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GRAVITATION

Unit: - 6

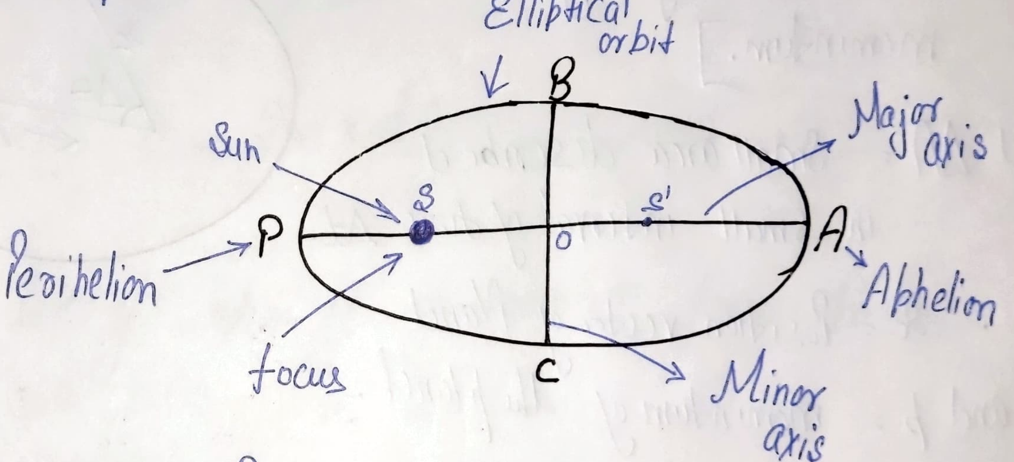
CLASS: - 11TH
PHYSICS
Education source

GRAVITATION ①

Kepler's laws of Planetary Motion:-

1) Kepler's First law (Law of orbits):-

According to this law, all planets moves in elliptical orbits with the sun situated at one of the foci of the ellipse.



Perihelion:- The Closest Point to the Sun.

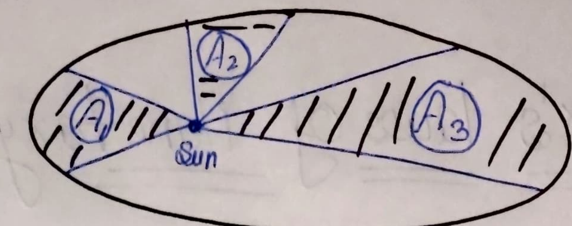
Aphelion:- The farthest Point to the sun.

2) Kebler's Second law (law of Area):-

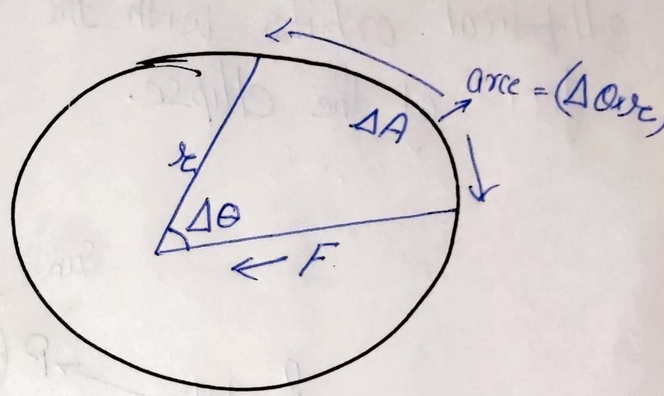
According to this law, the line joining a planet to the sun sweeps out equal areas in equal interval of time, i.e. the areal velocity of the planet around the sun is Constant.

The area A_1, A_2, A_3 are swept out by the Radius vector in equal interval of time

ie $\left[A_1 = A_2 = A_3\right]$



[This law is based on law of Conservation of angular momentum.]



Let ΔA = Small area described in Small interval of time Δt.

r = Position vector of Planet

and p = momentum of the planet.

Now

$$\Delta A = \frac{1}{2} r (r \Delta \theta)$$

dividing by Δt both side

$$\frac{\Delta A}{\Delta t} = \frac{1}{2} r^2 \frac{\Delta \theta}{\Delta t}$$

Limit $\Delta t \to 0$

$$\lim_{\Delta t \to 0} \frac{\Delta A}{\Delta t} = \frac{1}{2} r^2 \left(\lim_{\Delta t \to 0} \frac{\Delta \theta}{\Delta t} \right) \quad \left[\because \frac{d\theta}{dt} = \omega \right]$$

$$\frac{dA}{dt} = \frac{1}{2} r^2 \omega \quad \text{— (i)}$$

Instantaneous angular momentum

$$L = m r^2 \omega$$

$$r^2 \omega = \frac{L}{m} \quad \text{— (ii)}$$

using (ii) in (i)

$$\frac{dA}{dt} = \frac{1}{2} \frac{L}{m}$$

$$L = 2m \times \frac{dA}{dt} \quad \text{— (iii)}$$

Here the line of action of gravitational force passes through the axis ie $\theta = 180$

$$\tau_{ext} = r \times F = rF \sin\theta$$

$$\tau_{ext} = rF \sin(180)$$

$$\tau_{ext} = 0 \quad \text{— (IV)}$$

We know that

$$\frac{dL}{dt} = \tau_{ext} = 0$$

$$\frac{dL}{dt} = 0$$

Intigrating both side

$$L = \text{Constant} \quad \text{—— (V)}$$

from equation (III) and (V)

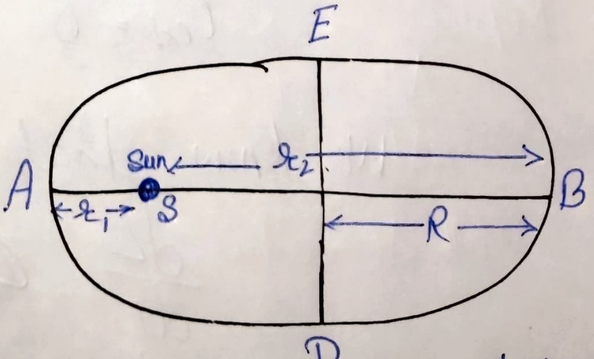
$$\boxed{\text{Areal velocity } \frac{dA}{dt} = \text{Constant}}$$

The Areal velocity of the planet around the sun is Constant.

3) Kepler's Third Law of Period :-

The Square of the time period of revolution of a planet around the Sun is directly proportional to the Cube of semi major axis of its elliptical orbit ie

$$T^2 \propto R^3 \quad \text{—— (1)}$$



$AB = AS + SB$

$2R = r_1 + r_2$

$R = \dfrac{r_1 + r_2}{2}$

using in (1)

$$T^2 \propto \left(\frac{r_1 + r_2}{2}\right)^3$$

[Here r_1 = Perigee (shortest distance
r_2 = (apogee)(longest distance)
of the Planet from the Sun.

Consider a planet
m = mass of Planet
M = Mass of sun
R = Radius of orbit.

Here

$$F = \frac{GMm}{R^2} \quad [\text{Gravitational force}] \text{ — (I)}$$

$$F = mR\omega^2 \quad [\text{Centrifugal force}] \text{ — (II)}$$

equating (I) and (II)

$$\frac{GMm}{R^2} = mR\omega^2 \quad \left[\omega = \frac{2\pi}{T}\right]$$

$$\frac{GM}{R^2} = R\left(\frac{4\pi^2}{T^2}\right)$$

$$GM = R^3 \frac{4\pi^2}{T^2}$$

$$T^2\left(\frac{GM}{4\pi^2}\right) = R^3 \quad \left[\frac{4\pi}{GM} = \text{Constant}\right]$$

$$T^2 = \left(\frac{4\pi^2}{GM}\right) R^3$$

$$\boxed{T^2 \propto R^3}$$

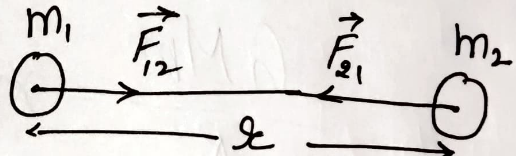
⇒ The Universal law of Gravitation OR Newton's law of Gravitation :-

Gravitation is the name given to the force of attraction between any two bodies of the universe.

or

Fone of Gravitation states that the force of attraction between the masses ~~of~~ is

Let m_1 and m_2 be the masses of two bodies and (r) be the saparation between them.



According to the law

$$F \propto m_1 m_2 \quad \text{—} \ (\text{I})$$

$$F \propto \frac{1}{r^2} \quad \text{—} \ (\text{II})$$

from (I) and (II)

$$F \propto \frac{m_1 m_2}{r^2}$$

$$F = G\frac{m_1 m_2}{r^2}$$

G = Gravitational Constant (6.67×10^{-8} dyne $cm^2 g^{-2}$)
(6.67×10^{-11} $N m^2 kg^{-2}$)

SI Unit and dimensional formula :-

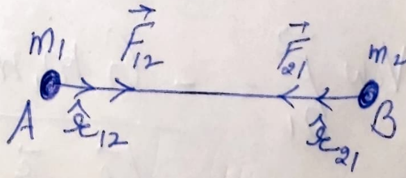
$$G = \frac{Fr^2}{m_1 m_2} = \frac{[MLT^{-2}][L^2]}{[M][M]}$$

$$= [M^{-1}L^3T^{-2}]$$

S.I Unit :- Nm^2kg^{-2}

⇒ **Vector form of Newton's law of Gravitation :-**

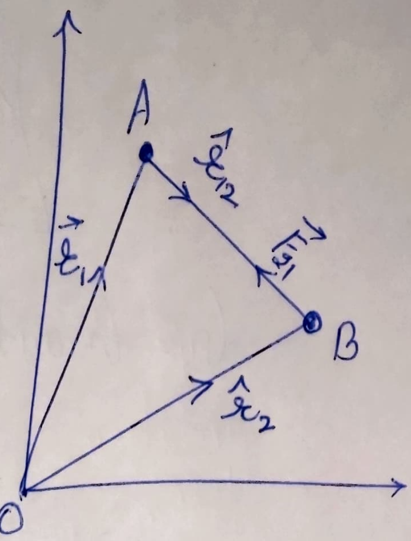
Let m_1 and m_2 masses of two objects placed at a distance r.



Let $\hat{r}_{12}$ = Unit vector from A to B
$\hat{r}_{21}$ = Unit vector from B to A
$\vec{F}_{12}$ = gravitational force on body A by body B
$\vec{F}_{21}$ = gravitational force on body B by body A.

According Newton's law

$$\vec{F}_{12} = -G \frac{m_1 m_2}{r^2} \hat{r}_{21}$$



(-ve) sign indicate the direction of $\vec{F}_{12}$ opposite to the direction of $\hat{r}_{21}$

and $r = |\vec{r}_{12}| = |\vec{r}_{21}|$

$$\vec{F}_{12} = -\frac{G m_1 m_2}{|\vec{r}_{21}|^2} \hat{r}_{21}$$

||rly $$\vec{F}_{21} = -\frac{G m_1 m_2}{|\vec{r}_{12}|^2} \hat{r}_{12}$$

Here $\vec{r}_{12} = \vec{r}_2 - \vec{r}_1$

[acc. to vector law
$\vec{OA} + \vec{AB} = \vec{OB}$
an $\vec{r}_1 + \vec{AB} = \vec{r}_2$
$\vec{AB} = \vec{r}_2 - \vec{r}_1$
ie $\vec{r}_{12} = \vec{r}_2 - \vec{r}_1$]

Now $$\vec{F}_{21} = -\frac{G m_1 m_2}{|\vec{r}_2 - \vec{r}_1|^2} \hat{r}_{12}$$

$$\boxed{\vec{F}_{21} = -\frac{G m_1 m_2}{|\vec{r}_2 - \vec{r}_1|^2} (\vec{r}_2 - \vec{r}_1)}$$

⇒ Acceleration Due to Gravity :-

Acceleration due to gravity is defined as the force of gravity acting on unit mass of a body placed on or near the surface of earth.

if a force (F) acts on a body of mass (m) produce an acceleration "a"

ie $F = ma$

if the force on body due to gravity of earth, then acceleration in body is known as acceleration due to gravity.

ie $F = mg$

$$g = F/m$$

if $(m = 1)$

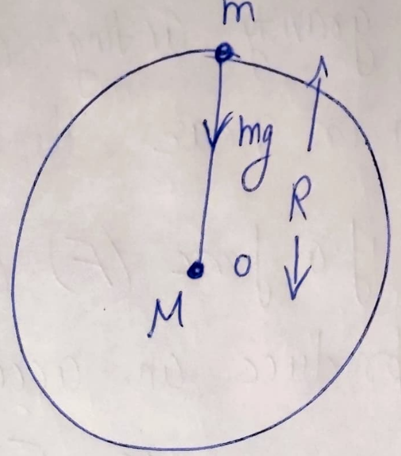
$$\boxed{g = F}$$

S.I Unit :- m/s^2

dimension formula: $[M^0 L T^{-2}]$

Relation between g and G :-

According Newton's law of Gravitation



$$F = \frac{GMm}{R^2} \quad \text{—(I)}$$

gravity pull :- $F = mg$ —(II)

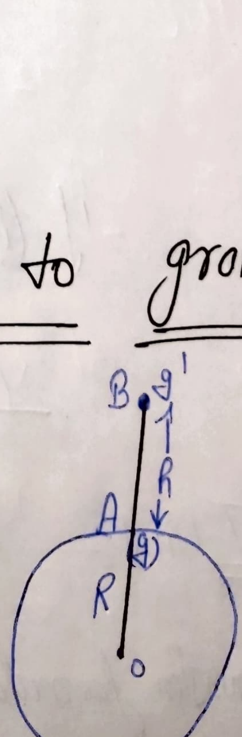
equating (I) and (II)

$$mg = \frac{GMm}{R^2}$$

$$\boxed{g = \frac{GM}{R^2}}$$

Variation of Acceleration due to gravity :-

a) Effect of Altitude :-



let R = Radius of earth
g = acceleration on surface of earth.

$$\therefore g = \frac{GM}{R^2} \quad \text{--- (I)}$$

if g' is the acceleration due to gravity at Point B, at a height h.

$$\therefore g' = \frac{GM}{(R+h)^2} \quad \text{--- (II)}$$

dividing (II) and (I)

$$\frac{g'}{g} = \frac{GM}{(R+h)^2} \Big/ \frac{GM}{R^2}$$

$$\frac{g'}{g} = \frac{GM}{(R+h)^2} \times \frac{R^2}{GM}$$

$$\frac{g'}{g} = \frac{R^2}{(R+h)^2} = \frac{R^2}{R^2\left(1+\frac{h}{R}\right)^2}$$

$$\frac{g'}{g} = \left(1+\frac{h}{R}\right)^{-2}$$

using Binomial theorm (neglecting heigher term $\because h << R$)

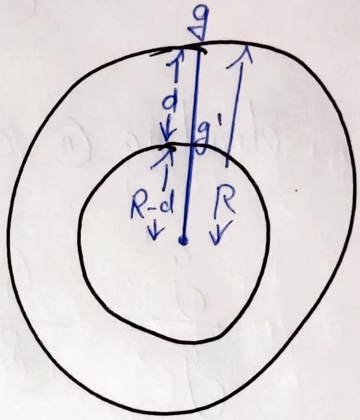
$$\frac{g'}{g} = 1 - \frac{2h}{R}$$

$$g' = g\left(1 - \frac{2h}{R}\right)$$

This equation shows that acceleration due to gravity decreases with height.

b) <u>Effect</u> of <u>depth</u> :-

acceleration due to gravity on the surface of earth



$$g = \frac{GM}{R^2} \quad \text{---} \text{ (I)}$$

if ρ is the density of material of the earth

$$\rho = \frac{M}{V}$$

$$M = \rho V = \rho\left(\frac{4}{3}\pi R^3\right)$$

using in (I)

$$g = \frac{G}{R^2}\rho\left(\frac{4}{3}\pi R^3\right)$$

$$g = G\rho\left(\frac{4}{3}\pi R\right) \quad \text{---} \text{ (II)}$$

let g' be the acceleration due to gravity inside the earth at depth (d)

$$g' = \frac{GM}{(R-d)^2}$$

IIrly $M' = \rho' V = \rho \frac{4}{3}\pi (R-d)^3$

$$g' = \frac{G}{(R-d)^2}\left(\rho \frac{4}{3}\pi (R-d)^3\right)$$

$$g' = \frac{4}{3} G\rho\pi (R-d) \quad \text{—— (III)}$$

dividing (III) and (II)

$$\frac{g'}{g} = \frac{\frac{4}{3} G\rho \pi (R-d)}{\frac{4}{3}\rho G \pi R}$$

$$\frac{g'}{g} = \frac{R-d}{R} = \frac{R}{R} - \frac{d}{R}$$

$$\boxed{g' = g\left(1 - \frac{d}{R}\right)}$$

Acc. due to gravity decreases with depth.

At the Centre of the earth :-

$$(d = R)$$

$$g_0 = g\left(1 - \frac{R}{R}\right)$$

$$g_0 = g(1-1)$$

$$\boxed{g_0 = 0}$$

Acceleration due to gravity zero at the Centre of the earth.

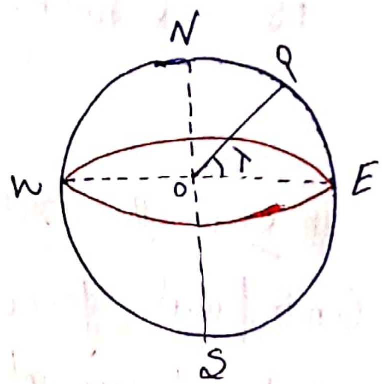
Gravitation

Effect of Latitude (Due to Rotational of earth about its own Axis) :-

Latitude at a place is defined as the angle which the line joining the place to the centre of the earth makes with the equatorial plane. And It is denoted by (λ)

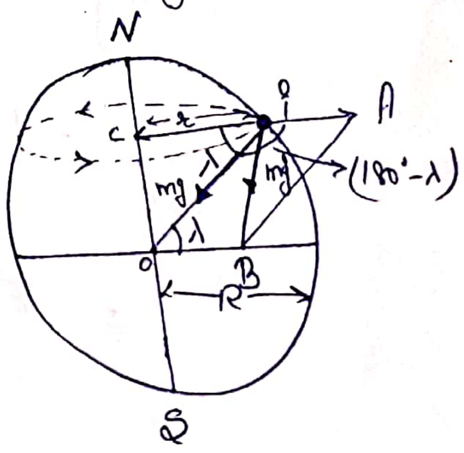
ie $\angle POE = \lambda$

if $\lambda = 90$ at the pole
and $\lambda = 0$ at equator.



Consider a earth of mass (M) and Radius (R) with centre (O). The whole mass is concentrated at the point (O). As the earth is Rotating (W to E). west to east.

Consider a particle of mass (m) at point (P) of latitude (λ). If the earth is Rotating about its fixed axis (N S) with constant angular velocity (ω), the particle (P) also Rotate and describes a horizontal circle of Radius (r).



ie $PC = r = OP\cos\lambda = R\cos\lambda$.

$\therefore$ The Centrifugal force acting on the particle at P is $(m r \omega^2)$.

and it act along (PA). directed away from the Centre (C) of the circle of Rotation.

(1) Suppose (g) is the acc. due to gravity when earth is stoped ie weight of the body

$(W = mg)$

(2) Suppose (g') is the acc. due to gravity when earth is Rotating ie weight of the body

$(W = mg')$

This is the resultant of the true weight and Centrifugal force acting on the particle at (P). and represented by diagonal POBA

Now using Parallogram law of Vector addition

$$mg' = \sqrt{(mg)^2 + (mr\omega^2)^2 + 2mg(mr\omega^2)\cos(180-\lambda)}$$

$$g' = \sqrt{g^2 + r^2\omega^4 \mp 2gr\omega^2\cos\lambda}$$

$$= \sqrt{g^2 + R^2\cos^2\lambda\,\omega^4 \mp 2gR\cos\lambda\,\omega^2\cos\lambda}$$

$$= \sqrt{g^2 + R^2\cos^2\lambda\,\omega^4 - 2gR\omega^2\cos^2\lambda}$$

$$= \sqrt{g^2 + R^2\omega^4\cos^2\lambda - 2Rg\omega^2\cos^2\lambda}$$

$$g' = g\left(1 + \frac{R^2\omega^4}{g^2}\cos^2\lambda - \frac{2R\omega^2\cos^2\lambda}{g}\right)^{1/2}$$ (1)

Here $R = 6.4\times10^6\,m$

$g = 9.8\,ms^{-2}$

$\omega = \frac{2\pi}{T} = \frac{2\pi}{24\times60\times60}\,rad\,s^{-1}$

$$\frac{R\omega^2}{g} = 6.4\times10^6 \times \left(\frac{2\pi}{24\times60\times60}\right)^2 \times \frac{1}{9.8}$$

$$= \frac{1}{289}$$

$\therefore \frac{R\omega^2}{g}$ is very small so that we can neglected square and higher ~~val~~ powers

$$g' = g\left(1 - \frac{2R\omega^2\cos^2\lambda}{g}\right)^{1/2}$$

applying Binomial theorem

$$g' = g\left(1 - \frac{1}{2}\times\frac{2R\omega^2\cos^2\lambda}{g}\right)$$

$$g' = g\left(1 - \frac{R\omega^2\cos^2\lambda}{g}\right)$$

$$\boxed{g' = g - R\omega^2\cos^2\lambda}$$

Here ω and λ is (+ve) so that $g' < g$.

∴ acceleration due to gravity

(i) decreases on account of Rotation of earth.

(ii) Increase with the increase in latitude of the place.

Case:- (i) At equator ($\lambda = 0$)

$$\cos\lambda = \cos(0) = 1$$

$$g' = g_e$$

$$g_e = g - R\omega \quad (\text{minimum})$$

(ii) at Poles ($\lambda = 90$)

$$\cos 90 = 0$$

$$g_p = g'$$

$$(g_P = g) \quad (\text{maximum})$$

∴ The value of acceleration due to gravity is maximum at the pole and remain unchanged whether the earth is Rotating or at rest.

Thus the effect of Rotation of the earth on the value of acc. due to gravity is maximum at the equator and minimum at the pole.

Gravitational field

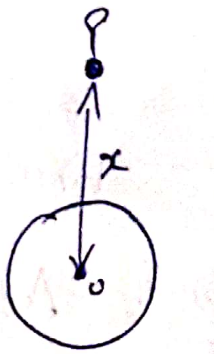
According to the Newton's law of gravitation, the force of attraction varies inversely as the square of the distance of the body from the centre of the earth. This show that in the space all around the earth, its gravitational pull can be experienced by other material bodies.

Hence the space around a material body in which its gravitational pull can be experienced is called its gravitational field.

Intensity of Gravitation field:

The intensity of the gravitational field of a body at a point in a field is defined as the force experienced by a body of unit mass placed at that point provided the presence of unit mass does not disturb the original gravitational field.

Let M be the mass of body (earth) and (m_0) be the mass of the particle at point (P) at a distance (x) from the centre of the earth.



Now acc. to the Newton's second law

$$F = G\frac{Mm_0}{x^2} \quad \text{---} (1)$$

Intensity of gravitational field at Point (P)

$$I = \frac{F_0}{m_0} = \frac{GMm_0/x^2}{m_0} = \frac{GM}{x^2} \quad \text{--- (2)}$$

In vector form $\Rightarrow \vec{I} = -\frac{GM}{x^2}\hat{x}$ --- (2a)

Here (-ve) sign shows that the gravitational intensity is of attraction.

from above equation x increase, I decreases and at $(x = \infty)$, $(I = 0)$

If the test mass is free to move at point (P). it will be accelerated due to force of attraction (F)

ie $F = m_0 a \Rightarrow \left(F/m_0 = a\right)$ --- (3)

$\therefore$ from equation (2) and (3)

$$(a = I)$$

It means that the Intensity of gravitational field at a Point in the gravitational field is equal to the acceleration of the test mass placed at that Point.

if the Particle at the Surface of the earth

$(x = R)$

$$I = \frac{GM}{R^2} = g$$

where g is acc. due to gravity of earth.

Unit:- $N kg^{-1}$

dimensional formula = $I = \frac{F}{m} = \frac{[MLT^{-2}]}{[M]} = [M^0 L T^{-2}]$.

Gravitational Potential : (4)

Gravitational potential at a point in a gravitational field of a body is defined as the amount of work done in bringing a body of unit mass from infinity to that point without acceleration.

If W = amount of work done
m_0 = mass of the body

∴ Gravitational Potential at $\left[P = \frac{W}{m_0} \right]$

Gravitational Potential is scalar quantity. Since work done is scalar.

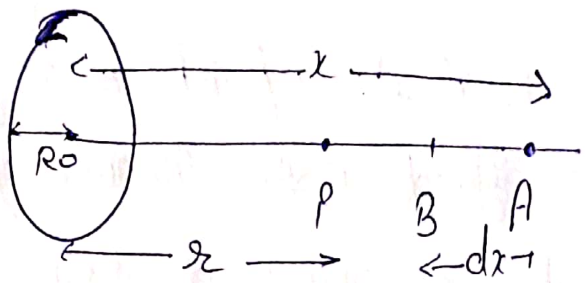
Unit :- $P = \frac{W}{M_0} = \frac{J}{kg} = J\,kg^{-1}$

Dimensional formula :- $G.P = \frac{W}{M_0} = \frac{[ML^2T^{-2}]}{[M]} = [M^0L^2T^{-2}]$

Expression for the gravitational Potential at a Point.

Let earth be a perfect sphere of Radius (R) and mass (M), supposed to be concentrated at its centre O.

Now let us Calculate the gravitational Potential at Point (P) ie OP = r and (r > R)



Take two points A and B on this line such that

$OA = x$ and $AB = dx$

The gravitational force of attraction on a body of unit mass at point A

$$F = \frac{GM \times 1}{x^2} = \frac{GM}{x^2}$$

Small amount work done in bringing the particle from point A to B (dx)

$$dw = F\,dx = \frac{GM\,dx}{x^2}$$

Now let us calculate the total work done bringing the body of unit mass from infinity to Point (P) ie $(x = \infty)$ to $(x = r)$

$$w = \int_{\infty}^{r} \frac{GM}{x^2}\,dx = GM \int_{\infty}^{r} (x^{-2})\,dx$$

$$-GM \left[\frac{x^{-1}}{-1} \right]_{\infty}^{r} = -GM \left[(r)^{-1} - (\infty)^{-1} \right]$$

$$= -GM \left[\frac{1}{r} - \frac{1}{\infty} \right] = -\frac{GM}{r}$$

$$w = -\frac{GM}{r}$$

This work done is the measure of gravitational potential at (P)

$$V_p = w = \frac{-GM}{r}$$

ie (i) the gravitational Potential is allways negative

(ii) when $(r = \infty)$ ie $(V_p) = 0$

(iii) at surface of earth. $(r = R)$ ∴ $V_p = \frac{-GM}{R}$

Gravitational Potential Energy:-

Gravitational Potential energy of a body at a Point in a gravitational field of a another body is defined as the amount of work done in bringing the given body from infinity to that point without accelerating.

Expression for gravitational Potential energy

Same as Previous expression :- we Consider point mass (m) as Compared to unit mass

The gravitational force of attraction on the body at A will be

$$F = \frac{GMm}{x^2} \quad \text{— (I)}$$

Small amount of work to bring a particle from point A to B.

$$dw = F\,dx = \frac{GMm}{x^2}\,dx \quad \text{— (II)}$$

Total work done in bringing the body from infinity to point (P)

$$W = \int_{\infty}^{r} \frac{GMm}{x^2}\,dx = GMm \int_{\infty}^{r} x^{-2}\,dx$$

$$= GMm \left[\frac{x^{-1}}{-1}\right]_{\infty}^{r} = -GMm\left[\frac{1}{r} - \frac{1}{\infty}\right]$$

$$W = -\frac{GMm}{r} \quad \text{— (III)}$$

Since this work done stored in the body as it gravitational potential energy (U)

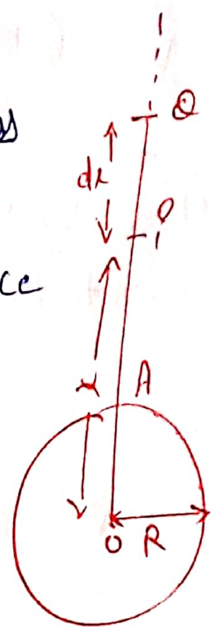
$$\text{ie } U = W = \frac{-GMm}{r} \quad \text{— (IV)}$$

$$\therefore \left(U = V_p \times m\right)$$

Gravitational potential energy = gravitational potential × mass of the body.

Escape Velocity.

Escape velocity on earth is defined as the minimum velocity with which the body has to be projected vertically upwards from the surface of earth so that it just cross the gravitational field of earth and never returns on its own.



Let us consider earth of Radius (R) Mass (M). Let a body of mass (m) to be projected from a point (A) on the surface of earth. Consider two Point (P) and (Q) at a distance (x) and (x+dx) from the centre.

Now Gravitational force of attraction on the body at (P) is

$$F = \frac{GMm}{x^2} \quad \text{---} (1)$$

Now work done in talking the body against gravitational attraction from P to Q.

$$dw = Fdx = \frac{GMm}{x^2} dx$$

Total work done in taking the body against gravitational attraction from surface of earth ie from $(x = R)$ to $(x = \infty)$

$$W = \int_R^\infty \frac{GMm}{x^2} dx = GMm \int_R^\infty x^{-2} dx$$

$$= GMm \left[\frac{x^{-2+1}}{-2+1} \right]_R^\infty = -GMm \left[\frac{1}{x} \right]_R^\infty$$

$$= -GMm \left[\frac{1}{\infty} - \frac{1}{R} \right] = \frac{GMm}{R}$$

$$\boxed{W = \frac{GMm}{R}} \quad \text{—— (II)}$$

This work done is at the cost of kinetic energy given to the body at the surface of earth ie (V_e)

$$\therefore K.E = \frac{1}{2} m V_e^2 \quad \text{—— (III)}$$

Now Comparing (II) and (III)

$$\frac{GMm}{R} = \frac{1}{2} m V_e^2$$

$$V_e^2 = \frac{2GMm}{Rm} \Rightarrow V_e^2 = \frac{2GM}{R}$$

$$\boxed{V_e = \sqrt{2GM/R}} \quad \text{—— (IVa)}$$

as we know $g = GM/R^2$

$$GM = gR^2$$

$$V_e = \sqrt{2gR^2/R} = \sqrt{2gR}$$

$$\boxed{V_e = \sqrt{2gR}} \quad \text{—— (IVb)}$$

Weightlessness :

Weightlessness is a situation in which the effective weight of the body become zero.

The body can be in a weightlessness state in the following circumstances.

(1) **When the body is taken at the Centre of the earth :**

The effective value of acc. due to gravity there is zero. So the effective weight of the body at the centre of the earth = $mg' = m \times 0 = 0$.

(2) When the body is taken at null points, the effective value of acceleration due to gravity there is zero. So, the effective weight of the body there is zero.

(3) When a body is lying in a freely falling lift, for which acceleration, $a = g$, the effective acc. due to gravity in the lift : $g' = g - a = g - g = 0$. So, effective weight of body in the freely falling lift is zero.

(4) When the body is inside a space craft or satellite which is orbiting around the earth, the gravitational pull on the body due to earth is counter balanced by the ~~Centrifut~~ centrifugal force on the body. Due to which the effective weight of the body becomes zero.

if (ρ) is the mean density of the material of earth
then
$$M = \frac{4}{3}\pi R^3 \rho$$
Put in equation (IV(b))
$$V_e = \sqrt{\frac{2}{R} G \times \frac{4}{3}\pi R^3 \rho}$$
$$= \sqrt{\frac{8\pi \rho G R^2}{3}} \quad \text{—(IV c)}$$

∴ equation (IV) (a, b, and c) give different expressions for the escape velocity of the body.

(I) The value of escape velocity of a body does not depend upon upon the mass of the body and its angle of projection from the surface of earth or planet.

(II) The value of escape velocity depends upon the mass and Radius of the planet from the surface of which the body is to be projected.

Note escape velocity of earth
$$g = 9.8\,ms^{-1}, \quad R = 6.4 \times 10^6\,m$$
$$V_e = \sqrt{2(9.8)(6.4)\times 10^6}$$
$$= \sqrt{125.44} \times 10^3$$
$$= 11.2 \times 10^3\,ms^{-1} = 11.2\,km\,s^{-1}$$

Earth satellite :

(1) **Orbital velocity**:- Orbital velocity of a satellite is the minimum velocity required to put the satellite into a given orbit around earth.

The orbital velocity is different for different orbits around earth and is independent of the mass of satellite.

Let Consider

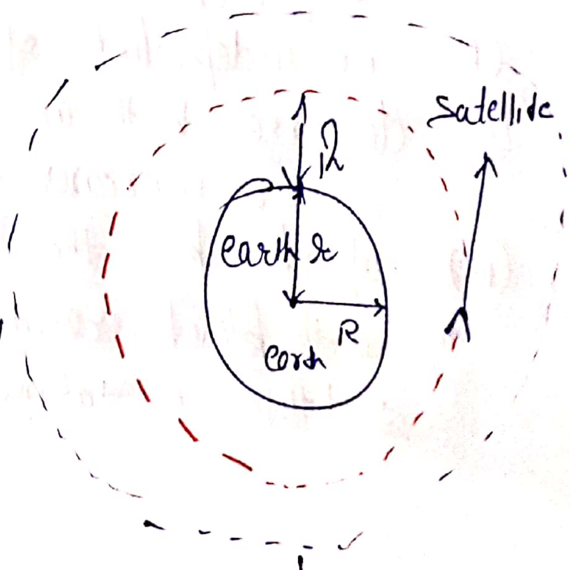
M = mass of earth
R = Radius of earth
m = mass of satellite
v = Orbital velocity of the satellite
h = height of satellite above the surface of earth

Now r = radius of the orbit of the satellite

$r = (R+h)$

Centripetal force required to keep the satellite

$$F = \frac{mv^2}{r} \quad \text{—(I)}$$



Acc. to Newton's gravitational pull acting on the satellite.

$$= \frac{GMm}{r^2} \quad \text{—(II)}$$

Comparing equation (I) and (II)

$$\frac{mv^2}{r} = \frac{GMm}{r^2}$$

$$v = \sqrt{\frac{GM}{r}} \quad \text{—— (III)}$$

if g is the value of acceleration due to gravity on the ~~required~~ surface of earth, then

$$g = GM/R^2$$

$$\text{or } GM = gR^2 \quad \text{—— (IV)}$$

Using in (III)

$$v = \sqrt{gR^2/r}$$

$$v = R\sqrt{g/r} \quad \left[\because r = R+h\right]$$

$$v = R\sqrt{g/(R+h)} \quad \text{—— (V)}$$

∴ from above equation ~~we~~ that orbital velocity of a satellite

(I) is independent of the mass of satellite.

(II) decreases with an increase in the Radius of orbit or increases in height of satellite

(III) depends upon the mass ~~of~~ and Radius ~~of~~ the earth/planet around which the revolution of satellite is taking place.

If the earth is supposed to be a sphere of mean density (ρ),

$$M = \frac{4}{3}\pi R^3 \rho$$

$$g = \frac{GM}{R^2} = \frac{G}{R^2}\left[\frac{4}{3}\pi R^3 \rho\right] = \frac{4\pi G R \rho}{3}$$

Substituting this value of (g) in (1)

$$T = \frac{2\pi}{R}\sqrt{\frac{3(R+h)^3}{4\pi G R \rho}}$$

$\because T = \frac{2\pi r}{\text{orbital velocity}}$ — $2\pi r$: circumference of orbit

$$= \sqrt{\frac{4\pi^2}{R^2} \times \frac{3}{4}\frac{(R+h)^3}{\pi G R \rho}}$$

$$= \sqrt{\frac{3\pi(R+h)^3}{G\rho R^3}} \quad \text{—— (II)}$$

for satellite orbiting close to the surface of earth, $h \ll R$

$$h + R \approx R$$

from equ (II) $T = \sqrt{\frac{3\pi R^3}{G\rho R^3}} = \sqrt{\frac{3\pi}{G\rho}}$

from eq (1) $T = \frac{2\pi}{R}\sqrt{\frac{R^3}{g}}$

$$= 2\pi\sqrt{\frac{R}{g}}$$

The direction of orbital velocity of a satellite at any instant is along the tangent to the orbital path of satellite at that instant.

When a satellite is orbiting very close to the surface of earth ($h \ll R$),

$$r = R + h \simeq R$$

$$v = V_0$$

$$V_0 = R\sqrt{g/R} = \sqrt{gR}$$

$$g = 9.8\ ms^{-2}, \quad R = 6.4 \times 10^6\ m$$

$$V_0 = \sqrt{9.8 \times 6.4 \times 10^6} = \sqrt{62.72} \times 10^3\ ms^{-1}$$

$$V_0 = 7.92\ km\ s^{-1}$$

(II) Time period of satellite :- It is the time taken by satellite to complete one revolution around the earth and is denoted by T.

$$T = \frac{\text{distance travelled in one revolution}}{\text{orbital velocity}}$$

$$T = \frac{2\pi r}{v}$$

$$T = \frac{2\pi r}{R}\sqrt{\frac{r}{g}} = \frac{2\pi}{R}\sqrt{\frac{r^3}{g}}$$

$$= \frac{2\pi}{R}\sqrt{\frac{(R+h)^3}{g}} \quad \text{--- (1)}$$

if $g = 9.8\ ms^{-1}$ and $R = 6.4 \times 10^6 m$

$$T = 2(3.14)\sqrt{\frac{6.4 \times 10^6}{9.8}}$$

$$T = 5.08 \times 10^3 \ sec$$

$$\boxed{T = 84.6 \ minutes}$$

(III) Altitude or Height of satellite above the earth's surface.

Squaring both side equation (1)

$$T^2 = \frac{4\pi^2 (R+h)^3}{R^2 g}$$

$$(R+h)^3 = \frac{T^2 R^2 g}{4\pi^2}$$

$$(R+h) = \left(\frac{T^2 R^2 g}{4\pi^2}\right)^{1/3}$$

$$\boxed{h = \left(\frac{T^2 R^2 g}{4\pi^2}\right)^{1/3} - R} \quad \text{—— (III)}$$

(IV) Angular momentum :-

When a satellite of mass (m) is orbiting with linear speed (v) on the orbital path of Radius (r) around the earth its angular momentum is

$$L = mvr = mr\sqrt{\frac{GM}{r}} = [m^2 GMr]^{1/2}$$

it is clear that angular momentum of a satellite depends on both, the mass of the satellite (m) and the mass of planet (M).

(v) Energy of satellite :- The Total mechanical energy of a satellite revolving around the earth is the sum of its Potential energy (U) and kinetic energy (K).

P.E. due the its position w.r.t earth

$$U = -\frac{GMm}{r}$$ (-ve due to attractive pull)

$$K.E = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{GM}{r}\right)$$

Total Mechanical energy

$$E = U + K$$

$$= -\frac{GMm}{r} + \frac{1}{2}\frac{GMm}{r}$$

$$E = -\frac{GMm}{2r}$$

if satellite is closed ie $r = R$

$$E = -\frac{GMm}{...}$$

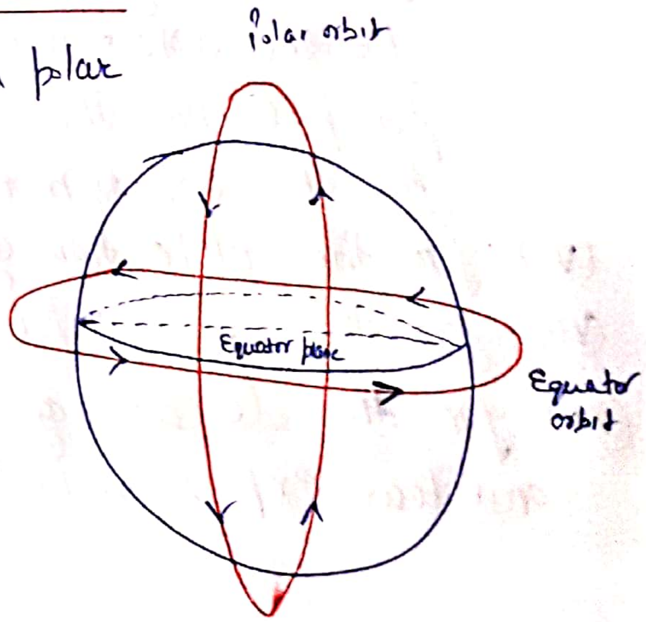
Binding energy of the satellite : The energy required to remove the satellite from its orbit around the earth to infinity is called Binding energy of the satellite.

Binding energy is equal to negative of total Mechanical energy

$$-E = -\left(-\frac{GMm}{2r}\right)$$

$$\boxed{-E = \frac{GMm}{2r}}$$

Polar Satellite :



(1) It is that satellite which revolve in polar orbit around earth.

(2) A Polar orbit is that orbit whose angle of inclination with equatorial plane of earth is 90° and the satellite in polar orbit will pass over both the north and south geographic poles once per orbit

(3) A single polar satellite can monitor 100% earth's surface. Every location on earth lies within the observation of polar satellite twice each day.

The Polar satellites are not used for Communication purposes but are used for remote sensing. That is why they are also known as remote sensing satellites.

The Polar satellites are used for getting the informa about the images of cloud, atmospheric data, ozone layer in the atmosphere and to detect the ozone hole over Antarctica.

India has already launched four remote sensing satellites: IRS-1A, IRS-1B, IRS-1C and IRS-1D.

In India the polar satellites are used (n)

(i) for the survey of ground water.
(ii) for detecting the area under forest.
(iii) for the assessnet of drought.
(iv) for preparing the waste land maps.
(v) for the assessment of crop diseases.
(vi) for the detection of good fishing zones in river/ocen.
(vii) for the spying purpose.
(viii) for the location of the place and time when any nuclear explosion is conducted.

Important formulas

1) Kepler's second law of Areas:-

Area velocity $\frac{dA}{dt}$ = Constant.

2) Kepler's ~~third~~ law of Period:-

$$T^2 \propto R^3$$

3) The universal law of Gravity:-

$$F = G\frac{m_1 m_2}{R^2}$$

4) Relation between acceleration due to gravity and gravitational force.

$$g = \frac{GM}{R^2}$$

5) Acceleration due to gravity above the surface of earth:-

$$g' = g\left(1 - \frac{2h}{R}\right) \text{ or } g\left(1 + \frac{h}{g}\right)^{-2}$$

⑥ Acceleration due to gravity below the Earth Surface.

$$g' = g\left(1 - \frac{d}{R}\right)$$

d → depth

R → Radius.

⑦ Acceleration due to gravity (Effect of latitude)

$$g' = g - R\omega^2 \cos^2\theta$$

⑧ **Gravitational Potential :-**

$$V = -\frac{GM}{r}$$

⑨ **Gravitational Potential Energy :-**

$$U = -\frac{GMm}{r}$$

Gravitational Potential energy = Gravitational Potential × mass of the body.

(10) **Escape velocity :-**

$$V_e = \sqrt{\frac{2gR^2}{R}} \text{ or } \sqrt{2gR} \text{ or } \sqrt{\frac{2GM}{R}}$$

(11) **Orbital speed :-**

$$v = \sqrt{\frac{GM}{r}}$$

(12) **Time period of satellite :-**

$$T = \frac{2\pi r}{v} = 2\pi\sqrt{R/g}$$

(13) **height of satellite :-**

$$(R+h)^3 = \frac{T^2 R^2 g}{4\pi^2}$$

$$\boxed{h = \left(\frac{T^2 R^2 g}{4\pi^2}\right)^{1/3} - R}$$

(14) **Angular Momentum (L) :-**

$$L = mvr = \sqrt{m^2 GMr}$$

(15) **Energy of an orbitting satellite :-**

$$E = -\frac{GMm}{2r} = -\frac{GMm}{2(R+h)}$$