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KINETIC THEORY

[Document subtitle]

CLASS: - 11TH

PHYSICS

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Chapter – 13
Kinetic Theory

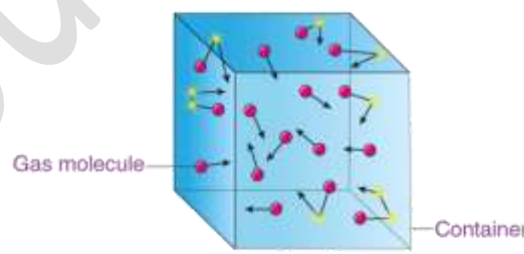
- **Kinetic theory of gases: -** The kinetic theory of gases provides a microscopic explanation for the macroscopic behaviour of gases. It's based on the premise that gases consist of a large number of tiny particles (atoms or molecules) that are in constant, random motion.

  Kinetic theory of gases is based on the molecular picture of matter. It correlates the macroscopic properties (e.g., pressure and temperature) of gases to microscopic properties (e.g., speed and kinetic energy) of gas molecules.

- **Assumptions of Kinetic Theory of Gases**

(a) A given amount of gas consists of a very large number of molecules (of the order of Avogadro's number 10^{23}) and all molecules are identical in all respect. The molecules are rigid, elastic, spheres identical in all respect for given gas.

(b) The molecules of a gas are in a state of random motion in all directions with different speeds, move freely in straight lines following Newton's first law.



(c) The size of a molecule is much smaller than the average separation or distance between the molecules. At ordinary pressure and temperature, the average distance between molecules is about 20 Å whereas size of a molecule is 2 Å.

(d) There are no intermolecular forces between molecules of gas except during collision.

(e) The collision between molecules among themselves or between molecules and walls are perfectly elastic (i.e., total momentum and total kinetic energy of molecules are conserved, however only their velocities will change).

(f) The duration of collision between two molecules is negligible as compared to time interval of two successive collisions, i.e., collisions are instantaneous.

(g) The density and the distribution of molecules is uniform throughout the gas.

(h) Between two collisions a molecule moves in a straight path with a uniform velocity. This average distance between two successive collisions is called **mean free path.**

Concept of Pressure:- The molecules of a gas are in a state of continuous random motion. They collid continuously against the wall of the container even at ordinary temperature.

Hence, the pressure exerted by the gas is due to the continuous bombardment of gas molecules against the walls of the container.

⇒ **Pressure of an Ideal Gas**:-

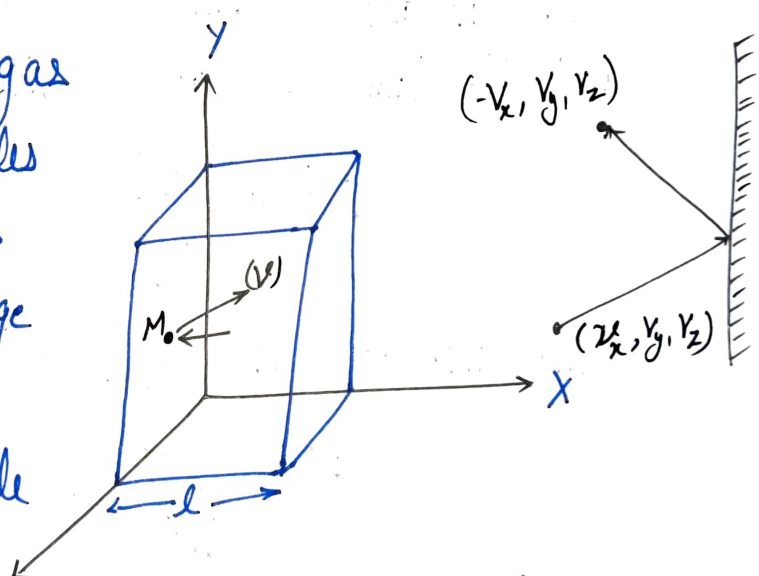
Consider an ideal gas

N = number of molecules

V = Volume of the gas

l = length of the edge of the container.

m = mass of molecule



(V_x, V_y, V_z) = be the Z velocity of the molecule in x, y and Z direction.

As molecule collides with the wall yz-plane elastically, its (x-component) of velocity reversed

ie $(-V_x, V_y, V_z)$

Change in momentum of the molecule is

$$\Delta P_x = (P_x)_{final} - (P_{(x)})_{initial}$$

$$\Delta P_x = (-mV_x) - (mV_x)$$

$$\Delta P_x = -2mV_x$$

Now Impulse = Change in momentum

$$I_x = F\Delta t = -2mV_x \quad \text{--- (1)}$$

Now if molecule collide with the same wall twice.

$\therefore$ distance travelled by the molecule is $(2l)$
and the time for two collision is

$$\Delta t = \frac{2l}{V_x}$$

The average force exerted on a molecule

$$F = \frac{I}{\Delta t} = \frac{-2mV_x}{\Delta t}$$

$$F = \frac{-2mV_x}{\frac{2l}{V_x}} = -\frac{mV_x^2}{l}$$

$$\boxed{F = -\frac{mV_x^2}{l}}$$

if there are (N) number of molecules collide with the wall of the container.

$$F_{wall} = \frac{m}{l}\left[V_{x_1}^2 + V_{x_2}^2 + \cdots + V_{x_N}^2\right]$$

multiplying and dividing by (N)

$$F_{wall} = \frac{Nm}{l}\left[\frac{V_{x_1}^2 + V_{x_2}^2 + \cdots + V_{x_N}^2}{N}\right]$$

$$F_{wall} = \frac{Nm}{l}\left(\bar{V}_x^2\right) \quad \text{— (II)} \quad \left[\because \bar{V}_x^2 = \frac{V_{x_1}^2 + V_{x_2}^2 + \cdots + V_{x_N}^2}{N}\right.$$

for (velocity in three coordinate)

$$\bar{V}^2 = \bar{V}_x^2 + \bar{V}_y^2 + \bar{V}_z^2 \quad \left[\bar{V}^2 = 3\bar{V}_x^2 \quad \because V_x = V_y = V_z\right]$$

$$F_{wall} = \frac{Nm}{l}\left(\frac{\bar{V}^2}{3}\right) \quad \left[\because V_x^2 = \frac{1}{3}\bar{V}^2\right]$$

$$F_{wall} = \frac{N}{3}\left(\frac{m}{l}\bar{V}^2\right) \quad \text{— (III)}$$

Here, F_{wall} is Normal force on the wall

Pressure on the wall

$$P = \frac{F_{wall}}{Area} = \frac{F_{wall}}{l^2}$$

$$P = \frac{1}{l^2}\left[\frac{Nm\bar{v}^2}{3\,l}\right]$$

$$P = \frac{1}{l^3}\left[\frac{Nm\bar{v}^2}{\cancel{3}}\right]$$

$$P = \frac{1}{V}\left[\frac{Nm\bar{v}^2}{3}\right] \qquad \left[\because l^3 = V\right]$$

$$P = \frac{1}{3}\left[\frac{N}{V}\right]m\bar{v}^2$$

$$\boxed{P = \frac{1}{3}nm\bar{v}^2} \qquad \left[\because n = \frac{N}{V}\right]$$

n = number density (no. of molecule per unit volume)

m = mass of molecule

$\bar{v}^2$ = squared mean speed

→ Kinetic gas equation.

Equation of State of a Perfect gas :-

Pressure of an ideal gas is given by

$$P = \frac{1}{3} \frac{mN}{V} \bar{v}^2$$

(multiplying and dividing by 2)

$$PV = \frac{2N}{3}\left(\frac{1}{2} m\bar{v}^2\right)$$

for ideal gas kinetic energy of gas directly protional to temperature.

$$\therefore \frac{1}{2} m\bar{v}^2 \propto T$$

$$\bar{v}^2 \propto T$$

$$\bar{v}^2 = KT$$

/ where K = Constant /

$$PV = \frac{2}{3} N\left(\frac{1}{2} mKT\right)$$

$$\therefore PV = RT$$

(dividing and multiplying by N)

$$PV = \frac{NRT}{N}$$

$$\left(\frac{R}{N} = k\right)$$

$$\boxed{PV = NkT}$$

$$\boxed{k = \text{Boltzmann's Constant}}$$

from above we conclude that

1) The shape of vessel is immaterial because area A and time interval (Δt) do not appear in the final result.

2) As per pascal's law, pressure in one portion of the gas in equilibrium is the same any where else.

Relation between Pressure Exerted by an Ideal Gas and its density :-

M = mass of all molecule in ideal gas in a vessel of volume (V)

n = number of molecules per unit volume

ρ = density of the gas.

$$n \times m = \frac{M}{V}$$ (m = mass of one molecule)

$$n \times m = \rho \quad \text{— (I)}$$ $\left[\rho = \frac{\text{mass}}{\text{Volume}}\right]$.

Pressure exerted by ideal gas

$$P = \frac{1}{3} n m \bar{v}^2 \quad \text{— (II)}$$

from equation ① and ⑪

$$\boxed{P = \frac{1}{3}\rho\bar{v}^2}$$

Relation between Pressure and K.E. per Unit Volume :-

Pressure exerted by a gas of density ρ.

$$P = \frac{1}{3}\rho v^2 \text{ ——— ①}$$

Average K.E. of translations per unit volume

$$E = \frac{1}{2}\rho v^2 \text{ — ⑪} \quad \left[\because \rho = \frac{m}{V}\right]$$

dividing equation ① and ⑪

$$\frac{P}{E} = \frac{\frac{1}{3}\cancel{\rho}\cancel{v^2}}{\frac{1}{2}\cancel{\rho}\cancel{v^2}}$$

$$\frac{P}{E} = \frac{2}{3}$$

$$\boxed{P = \frac{2}{3}E}$$

Pressure $= \frac{2}{3}$ (Average K.E. per unit Volume)

Kinetic Interpretation of Temprature :-

The average of a molecule depends on the absolute temperature of the gas.

Pressure exerted by the 1 mole of ideal gas

$$P = \frac{1}{3} \frac{M}{V} (V_{rms})^2$$

$$(V_{rms})^2 = \frac{3PV}{M}$$

we have $PV = RT$

$$(V_{rms})^2 = \frac{3RT}{M}$$

$$\frac{1}{3} M V_{rms}^2 = RT$$

dividing by (N_0) both side

$$\frac{1}{3} \frac{M V_{rms}^2}{N_0} = \frac{RT}{N_0}$$

$$\left[\frac{M}{N_0} = m , \frac{R}{N_0} = k_B \right]$$

$$\frac{1}{3} m V_{rms}^2 = k_B T$$

$$m V_{rms}^2 = 3 \cdot k_B T$$

dividing by (2) both side

$$\left[\frac{1}{2} m v_{rms}^2 = \frac{3}{2} K_B T\right]$$

m = mass of each molecule

$$(KE)_T = \frac{3}{2} K_B T$$

↳ Translational kinetic energy associated with each other.

Root mean square speed of gas molecule :-

It is defined as the mean of the squares of the random velocities of the Individual molecules of a gas.

From kinetic Interpretation

$$\frac{1}{2} m \bar{v}^2 = \frac{3}{2} K_B T$$

$$\bar{v}^2 = \frac{3 K_B T}{m}$$

$$\boxed{\bar{v} = \sqrt{\frac{3 K_B T}{m}}}$$

m = mass of one molecule

K_B = Boltzmann Constant

T = absolute temperature.

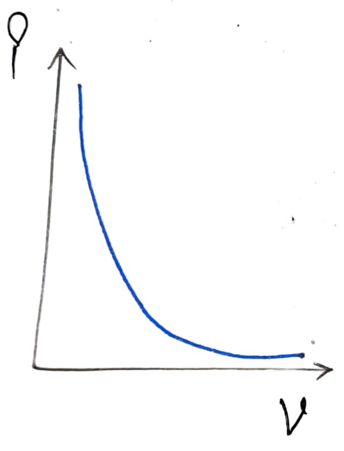
$$\boxed{v_{rms} \propto \sqrt{T}}$$

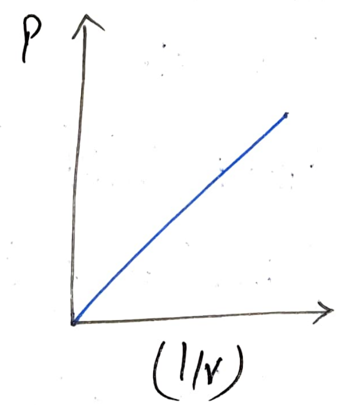
Hence faster the motion of molecules of a gas, higher will be their kinetic energy and higher will be the temperature of the gas.

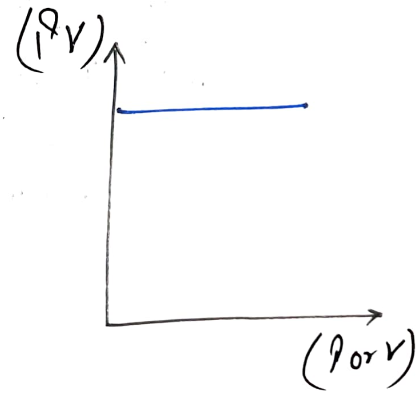
⇒ **GAS laws from Kinetic theory :-**

(1) **Boyle's law** :- It states that for a given mass of an ideal gas at constant temperature the volume of a gas is inversely proportional to its pressure.

$$V \propto \frac{1}{P} \quad [T = \text{Constant}]$$



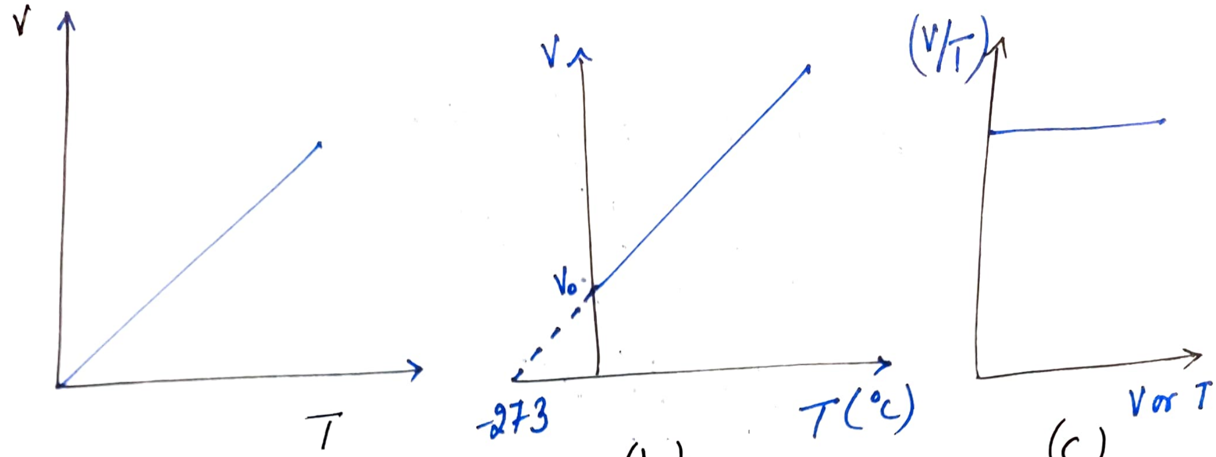




(2) **Charles' law** :- It states that for a given mass of an ideal gas at constant pressure volume of a gas directly proportional to the temperature.

$$[V \propto T]$$

$$\left[\frac{V}{T} = \text{Constant}\right]$$



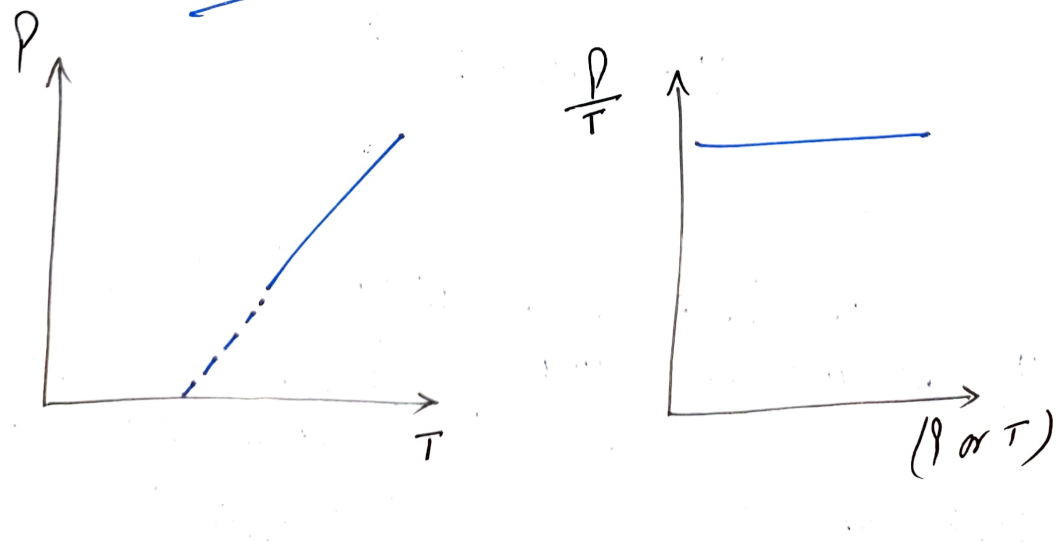
(a) (b) (c)

3) Gray - Lussac's law or pressure law :-

It states that for a given mass of an ideal gas at Constant volume pressure of a gas is directly proportional to the absolute temperature.

$$P \propto T$$

$$\boxed{\frac{P}{T} = \text{Constant}}$$



(4) <u>Perfect gas equation</u> :-

According kinetic theory

$$P = \frac{1}{3} \frac{M}{V} v^2$$

$$PV = \frac{M}{3} v^2$$

$$PV = v^2 \quad \text{— (I)}$$

and $v^2 \propto T$ — (II)

$\therefore$ $PV \propto T$ or $\boxed{PV = RT}$

(5) <u>Avogadro's law</u>:- It states that equal volumes of all gases, under identical conditions of temperature and pressure contain the same number of molecules. ie $N_1 = N_2$

$[N = 6.023 \times 10^{23}]$

⇒ <u>Law of Equipartition of Energy</u>:-

It states that on on average, the energy of a gas molecule is equally distributed amongst its degree of freedom and the mean energy associated with each degree of freedom is $(\frac{1}{2} K_B T)$

$[K_B =$ Boltzman Constant$]$ ie $\boxed{U = \frac{n}{2} f R T}$

⇒ • Specific Heat Capacity :-

It is the amount of heat required to raise the temperature of Unit mass of a substance through 1°C

$$\text{Specific heat, } c = \frac{1}{m}\frac{\Delta Q}{\Delta T}$$

S.I. $[J\,kg^{-1}K^{-1}]$

• Molar Specific Heat of the Gases :-

Molar specific heat (c) of a substance is defined as the amount of heat required to raise the temperature of 1 mole of substance through one degree Celsius.

$$\boxed{C = \frac{1}{n}\frac{\Delta Q}{\Delta T}}$$

Specific heat at constant pressure (C_p)

$$C_p = \frac{1}{n}\left(\frac{\Delta Q}{\Delta T}\right)_p$$

Specific heat at constant volume (C_v)

$$C_v = \frac{1}{n}\left(\frac{\Delta Q}{\Delta T}\right)_v$$

at constant pressure, to increase internal energy of gas by the same amount, more amount of heat has to be supplied.

i.e $\boxed{C_p > C_v}$

Determination of γ from the degree of freedom.

$$\gamma = \frac{C_p}{C_v} \Rightarrow \boxed{\gamma = 1 + \frac{2}{f}}$$

Values of f, U, C_v, C_p and γ for different gases

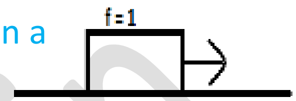
Nature of gas	f	$U = \frac{f}{2}RT$	$C_v = \frac{dU}{dT} = \frac{f}{2}R$	$C_p = C_v + R$	$\gamma = \frac{C_p}{C_v}$
Monoatomic	3	$\frac{3}{2}RT$	$\frac{3}{2}R$	$\frac{5}{2}R$	1.67
Diatomic and linear polyatomic	5	$\frac{5}{2}RT$	$\frac{5}{2}R$	$\frac{7}{2}R$	1.4
Non-linear Polyatomic	6	$3RT$	$3R$	$4R$	1.33

- **<u>Degree of Freedom: -</u>**

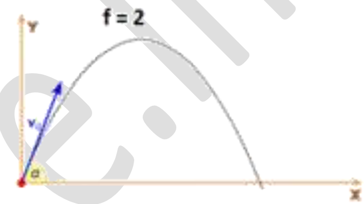
The term degree of freedom refers to the number of possible independent ways in which a system can absorb energy. It can also be defined as the *total number of independent quantities or coordinates required to describe position and configuration of the system.*

e.g.,

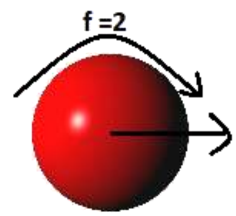
(a) Block has one degree of freedom, because it is confined to move in a straight line and has only one translational degree of freedom.



(b) The projectile has two degrees of freedom because it is confined to move in a plane and so it has two translational degrees of freedom.



(c) The sphere has two degrees of freedom one rotational and another translational. Similarly, a particle free to move in space will have three translational degrees of freedom.



- **<u>Degree of Freedom of Gas Molecules: -</u>**

A gas molecule can have the following types of energies: -

1. translational kinetic energy
2. rotational kinetic energy
3. vibrational energy **(potential + kinetic)**

Mathematically, we can say the number of degrees of freedom of a system is equal to the total number of coordinates required to specify the positions of the constituent particles of the system minus the number of independent relations existing between the particles.

If N = number of particles in the system,
k=number of independent relations between the particles, then the number of degrees of freedom of the system is

$$f=3N-k$$

- **Degree of Freedom of Monoatomic Gas**

A monoatomic gas molecule (like He) consists of a single atom. It can have translational motion in any direction in space. Thus, it has 3 translational degrees of freedom.

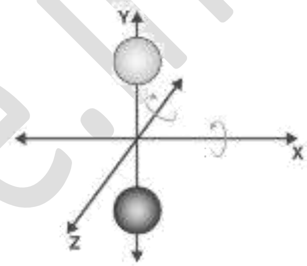
$$f=3 \qquad \text{(all translational)}$$

It can also rotate but due to its small moment of inertia, rotational kinetic energy is neglected. For monoatomic, here, **N = 1, k = 0,**

so, f = 3x1-0 = 3.

- **Degree of Freedom of a diatomic and linear polyatomic Gas**

The molecules of a diatomic and linear polyatomic gas (like O_2, CO, and H_2) cannot only move bodily but also rotate about any one of the three coordinate axes. However, its moment of inertia about the axis joining two atoms (x-axis) is negligible. Hence, it can have only rotational degrees of freedom. Thus, a diatomic molecule has 5 degrees of freedom, for diatomic molecule

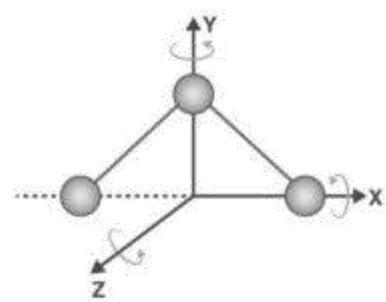


N = 2, k = 1
f = 3N – K
f = 3x2 – 1 = 5

if f=5
(3 translational + 2 rotational) at room temperatures
And
If f =7
(3 translational + 2 rotational + 2 vibrational) at high temperatures.

- **Degree of Freedom of Non-linear Polyatomic Gas**

A non-linear polyatomic molecule (such as NH_3) can rotate about any of three coordinate axes. Hence, it has 6 degrees of freedom 3 translational and 3 rotational. At room temperatures, a polyatomic gas molecule has vibrational energy greater than that of a diatomic gas. But at high enough temperatures it is also significant. So, it has 8 degrees of freedom 3 rotational, 3 translational and 2 vibrational.

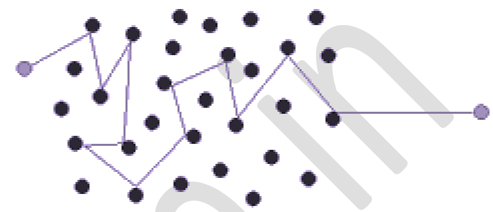


At room temperature (f = 6)
At high temperature (f = 8)

Mean Free Path (λ)

Every gas consists of a large number of molecules undergoing frequent collisions. These molecules are in a perfectly elastic collision against one another. The zig-zag path of different lengths is called free path and their mean state of continuous random motion. They undergo called mean free path.

$$\lambda = {}^{1}\!/_{\sqrt{2}\pi d^2 n}$$



$$\lambda = \frac{RT}{\sqrt{2}\pi d^2 p}$$

n = number of molecules per unit volume.
d = diameter of molecule
T = temperature (K)
P = pressure of the gas
K = Boltzmann constant

Formulas

1. Kinetic Gas Equation :-

$$P = \frac{1}{3} n m \bar{v}^2$$

n = number density
m = mass of molecule
$\bar{v}^2$ = squared mean speed.

2. Equation of state of Perfect gas:-

$$PV = NKT$$

3. Relation between Pressure exerted by ideal gas and density:-

$$P = \frac{1}{3} \rho \bar{v}^2$$

4. Relation between Pressure and Kinetic energy

$$\boxed{P = \frac{2}{3} E}$$

5. Kinetic Interpretation of Temperature:-

$$(K.E)_T = \frac{3}{2} K_B T$$

⑥ Tb Root mean square speed of gas molecule :-

$$\bar{V} = \sqrt{\frac{3k_BT}{m}}$$

⑦ Specific heat Capacity :-

$$\boxed{C = \frac{1}{m}\frac{\Delta Q}{\Delta T}}$$

⑧ Molar specific heat of the gas :-

$$\boxed{C = \frac{1}{n}\frac{\Delta Q}{\Delta T}}$$

at Constant Pressure

$$C_p = \frac{1}{n}\left(\frac{\Delta Q}{\Delta T}\right)_P$$

at constant volume

$$C_V = \frac{1}{n}\left(\frac{\Delta Q}{\Delta T}\right)_V$$

⑨ Determination of γ from degree of freedom

$$\boxed{\gamma = 1 + \frac{2}{f}}$$

⑩ Degree of freedom :- $\boxed{f = 3N - k}$