2)^3 = 8$

(*iv*) We have, $p(x) = (x - 1)(x + 1)$

$\therefore$ $p(0) = (0 - 1)(0 + 1) = (-1)(1) = -1,$

$p(1) = (1 - 1)(1 + 1) = (0)(2) = 0$

and $p(2) = (2 - 1)(2 + 1) = (1)(3) = 3$

3. Verify whether the following are zeroes of the polynomial, indicated against them,

(*i*) $p(x) = 3x + 1, x = -\frac{1}{3}$

(*ii*) $p(x) = 5x - \pi, x = \frac{4}{5}$

(*iii*) $p(x) = x^2 - 1, x = 1, -1$

(*iv*) $p(x) = (x + 1)(x - 2), x = -1, 2$

(*v*) $p(x) = x^2, x = 0$

(*vi*) $p(x) = lx + m, x = -\frac{m}{l}$

(*vii*) $p(x) = 3x^2 - 1, x = -\frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}$

(*viii*) $p(x) = 2x + 1, x = \frac{1}{2}$

Sol. (*i*) We have, $p(x) = 3x + 1$

At $x = -\frac{1}{3}$, $p\left(-\frac{1}{3}\right) = 3\left(-\frac{1}{3}\right) + 1 = -1 + 1 = 0$

$\therefore -\frac{1}{3}$ is a zero of $p(x)$.

(*ii*) We have, $p(x) = 5x - \pi$

At $x = \frac{4}{5}$, $p\left(\frac{4}{5}\right) = 5\left(\frac{4}{5}\right) - 4 = 4 - 4 = 0$

$\therefore \frac{4}{5}$ is a zero of $p(x)$.

(*iii*) We have, $p(x) = x^2 - 1$

At $x = 1$, $p(1) = (1)^2 - 1 = 1 - 1 = 0$

$\therefore$ 1 is a zero of $p(x)$.

Also, at $x = -1$ $p(-1) = (-1)^2 - 1$

$= 1 - 1 = 0$

$\therefore -1$ is a zero of $p(x)$.

(*iv*) We have, $p(x) = (x + 1)(x - 2)$

At $x = -1$, $p(-1) = (-1 + 1)(-1 - 2)$

$= (0)(-3) = 0$

$\therefore -1$ is a zero of $p(x)$.

Also, at $x = 2$, $p(2) = (2 + 1)(2 - 2) = (3)(0) = 0$

$\therefore$ 2 is zero of $p(x)$.

(*v*) We have, $p(x) = x^2$

At $x = 0$, $p(0) = (0)^2 = 0$

$\therefore$ 0 is a zero of $p(x)$.

(*vi*) We have, $p(x) = lx + m$

At $x = -\dfrac{m}{l}$, $p\left(-\dfrac{m}{l}\right) = l\left(\dfrac{-m}{l}\right) + m$

$= -m + m = 0$

$\therefore \dfrac{-m}{l}$ is a zero of $p(x)$.

(*vii*) We have, $p(x) = 3x^2 - 1$

At $x = -\dfrac{1}{\sqrt{3}}$, $p\left(-\dfrac{1}{\sqrt{3}}\right) = 3\left(\dfrac{1}{\sqrt{3}}\right)^2 - 1$

$= 3 \times \dfrac{1}{3} - 1 = 1 - 1 = 0$

At $x = \dfrac{2}{\sqrt{3}}$, $p\left(\dfrac{2}{\sqrt{3}}\right) = 3\left(\dfrac{2}{\sqrt{3}}\right)^2 - 1 = 3 \times \dfrac{4}{3} - 1$

$= 4 - 1 = 3$

(*viii*) We have, $p(x) = 2x + 1$

At $x = \frac{1}{2}$, $p\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right) + 1 = 1 + 1 = 2$

4. Find the zero of the polynomial in each of the following cases :

(*i*) $p(x) = x + 5$ **(*ii*) $p(x) = x - 5$**

(*iii*) $p(x) = 2x + 5$ **(*iv*) $p(x) = 3x - 2$**

(*v*) $p(x) = 3x$ **(*vi*) $p(x) = ax, a \neq 0$**

(*vii*) $p(x) = cx + d, c \neq 0, c, d$ are real numbers.

Sol. (*i*) We have to solve $p(x) = 0$

$\Rightarrow$ $x + 5 = 0 \Rightarrow x = -5$

$\therefore -5$ is a zero of the polynomial $x + 5$.

(*ii*) We have to solve $p(x) = 0$

$\Rightarrow$ $x - 5 = 0 \Rightarrow x = 5$

$\therefore$ 5 is a zero of the polynomial $x - 5$.

(*iii*) We have to solve $p(x) = 0$

$\Rightarrow$ $2x + 5 = 0 \Rightarrow x = -\frac{5}{2}$

$\therefore -\frac{5}{2}$ is a zero of the polynomial $2x + 5$.

(*iv*) We have to solve $p(x) = 0$

$\Rightarrow$ $3x - 2 = 0 \Rightarrow x = \frac{2}{3}$

$\therefore \frac{2}{3}$ is a zero of the polynomial $3x - 2$.

(*v*) We have to solve $p(x) = 0$

$\Rightarrow$ $3x = 0 \Rightarrow x = 0$

$\therefore$ 0 is a zero of the polynomial $3x$.

(*vi*) We have to solve $p(x) = ax, a \neq 0$

$\Rightarrow$ $ax = 0 \Rightarrow x = 0$

$\therefore$ 0 is a zero of the polynomial ax.

(*vii*) We have to solve $p(x) = 0, c \neq 0$

$$\Rightarrow \quad cx + d = 0 \Rightarrow x = -\frac{d}{c}$$

$\therefore -\frac{d}{c}$ is a zero of the polynomial $cx + d$.

EXERCISE 2.3

1. Find the remainder when $x^3 + 3x^2 + 3x + 1$ is divided by

(*i*) $x + 1$ (*ii*) $x - \frac{1}{2}$ (*iii*) x

(*iv*) $x + \pi$ (*v*) $5 + 2x$

Sol. (*i*) By remainder theorem, the required remainder is equal to $p(-1)$.

Now, $p(x) = x^3 + 3x^2 + 3x + 1$

$\therefore$ $p(-1) = (-1)^3 + 3(-1)^2 + 3(-1) + 1$

$= -1 + 3 - 3 + 1 = 0$

Hence, required remainder $= p(-1) = 0$

(*ii*) By remainder theorem, the required remainder is equal to $p\left(\frac{1}{2}\right)$.

Now, $p\left(\frac{1}{2}\right) = x^3 + 3x^2 + 3x + 1$

$$= \left(\frac{1}{2}\right)^3 + 3\left(\frac{1}{2}\right)^2 + 3\left(\frac{1}{2}\right) + 1$$

$$= \frac{1}{8} + \frac{3}{4} + \frac{3}{2} + 1 = \frac{1 + 6 + 12 + 8}{8}$$

$$= \frac{27}{8}$$

(*iii*) By remainder theorem, the required remainder is equal to $p(0)$.

Now, $p(x) = x^3 + 3x^2 + 3x + 1$

$\therefore$ $p(0) = 0 + 0 + 0 + 1 = 1$

Hence, the required remainder $= p(0) = 1$

(*iv*) By remainder theorem the required remainder is $p(-\pi)$

Now, $p(x) = x^3 + 3x^2 + 3x + 1$

$\therefore$ $p(-\pi) = (-\pi)^3 + 3(-\pi)^2 + 3(-\pi) + 1$

$= -\pi^3 + 3\pi^2 - 3\pi + 1$

(*v*) By remainder theorem, the required remainder is

$p\left(-\frac{5}{2}\right)$

Now, $p(x) = x^3 + 3x^2 + 3x + 1$

$\therefore$ $p\left(-\frac{5}{2}\right) = \left(-\frac{5}{2}\right)^3 + 3\left(\frac{-5}{2}\right)^2 + 3\left(\frac{-5}{2}\right) + 1$

$= -\frac{125}{8} + \frac{75}{4} - \frac{15}{2} + 1$

$= \frac{-125 + 150 - 60 + 8}{8} = \frac{-27}{8}$

2. Find the remainder when $x^3 - ax^2 + 6x - a$ is divided by $x - a$.

Sol. Let $p(x) = x^3 - ax^2 + 6x - a$

By remainder theorem, when $p(x)$ is divided by $x - a$. Then, remainder $= p(a)$

$\therefore$ $p(x) = a^3 - a \cdot a^2 + 6a - a$

$= a^3 - a^3 + 6a - a = 5a$

3. Check whether $7 + 3x$ is a factor of $3x^3 + 7x$

Sol. $7 + 3x$ will be a factor of $p(x) = 3x^3 + 7x$ if $p\left(-\frac{7}{3}\right) = 0$

Now, $p\left(-\frac{7}{3}\right) = 3\left(-\frac{7}{3}\right)^3 + 7\left(-\frac{7}{3}\right)$

$= 3 \times \frac{-343}{27} - \frac{49}{3} = -\frac{343}{9} - \frac{49}{3} \neq 0$

$\therefore$ $7 + 3x$ is not a factor $3x^3 + 7x$

EXERCISE 2.4

1. Determine which of the following polynomials has $(x + 1)$ a factor :

(*i*) $x^3 + x^2 + x + 1$ (*ii*) $x^4 + x^3 + x^2 + x + 1$

(*iii*) $x^4 + 3x^3 + 3x^2 + x + 1$

(*iv*) $x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}$

Sol. (*i*) In order to prove that $x + 1$ is a factor of $p(x) = x^3 + x^2 + x + 1$, it is sufficient to show that $p(-1) = 0$.

Now, $p(-1) = (-1)^3 + (-1)^2 + (-1) + 1$

$= -1 + 1 - 1 + 1 = 0$

Hence, $(x + 1)$ is a factor of $p(x) = x^3 + x^2 + x + 1$.

(*ii*) In order to prove that $(x + 1)$ is a factor of $p(x) = x^4 + x^3 + x^2 + x + 1$, it is sufficient to show that $p(-1) = 0$.

Now, $p(-1) = (-1)^4 + (-1)^3 + (-1)^2 + (-1) + 1$

$= 1 - 1 + 1 - 1 + 1 = 1 \neq 0$

$\therefore$ $(x + 1)$ is not a factor of $x^4 + x^3 + x^2 + x + 1$.

(*iii*) In order to prove that $(x + 1)$ is a factor of $p(x) = x^4 + 3x^3 + 3x^2 + x + 1$, it is sufficient to show that $p(-1) = 0$.

Now, $p(-1) = (-1)^4 + 3(-1)^3 + 3(-1)^2 + (-1) + 1$

$= 1 - 3 + 3 - 1 + 1 = 1 \neq 0$

$\therefore$ $(x + 1)$ is not a factor of $x^4 + 3x^3 + 3x^2 + x + 1$.

(*iv*) In order to prove that $(x + 1)$ is a factor of $p(x) = x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}$, it is sufficient to show that $p(-1) = 0$.

$p(-1) = (-1)^3 - (-1)^2 - (2 + \sqrt{2})(-1) + \sqrt{2}$

$= -1 - 1 + 2 + \sqrt{2} + \sqrt{2} = 2\sqrt{2} \neq 0$

$\therefore$ $(x + 1)$ is not a factor of $x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}$

2. Use the Factor Theorem to determine whether $g(x)$ is a factor of $p(x)$ in each of the following cases :

(i) $p(x) = 2x^3 + x^2 - 2x - 1, g(x) = x + 1$

(ii) $p(x) = x^3 + 3x^2 + 3x + 1, g(x) = x + 2$

(iii) $p(x) = x^3 - 4x^2 + x + 6, g(x) = x - 3$

Sol. (i) In order to prove that $g(x) = x + 1$ is a factor of $p(x) = 2x^3 + x^2 - 2x - 1$, it is sufficient to show that $p(-1) = 0$

Now, $p(-1) = 2(-1)^3 + (-1)^2 - 2(-1) - 1$

$= -2 + 1 + 2 - 1 = 0$

$\therefore$ $g(x)$ is a factor of $p(x)$.

(ii) In order to prove that $g(x) = x + 2$ is a factor of $p(x) = x^3 + 3x^2 + 3x + 1$, it is sufficient to show that $p(-2) = 0$

Now, $p(-2) = (-2)^3 + 3(-2)^2 + 3(-2) + 1$

$= -8 + 12 - 6 + 1$

$= -1 \neq 0$

$\therefore$ $g(x)$ is not a factor of $p(x)$.

(iii) In order to prove that $g(x) = x - 3$ is a factor of $p(x) = x^3 - 4x^2 + x - 6$, it is sufficient to show that $p(+3) = 0$

Now, $p(3) = (3)^3 - 4(3)^2 + 3 - 6$

$= 27 - 36 + 3 - 6$

$= -12 \neq 0$

$\therefore$ $g(x)$ is not a factor of $p(x)$.

3. Find the value of k, if $x - 1$ is a factor of $p(x)$ in each of the following cases :

(i) $p(x) = x^2 + x + k$ **(ii) $p(x) = 2x^2 + kx + \sqrt{2}$**

(iii) $p(x) = kx^2 - \sqrt{2}x + 1$ **(iv) $p(x) = kx^2 - 3x + k$**

Sol. (i) If $(x - 1)$ is a factor of $p(x) = x^2 + x + k$, then

$p(1) = 0$

$\Rightarrow$ $(1)^2 + 1 + k = 0$

$\Rightarrow$ $1 + 1 + k = 0$

$\Rightarrow$ $k = -2$

(*ii*) If $(x - 1)$ is a factor of $p(x) = 2x^2 + kx + \sqrt{2}$, then

$$p(1) = 0$$

$$\Rightarrow \quad 2(1)^2 + k(1) + \sqrt{2} = 0 \quad \Rightarrow \quad 2 + k + \sqrt{2} = 0$$

$$\Rightarrow \quad k = -(2 + \sqrt{2})$$

(*iii*) If $(x - 1)$ is a factor of $p(x) = kx^2 - \sqrt{2}\,x + 1$, then

$$p(1) = 0$$

$$\Rightarrow \quad k(1)^2 - \sqrt{2}(1) + 1 = 0 \quad \Rightarrow \quad k - \sqrt{2} + 1 = 0$$

$$\Rightarrow \quad k = \sqrt{2} - 1$$

(*iv*) If $(x - 1)$ is a factor of $p(x) = kx^2 - 3x + k$, then

$$p(1) = 0$$

$$\Rightarrow \quad k(1)^2 - 3(1) + k = 0 \quad \Rightarrow \quad k - 3 + k = 0$$

$$\Rightarrow \quad 2k = 3 \quad \Rightarrow \quad k = \frac{3}{2}$$

4. Factorise :

(*i*) $12x^2 - 7x + 1$ **(*ii*) $2x^2 + 7x + 3$**

(*iii*) $6x^2 + 5x - 6$ **(*iv*) $3x^2 - x - 4$**

Sol. (*i*) Here

$$p + q = \text{coeff. of } x = -7$$

$$pq = \text{coeff. of } x^2 \times \text{constant term}$$

$$= 12 \times 1 = 12$$

$$\therefore \quad p + q = -7 = -4 - 3$$

and

$$pq = 12 = (-4)(-3)$$

$$\therefore \quad 12x^2 - 7x + 1 = 12x^2 - 4x - 3x + 1$$

$$= 4x(3x - 1) - 1(3x - 1)$$

$$= (3x - 1)(4x - 1)$$

(*ii*) Here

$$p + q = \text{coeff. of } x = 7$$

$$pq = \text{coeff. of } x^2 \times \text{constant term}$$

$$= 2 \times 3 = 6$$

$$\therefore \quad p + q = 7 = 1 + 6$$

and

$$pq = 6 = 1 \times 6$$

$$\therefore \quad 2x^2 + 7x + 3 = 2x^2 + x + 6x + 3$$
$$= x(2x + 1) + 3(2x + 1)$$
$$= (2x + 1)(x + 3)$$

(*iii*) Here $\quad p + q = \text{coeff. of } x = 5$

$$pq = \text{coeff. of } x^2 \times \text{constant term}$$
$$= 6 \times -6 = -36$$

$$\therefore \quad p + q = 5 = 9 + (-4)$$

and $\quad pq = -36 = 9 \times (-4)$

$$\therefore \quad 6x^2 + 5x - 6 = 6x^2 + 9x - 4x - 6$$
$$= 3x(2x + 3) - 2(2x + 3)$$
$$= (2x + 3)(3x - 2)$$

(*iv*) Here $\quad p + q = \text{coeff. of } x = -1$

$$pq = \text{coeff. of } x^2 \times \text{constant term}$$
$$= 3 \times -4 = -12$$

$$\therefore \quad p + q = -1 = 3 + (-4)$$

and $\quad pq = -12 = 3 \times (-4)$

$$\therefore \quad 3x^2 - x - 4 = 3x + 3x - 4x - 4$$
$$= 3x(x + 1) - 4(x + 1)$$
$$= (x + 1)(3x - 4)$$

5. Factorise :

(*i*) $x^3 - 2x^2 - x + 2$ **(*ii*) $x^3 - 3x^2 - 9x - 5$**

(*iii*) $x^3 + 13x^2 + 32x + 20$ **(*iv*) $2y^3 + y^2 - 2y - 1$**

Sol. (*i*) Let $\quad f(x) = x^3 - 2x^2 - x + 2$

The constant term in $f(x)$ is $+ 2$ and factors of $- 2$ are ± 1, ± 2.

Putting $x = 1$ in $f(x)$, we have

$$f(1) = (1)^3 - 2(1)^2 - 1 + 2$$
$$= 1 - 2 - 1 + 2 = 0$$

$\therefore$ $(x - 1)$ is a factor of $f(x)$.

Putting $x = -1$ in $f(x)$, we have

$$f(-1) = (-1)^3 - 2(-1)^2 - (-1) + 2$$
$$= -1 - 2 + 1 + 2 = 0$$

$\therefore$ $x + 1$ is a factor of $f(x)$.

Putting $x = 2$ in $f(x)$, we have

$$f(2) = (2)^3 - 2(2)^2 - (2) + 2$$
$$= 8 - 8 - 2 + 2 = 0$$

$\therefore$ $(x + 2)$ is a factor of $f(x)$

Putting $x = -2$ in $f(x)$, we have

$$f(-2) = (-2)^3 - 2(-2)^2 - (-2) + 2$$
$$= -8 - 8 + 2 + 2$$
$$= -12 \neq 0$$

$\therefore$ $x + 2$ is not a factor of $f(x)$.

$\therefore$ The factors of $f(x)$ are $(x - 1)$, $(x + 1)$ and $(x - 2)$.

Let $\quad f(x) = k(x - 1)(x + 1)(x - 2)$

$$\Rightarrow \quad x^3 - 2x^2 - x + 2 = k(x - 1)(x + 1)(x - 2)$$

Putting $x = 0$ on both sides, we have

$$2 = k(-1)(1)(-2) \quad \Rightarrow \quad k = 1$$

$$\therefore \quad x^3 - 2x^2 - x + 2 = (x - 1)(x + 1)(x - 2)$$

(*ii*) Let $p(x) = x^3 - 3x^2 - 9x - 5$

We shall look for all factors of -5. These are ± 1, ± 5.

By trial, we find $p(-1) = -1 - 3 + 9 - 5 = 0$. So, $(x + 1)$ is a factor of $p(x)$.

$$
\begin{array}{r|rrrr}
 & x^2 & -4x & -5 & \\
\hline
x+1 & x^3 - & 3x^2 - & 9x - & 5 \\
 & x^3 + & x^2 & & \\
 & - & - & & \\
\hline
 & & -4x^2 - & 9x & \\
 & & -4x^2 - & 4x & \\
 & & + & + & \\
\hline
 & & & -5x - & 5 \\
 & & & -5x - & 5 \\
 & & & + & + \\
\hline
 & & & & 0
\end{array}
$$

$$\therefore \quad p(x) = (x + 1)(x^2 - 4x - 5)$$
$$= (x + 1)(x^2 + x - 5x - 5)$$
$$= (x + 1)[x(x + 1) - 5(x + 1)]$$
$$= (x + 1)(x + 1)(x - 5)$$

(*iii*) Let $p(x) = x^3 + 13x^2 + 32x + 20$

We shall look for all facotrs of + 20, these are ± 1, ± 2, ± 4, ± 5, ± 10 and ± 20.

By trial, we find

$$p(-2) = -8 + 52 - 64 + 20 = 0$$

$\therefore$ $(x + 2)$ is a factor of $p(x)$

Now, divide $p(x)$ by $x + 2$.

$$\begin{array}{r|l} & x^2 + 11x + 10 \\ \hline x + 2 & x^3 + 13x^2 + 32x + 20 \\ & x^3 + 2x^2 \\ & - \quad - \\ \hline & 11x^2 + 32x \\ & 11x^2 + 22x \\ & - \quad - \\ \hline & 10x + 20 \\ & 10x + 20 \\ & - \quad - \\ \hline & 0 \end{array}$$

$$\therefore \quad p(x) = (x + 2)(x^2 + 11x + 10)$$
$$= (x + 2)(x^2 + x + 10x + 10)$$
$$= (x + 2)[x(x + 1) + 10(x + 1)]$$
$$= (x + 2)(x + 1)(x + 10)$$

(*iv*) Let $p(y) = 2y^3 + y^2 - 2y - 1$

By trial, we find $p(1) = 2 + 1 - 2 - 1 = 0$.

So, $(y - 1)$ is a factor of $p(y)$.

Now, divide $p(y)$ by $y - 1$

$$
\begin{array}{r|l}
 & 2y^3 + 3y + 1 \\
\hline
y - 1 & 2y^3 + y^2 - 2y - 1 \\
 & 2y^3 - 2y^2 \\
 & - \quad + \\
\hline
 & \quad 3y^2 - 2y \\
 & \quad 3y^2 - 3y \\
 & \quad - \quad + \\
\hline
 & \qquad y - 1 \\
 & \qquad y - 1 \\
 & \qquad\quad 0 \\
\hline
\end{array}
$$

$$
\begin{aligned}
\therefore \quad p(y) &= (y-1)(2y^2+3y+1) \\
&= (y-1)(2y^2+2y+y+1) \\
&= (y-1)[2y(y+1)+1(y+1)] \\
&= (y-1)(y+1)(2y+1)
\end{aligned}
$$

EXERCISE 2.5

1. Use suitable identities to find the following products :

(i) $(x+4)(x+10)$ **(ii)** $(x+8)(x-10)$

(iii) $(3x+4)(3x-5)$

(iv) $\left(y^2+\frac{3}{2}\right)\left(y^2-\frac{3}{2}\right)$ **(v)** $(3-2x)(3+2x)$

Sol. (*i*)
$$
\begin{aligned}
(x+4)(x+10) &= x^2+(4+10)x+4\times 10 \\
&= x^2+14x+40
\end{aligned}
$$

(*ii*)
$$
\begin{aligned}
(x+8)(x-10) &= x^2+(8-10)x+8\times -10 \\
&= x^2-2x-80
\end{aligned}
$$

(*iii*)
$$
\begin{aligned}
(3x+4)(3x-5) &= 3x(3x-5)+4(3x-5) \\
&= 3x\times 3x-3x\times 5+4\times 3x-4\times 5 \\
&= 9x^2-15x+12x-20 \\
&= 9x^2-3x-20
\end{aligned}
$$

(*iv*)
$$
\left(y^2+\frac{3}{2}\right)\left(y^2-\frac{3}{2}\right) = (y^2)^2-\left(\frac{3}{2}\right)^2 = y^4-\frac{9}{4}
$$

(e) $(3 - 2x)(3 + 2x) = (3)^2 - (2x)^2$

$= 9 - 4x^2$

2. Evaluate the following products without multiplying directly :

(i) 103 × 107 **(ii) 95 × 96** **(iii) 104 × 96**

Sol. (i) $103 \times 107 = (100 + 3)(100 + 7)$

$= (100)^2 + (3 + 7)(100) + 3 \times 7$

$= 100 \times 100 + (10)(100) + 21$

$= 10000 + 1000 + 21 = 11021$

(ii) $95 \times 96 = (100 - 5)(100 - 4)$

$= (100)^2 + (-5 - 4)(100) + (-5)(-4)$

$= 100 \times 100 + (-9)(100) + 20$

$= 10000 - 900 + 20 = 9120$

(iii) $104 \times 96 = (100 + 4)(100 - 4)$

$= (100)^2 - (4)^2$

$= 10000 - 16 = 9984$

3. Factorise the following using appropriate identities :

(i) $9x^2 + 6xy + y^2$ **(ii)** $4y^2 - 4y + 1$

(iii) $x^2 - \dfrac{y^2}{100}$

Sol. (a) $9x^2 + 6xy + y^2 = (3x)^2 + 2(3x)(y) + (y)^2$

$= (3x + y)^2 = (3x + y)(3x + y)$

(b) $4y^2 - 4y + 1 = (2y)^2 - 2(2y)(1) + (1)^2$

$= (2y - 1)^2 = (2y - 1)(2y - 1)$

(c) $x^2 - \dfrac{y^2}{100} = (x)^2 - \left(\dfrac{y}{10}\right)^2$

$= \left(x - \dfrac{y}{10}\right)\left(x + \dfrac{y}{10}\right)$

4. Expand each of the following, using suitable identifies :

(i) $(x + 2y + 4z)^2$

(ii) $(2x - y + z)^2$

(iii) $(-2x + 3y + 2z)^2$

(iv) $(3a - 7b - c)^2$

(v) $(-2x + 5y - 3z)^2$

(vi) $\left[\frac{1}{4}a - \frac{1}{2}b + 1\right]^2$

Sol. (*i*) $(x + 2y + 4z)^2$

$= x^2 + (2y)^2 + (4z)^2 + 2(x)(2y) + 2(2y)(4z) + 2(4z)(x)$

$= x^2 + 4y^2 + 16z^2 + 4xy + 16yz + 8zx$

(*ii*) $(2x - y + z)^2$

$= [2x + (-y) + z]^2$

$= (2x)^2 + (-y)^2 + z^2 + 2(2x)(-y) + 2(-y)(z) + 2(z)(2x)$

$= 4x^2 + y^2 + z^2 - 4xy - 2yz + 4zx$

(*iii*) $(-2x + 3y + 2z)^2$

$= [(-2x) + 3y + 2z]^2$

$= (-2x)^2 + (3y)^2 + (2z)^2 + 2(-2x)(3y) + 2(3y)(2z) + 2(2z)(-2x)$

$= 4x^2 + 9y^2 + 4z^2 - 12xy + 12yz - 8zx$

(*iv*) $(3a - 7b - c)^2$

$= [3a + (-7b) + (-c)]^2$

$= (3a)^2 + (-7b)^2 + (-c)^2 + 2(3a)(-7b) + 2(-7b)(-c) + 2(-c)(3a)$

$= 9a^2 + 49b^2 + c^2 - 42ab + 14bc - 6ca$

(*v*) $(-2x + 5y - 3z)^2$

$= [(-2x)^2 + 5y + (-3z)]^2$

$= (-2x)^2 + (5y)^2 + (-3z)^2 + 2(-2x)(5y) + 2(5y)(-3z) + 2(-3z)(-2x)$

$= 4x^2 + 25y^2 + 9z^2 - 20xy - 30yz + 12zx$

(vi) $\left[\frac{1}{4}a - \frac{1}{2}b + 1\right]^2$

$= \left[\frac{1}{4}a + \left(-\frac{1}{2}b\right) + 1\right]^2$

$= \left(\frac{1}{4}a\right)^2 + \left(-\frac{1}{2}b\right)^2 + (1)^2 + 2\left(\frac{1}{4}a\right)\left(-\frac{1}{2}b\right)$

$+ 2\left(-\frac{1}{2}b\right)(1) + 2(1)\left(\frac{1}{4}a\right)$

$= \frac{1}{16}a^2 + \frac{1}{4}b^2 + 1 - \frac{1}{4}ab - b + \frac{1}{2}a$

5. Factorise :

(i) $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$

(ii) $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz$

Sol. (i) $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$

$= (2x)^2 + (3y)^2 + (-4z)^2 + 2(2x)(3y) + 2(3y)(-4z) + 2(2x)(-4z)$

$= [2x + 3y + (-4z)]^2 = (2x + 3y - 4z)^2$

(ii) $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz$

$= (\sqrt{2}x)^2 + (-y)^2 + (-2\sqrt{2}z)^2 + 2(\sqrt{2}x)(-y)$

$+ 2(-y)(-2\sqrt{2}z) + 2(\sqrt{2}x)(-2\sqrt{2}z)$

$= [\sqrt{2}x + (-y) + (-2\sqrt{2}z)]^2 = (\sqrt{2}x - y - 2\sqrt{z})^2$

6. Write the following cubes in expanded form :

(i) $(2x + 1)^3$ **(ii) $(2a - 3b)^3$**

(iii) $\left[\frac{3}{2}x + 1\right]^3$ **(iv) $\left[x - \frac{2}{3}y\right]^3$**

Sol. (i) $(2x + 1)^3 = (2x)^3 + 3(2x)^2(1) + 3(2x)(1)^2 + (1)^3$

$= 8x^3 + 12x^2 + 6x + 1$

(ii) $(2a - 3b)^3 = (2a)^3 - 3(2a)^2(3b) + 3(2a)(3b)^2 - (3b)^3$

$= 8a^3 - 36a^2b + 54ab^2 - 27b^3$

(iii) $\left[\frac{3}{2}x + 1\right]^3 = \left(\frac{3}{2}x\right)^3 + 3\left(\frac{3}{2}x\right)^2(1) + 3\left(\frac{3}{2}x\right)(1)^2 + 1^3$

$= \frac{27}{8}x^3 + \frac{27}{4}x^2 + \frac{9}{2}x + 1$

(iv) $\left[x - \frac{2}{3}y\right]^3 = x^3 - 3(x)^2\left(\frac{2}{3}y\right) + 3(x)\left(\frac{2}{3}y\right)^2 - \left(\frac{2}{3}y\right)^3$

$= x^3 - 2x^2y + \frac{4}{3}xy^2 - \frac{8}{27}y^3$

7. Evaluate the following using suitable identities :

(i) $(99)^3$ **(ii) $(102)^3$** **(iii) $(998)^3$**

Sol. (i) $(99)^3 = (100 - 1)^3$

$= (100)^3 - 1^3 - 3(100)(1)(100 - 1)$

$= 1000000 - 1 - 29700 = 970299$

(ii) $(102)^3 = (100 + 2)^3$

$= (100)^3 + (2)^3 + 3(100)(2)(100 + 2)$

$= 1000000 + 8 + 61200 = 1061208$

(iii) $(998)^3 = (1000 - 2)^3$

$= (1000)^3 - (2)^3 - 3(1000)(2)(1000 - 2)$

$= 1000000000 - 8 - 5988000$

$= 994011992$

8. Factorise each of the following :

(i) $8a^3 + b^3 + 12a^2b + 6ab^2$

(ii) $8a^3 - b^3 - 12a^2b + 6ab^2$

(iii) $27 - 125a^3 - 135a + 225a^2$

(iv) $64a^3 - 27b^3 - 144a^2b + 108ab^2$

(v) $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$

Sol. (*i*) $8a^3 + b^3 + 12a^2b + 6ab^2$

$= (2a)^3 + (b)^3 + 3(2a)(b)(2a + b)$

$= (2a + b)^3$

$= (2a + b)(2a + b)(2a + b)$

(*ii*) $8a^3 - b^3 - 12a^2b + 6ab^2$

$= (2a)^3 - b^3 - 3(2a)(b)(2a - b)$

$= (2a - b)^3$

$= (2a - b)(2a - b)(2a - b)$

(*iii*) $27 - 125a^3 - 135a + 225a^2$

$= (3)^3 - (5a)^3 - 3(3)(5a)(3 - 5a)$

$= (3 - 5a)^3$

$= (3 - 5a)(3 - 5a)(3 - 5a)$

(*iv*) $64a^3 - 27b^3 - 144a^2b + 108ab^2$

$= (4a)^3 - (3b)^3 - 3(4a)(3b)(4a - 3b)$

$= (4a - 3b)^3$

$= (4a - 3b)(4a - 3b)(4a - 3b)$

(*v*) $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$

$= (3p)^3 - \left(\frac{1}{6}\right)^3 - 3(3p)\left(\frac{1}{6}\right)\left(3p - \frac{1}{6}\right)$

$= \left(3p - \frac{1}{6}\right)^3 = \left(3p - \frac{1}{6}\right)\left(3p - \frac{1}{6}\right)\left(3p - \frac{1}{6}\right)$

9. Verify : (*i*) $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

(*ii*) $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

Sol. (*i*) L.H.S. $= (x + y)(x^2 - xy + y^2)$

$= x(x^2 - xy + y^2) + y(x^2 - xy + y^2)$

$= x^3 - x^2y + xy^2 + x^2y - xy^2 + y^3$

$= x^3 + y^3 =$ R.H.S.

Thus, verified.

(*ii*) L.H.S. $= (x - y)(x^2 + xy + y^2)$

$= x(x^2 + xy + y^2) - y(x^2 + xy + y^2)$

$= x^3 + x^2y + xy^2 - x^2y - xy^2 - y^3$

$= x^3 - y^3 =$ R.H.S.

Thus, verified.

10. Factorise each of the following :

(*i*) $27y^3 + 125z^3$ **(*ii*) $64m^3 - 343n^3$**

Sol. (*i*) $27y^3 + 125z^3 = (3y)^3 + (5z)^3$

$= (3y + 5z)[(3y)^2 - (3y)5z) + (5z)^2]$

$= (3y + 5z)(9y^2 - 15yz + 25z^2)$

(*b*) $64m^3 - 343n^3 = (4m)^3 - (7n)^3$

$= (4m - 7n)[(4m)^2 + (4m)(7n) + (7n)^2]$

$= (4m - 7n)(16m^2 + 28mn + 49n^2)$

11. Factorise : $27x^3 + y^3 + z^3 - 9xyz$

Sol. $27x^3 + y^3 + z^3 - 9xyz$

$= (3x)^3 + y^3 + z^3 - 3(3x)(y)(z)$

$= (3x + y + z)[(3x)^2 + y^2 + z^2 - (3x)y - yz - z(3x)]$

$= (3x + y + z)(9x^2 + y^2 + z^2 - 3xy - yz - 3zx)$

12. Verify that $x^3 + y^3 + z^3 - 3xyz = \frac{1}{2}(x + y + z)[(x - y)^2 + (y - z)^2 + (z - x)^2]$

Sol. L.H.S. $= \frac{1}{2}(x + y + z)[(x - y)^2 + (y - z)^2 + (z - x)^2]$

$= \frac{1}{2}(x + y + z)(x^2 - 2xy + y^2 + y^2 - 2yz$

$+ z^2 + z^2 - 2zx + x^2)$

$= (x + y + z)(x^2 + y^2 + z^2 - yz - zx - xy)$

$= x^3 + y^3 + z^3 - 3xyz$

$=$ R.H.S.

Hence verified.

13. If $x + y + z = 0$, show that $x^3 + y^3 + z^3 = 3xyz$.

Sol. We have, $\quad x + y + z = 0$

$$\Rightarrow \quad x + y = -z$$

Cubing both sides, we have

$$(x + y)^3 = (-z)^3$$

$$\Rightarrow \quad x^3 + y^3 + 3xy(x + y) = -z^3$$

$$\Rightarrow \quad x^3 + y^3 - 3xyz = -z^3 \qquad [\because x + y = -z]$$

$$\Rightarrow \quad x^3 + y^3 + z^3 = 3xyz, \text{ which stands proved.}$$

14. Without actually calculating the cubes, find the value of each of the following :

(*i*) $(-12)^3 + (7)^3 + (5)^3$ (*ii*) $(28)^3 + (-15)^3 + (-13)^3$

Sol. (*i*) Let $x = -12$, $y = 7$ and $z = 5$

Here, $\quad x + y + z = -12 + 7 + 5 = 6$

$$\Rightarrow \quad x^3 + y^3 + z^3 = 3xyz$$

$$\Rightarrow \quad (-12)^3 + (7)^3 + (5)^3 = 3 \times -12 \times 7 \times 5$$

$$= -1260$$

(*ii*) Let $x = 28$, $y = -15$ and $z = -13$

Here, $\quad x + y + z = 28 - 15 - 13 = 0$

$$\Rightarrow \quad x^3 + y^3 + z^3 = 3xyz$$

$$\Rightarrow \quad (28)^3 + (-15)^3 + (-13)^3 = 3(28)(-15)(-13)$$

$$= 16380$$

15. Give possible expressions for the length and breadth of each of the following rectangles, in which their areas are given

Area : $25a^2 - 35a + 12$	Area : $35y^2 + 13y - 12$
(*i*)	(*ii*)

Sol. Possible length and breadth of the rectangle are the factors of its given area.

(*i*) $\quad$ Area $= 25a^2 - 35a + 12 = 25a^2 - 15a - 20a + 12$

$$= 5a(5a - 3) - 4(5a - 3) = (5a - 3)(5a - 4)$$

∴ Possible length and breadth are $(5a - 3)$ and $(5a - 4)$ units.

(*ii*)

$$\begin{aligned} \text{Area} &= 35y^2 + 13y - 12 \\ &= 35y^2 + 28y - 15y - 12 \\ &= 7y(5y + 4) - 3(5y + 4) \\ &= (5y + 4)(7y - 3) \end{aligned}$$

∴ Possible length and breadth are $(5y + 4)$ and $(7y - 3)$ units.

16. What are the possible expressions for the dimensions of the cuboids whose volumes are given below :

Volume : $3x^2 - 12x$	**Volume : $12ky^2 + 8ky - 20k$**
(*i*)	**(*ii*)**

Sol. Possible expressions for the dimensions of the cuboids are the factors of their volumes.

(*i*)

$$\text{Volume} = 3x^2 - 12x = 3x(x - 4)$$

∴ Possible dimensions of cuboid are 3, x and $(x - 4)$ units.

(*ii*)

$$\begin{aligned} \text{Volume} &= 12ky^2 + 8ky - 20k = 4k(3y^2 + 2y - 5) \\ &= 4k(3y^2 - 3y + 5y - 5) \\ &= 4k[3y(y - 1) + 5(y - 1)] \\ &= 4k(y - 1)(3y + 5) \end{aligned}$$

∴ Possible dimensions of cuboid are

$4k$, $(y - 1)$ and $(3y + 5)$ units.