Physics

Ray Optics and optical Instruments

Chapter: - 9

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Unit: - 6

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Ray Optics and optical Instruments
Chapter: - 9

Reflection of light: -

Reflection of light is a phenomenon in which light waves encounter a surface and bounce back in same medium.

Laws of reflection: -

1) Angle of incident is equal to the angle of reflection.

$$\angle i = \angle r$$

2) The incident ray, reflected ray and normal lie on the same plane.

Types of Reflection:

Specular Reflection: The reflected light is parallel, and all rays reflect in the same direction, typically occurring on smooth surfaces like mirrors.

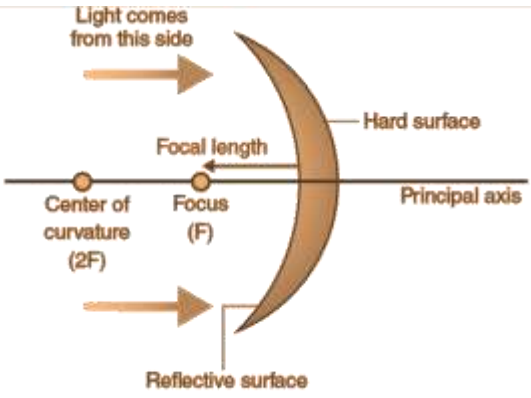
Diffuse Reflection: The reflected light is scattered in various directions, commonly observed on rough surfaces.

Spherical mirror: -

Spherical mirrors are whose reflecting surface is shaped like a section of a sphere. There are two main types of spherical mirrors:

- **Concave mirrors**
- **Convex mirrors.**

1. **Concave Mirrors:**
   - A concave mirror is curved inward, and its reflecting surface is on the inside of the curved shape.
   - **Focal Point (F):** The point where parallel rays of light either converge or appear to diverge after reflection from the mirror.
   - **Focal Length (f):** The distance between the focal point and the mirror's vertex(pole) (the center of curvature).
   - **Principal Axis:** An imaginary line that passes through the center of curvature (C), the vertex, and the focal point.



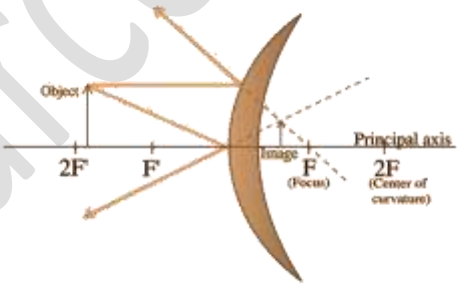
   - ***Characteristics:***
     - Concave mirrors can produce both real and virtual images.
     - Real images are formed when

the object is located beyond the focal point, and they are inverted.

- Virtual images are formed when the object is located between the mirror and the focal point, and they are erect.

2. Convex Mirrors:

- A convex mirror is curved outward, and its reflecting surface is on the outside of the curved shape.
- **Focal Point (F):** The point from which divergent rays of light appear to be coming after reflection from the mirror **(virtual focal point).**
- **Focal Length (f):** The distance between the focal point and the mirror's vertex (Pole).
- **Principal Axis:** An imaginary line that passes through the center of curvature (C), the vertex, and extends behind the mirror to the virtual focal point.



- ***Characteristics:***
  - Convex mirrors always produce virtual images.
  - These images are erect, diminished, and located behind the mirror.

Name	Symbol	Formulas	S.I. Unit
Focal length	f	$f = \frac{R}{2}$	m
Mirror formula	v,u,f	$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$	m
Magnification of spherical mirror	m	$m = \frac{h_i}{h_o} = -\frac{v}{u}$	Unit less

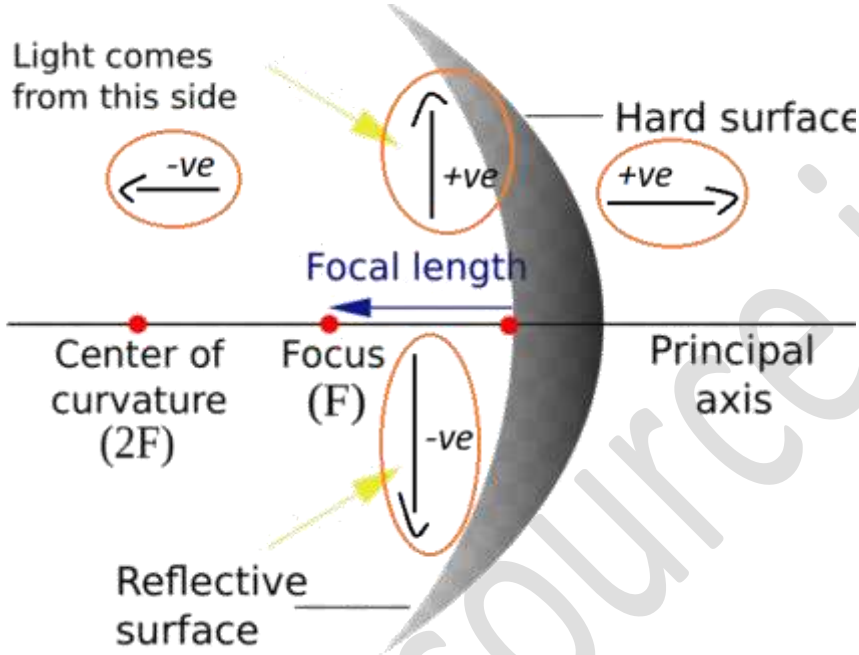
Sign Convention

(a) All distances measured in the direction of incident ray (along + X-axis) are taken as positive (right hand side of the mirror).

(b) All distance measured against the direction of incident ray (along – X-axis) are taken as negative (Left hand side of the mirror).

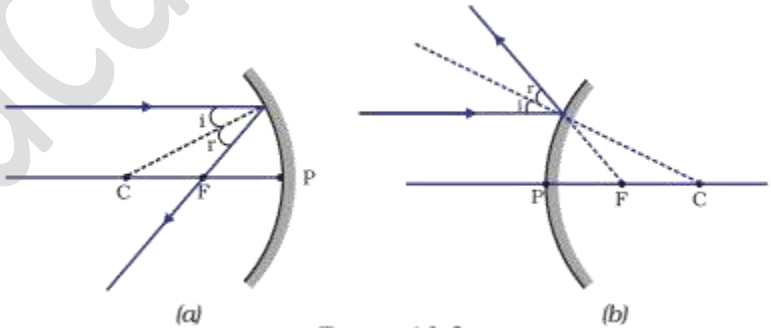
(c) Distance measured perpendicular to and above the principal axis are taken as positive.

(d) Distances measured perpendicular to and below the principal axis are taken as negative.



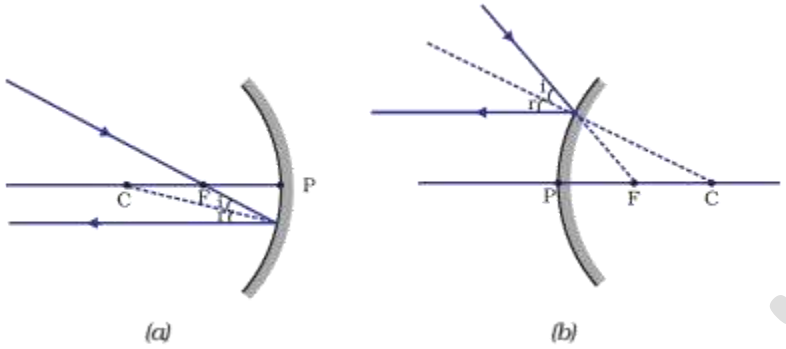
Representation of image formed by spherical Mirror

a) A ray parallel to the principal axis, after reflection, will pass through the principal focus in case of a concave mirror or appear to diverge from the principal focus in case of a convex mirror.



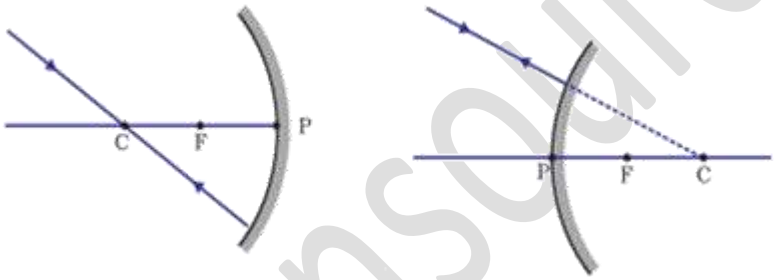
b) A ray passing through the principal focus of a concave mirror or a ray which is directed towards the principal focus of a convex mirror, after reflection, will emerge parallel to the principal axis.

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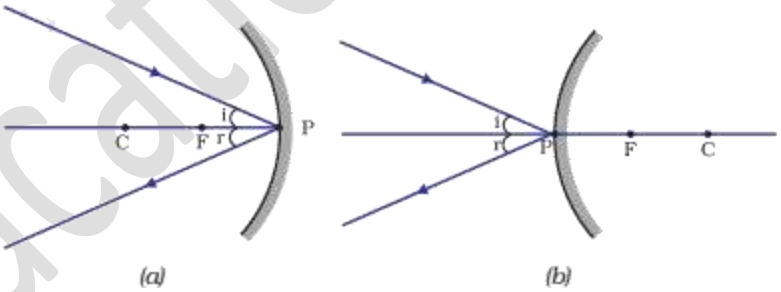


(a) (b)

c) A ray passing through the centre of curvature of a concave mirror or directed in the direction of the centre of curvature of a convex mirror, after reflection, is reflected back along the same path.



d) A ray incident obliquely to the principal axis, towards a point P (pole of the mirror), on the concave mirror or a convex mirror, is reflected obliquely.



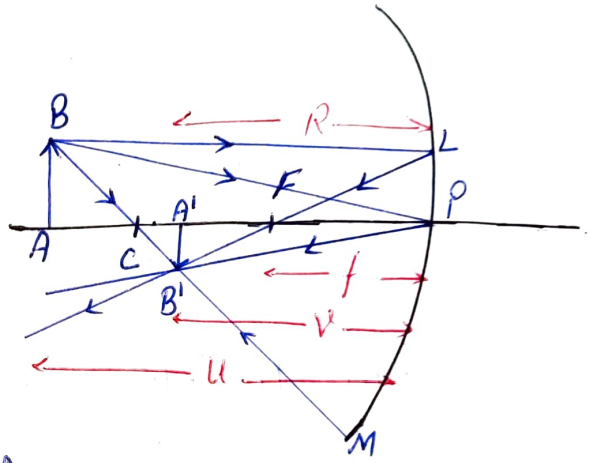
(a) (b)

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Mirror formula :-

a) For Concave mirror when Real Image is formed :-

⇒ A Real and inverted image is formed when a object is placed on the principal axis beyond the principal focus(F) of a Concave mirror.



Let us Consider a object AB in front of Concave mirror.

F = focal Point

f = focal length

C = Centre of curvature

R = Radius of curvature

U = distance of object from Pole.

V = distance of Image from Pole.

Here A'B' = Image formed by Concave mirror.

from ΔABC and $\Delta A'B'C$

[$\angle BAC = \angle B'A'C$ (each 90°) and $\angle ACB = \angle A'CB'$ (vertical opposite angle)

$\therefore \Delta ABC$ and $\Delta A'B'C$ are similar

ie $\frac{A'B'}{AB} = \frac{CA'}{CA}$ ——— (1)

Also from ΔABP and $\Delta A'B'P$ are similar

$\frac{A'B'}{AB} = \frac{PA'}{PA}$ — (II)

from equation (I) and (II)

$$\frac{CA'}{CA} = \frac{PA'}{PA} \quad \text{---(III)}$$

Acc. to New Cartesian Sign Convention

$$CA' = (PC - PA')$$

and $$CA = (PA - PC)$$

Using in (III)

$$\frac{PC - PA'}{PA - PC} = \frac{PA'}{PA}$$

Here $PC = -R$, $PA = -u$ and $PA' = -v$

$$\frac{-R + v}{-u + R} = \frac{+v}{+u}$$

$$(-R + v)u = v(-u + R)$$

$$-Ru + uv = -uv + vR$$

$$-Ru - vR = -uv - uv$$

$$-Ru - vR = -2uv$$

$$+(Ru + vR) = +2uv$$

dividing uvR on both side

$$\frac{Ru}{uvR} + \frac{vR}{uvR} = \frac{2uv}{uvR}$$

$$\frac{1}{v} + \frac{1}{u} = \frac{2}{R} \quad [\text{Here } R = 2f]$$

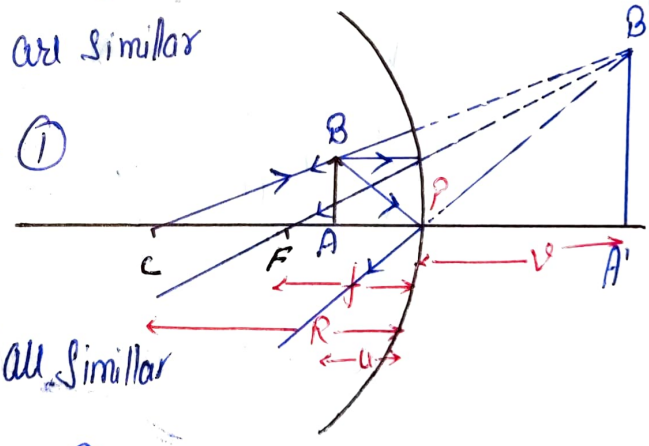
$$\frac{1}{v} + \frac{1}{u} = \frac{2}{2f}$$

$$\boxed{\frac{1}{v} + \frac{1}{u} = \frac{1}{f}}$$

Mirror formula or mirror equation

b) For Concave mirror when virtual Image is formed:-

When an object is placed between the pole and the focus of a Concave mirror, virtual, erect and enlarged Image is formed behind the mirror.



ΔABC and $A'B'C$ are similar

$$\frac{A'B'}{AB} = \frac{A'C}{AC} \quad \text{— (I)}$$

ΔABP and $A'B'P$ all Similar

$$\frac{A'B'}{AB} = \frac{A'P}{AP} \quad \text{— (II)}$$

from equation (I) and (II)

$$\frac{A'C}{AC} = \frac{A'P}{AP}$$

$$\frac{A'P + PC}{PC - PA} = \frac{A'P}{AP}$$

$$\boxed{\begin{aligned} A'C &= A'P + PC \\ AC &= PC - PA \end{aligned}}$$

Using New Cartesian Sign Convention

$PA' = v$, $PC = -R$, $PA = -u$

$$\frac{v-R}{-R+u} = \frac{v}{-u}$$

$$-uv + uR = v(-R+u)$$

$$-uv + uR = -Rv + vu \Rightarrow \qquad uR + Rv = +2uv$$

dividing by vuR

$$\frac{uR}{vuR} + \frac{Rv}{uvR} = \frac{+2uv}{uvR}$$

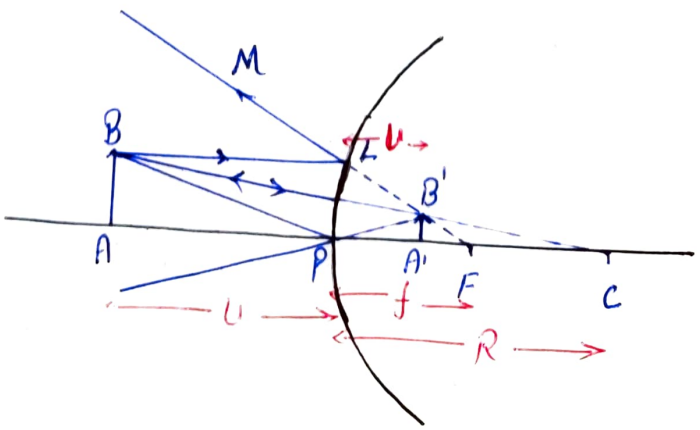
$$\frac{1}{v} + \frac{1}{u} = \frac{2}{R} \qquad [R = 2f]$$

$$\frac{1}{v} + \frac{1}{u} = \frac{2}{2f}$$

$$\boxed{\frac{1}{v} + \frac{1}{u} = \frac{1}{f}}$$

Mirror formula for Convex Mirror :-

AB = object
$A'B'$ = Image formed by the mirror



From ΔABC and $\Delta A'B'C$

$$\frac{A'B'}{AB} = \frac{A'C}{AC} \quad \text{---} \quad (I)$$

From ΔABP and $\Delta A'B'P$

$$\frac{A'B'}{AB} = \frac{PA'}{PA} \quad \text{---} \quad (II)$$

From equation (I) and (II)

$$\frac{A'C}{AC} = \frac{PA'}{PA} \quad \text{---} \quad (III)$$

$$A'C = PC - PA'$$

$$AC = AP + PC$$

Using (IV) (III)

$$\frac{PC - PA'}{AP + PC} = \frac{PA'}{PA}$$

Using Sign Convention.

$$PA' = v, \quad PC = R, \quad PA = -u$$

$$\frac{R - v}{-u + R} = \frac{v}{-u}$$

$$-uR + uv = -uv + Rv$$

$$-uR - Rv = -2uv$$

dividing by UVR

$$\frac{-UR}{UVR} - \frac{RV}{UVR} = \frac{-2UV}{UVR}$$

$$-\frac{1}{V} - \frac{1}{U} = -\frac{2}{R} \quad [R = 2f]$$

$$-\frac{1}{V} - \frac{1}{U} = -\frac{2}{2f}$$

$$\boxed{\frac{1}{V} + \frac{1}{U} = \frac{1}{f}}$$

Relative Refractive Index :-

If medium 1 has refractive index n_1 and medium 2 has refractive index n_2, then, $\frac{n_2}{n_1} = {}^1n_2$

${}^1n_2$ = Relative refractive index of medium 2 Relative to medium 1.

$${}^1n_2 = \frac{n_2}{n_1} = \frac{c/v_2}{c/v_1} = \frac{c}{v_2} \times \frac{v_1}{c} = \frac{v_1}{v_2}$$

$$\boxed{{}^1n_2 = \frac{v_1}{v_2}}$$

Higher refractive index is called optically denser medium. Any lower value of refractive index is called optically rarer medium.

⇒ What is the speed of light in medium whose refractive index is 3/2 ? Given speed of light in air = 3×10^8 m/s.

Here $n = 3/2$, $c = 3 \times 10^8$ m/s

$$n = \frac{c}{v}$$

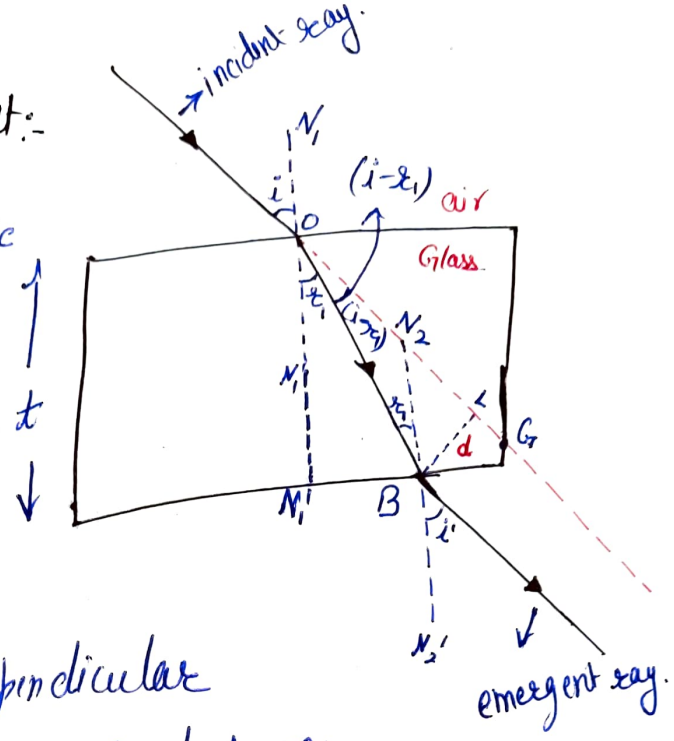
Speed of light in medium $v = \frac{c}{n} = \frac{3 \times 10^8}{3/2} = \frac{2}{3} \times 3 \times 10^8$ m/s

$$\boxed{v = 2 \times 10^8 \text{ m/s}}$$

Refraction through a Glass Slab (Lateral shift)

Lateral shift of displacement:-

The perpendicular distance between the direction of incident ray and the emergent ray is known as the lateral shift.



Here BL is the perpendicular distance (d) between incident ray and emergent ray.

From Δ BOL

$$\sin(i-r_1) = \frac{BL}{OB} = \frac{d}{OB}$$

$$d = OB \sin(i-r_1) \quad \text{---} \quad (1)$$

From Δ BO N'

$$\cos r_1 = \frac{ON_1'}{OB} = \frac{t}{OB}$$

$$OB = \frac{t}{\cos r_1} \quad \text{---} \quad (2)$$

Put the value of OB in (1)

$$d = \frac{t}{\cos r_1} \sin(i - r_1)$$

$$\boxed{d = \frac{t \sin(i - r_1)}{\cos r_1}}$$

Lateral shift will be maximum if

$\cos r_1 = 1$ or $r_1 = 0$

and $\sin(i - r_1) = 1$ or $i = 90°$

So maximum lateral shift = t (thickness of the slab)

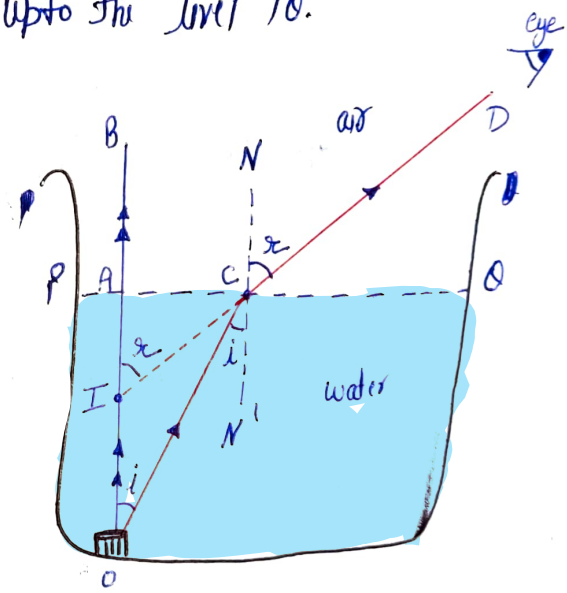
⇒ Formation of Image by refraction at plane surface (Real and Apparent depths)

Consider a bucket filled with water upto the level PQ.

Here AO = Real depth

AI = Apparent depth

of a coin at the bottom of the bucket.



According to Snell's law

$$w_{n_a} = \frac{\sin i}{\sin r} \quad \text{— (1)}$$

from ΔAOC

$$\sin i = \frac{AC}{OC}$$

and from ΔAIC

$$\sin r = \frac{AC}{IC}$$

use above values in (1)

$$w_{n_a} = \frac{AC/OC}{AC/IC} = \frac{AC}{OC} \times \frac{IC}{AC} = \frac{IC}{OC}$$

$$w_{n_a} = \frac{IC}{OC}$$

Since point C lies very close to A

i.e $IC \simeq AI$ and $OC \simeq AO$

$$\omega_{n_a} = \frac{AI}{AO} \quad \text{---} (11)$$

Since $\omega_{n_a} = \frac{1}{a_{n_w}}$

use in (11)

$$\frac{1}{a_{n_w}} = \frac{AI}{AO}$$

$$a_{n_w} = \frac{AO}{AI} = \left(\frac{\text{Real depth}}{\text{Apparent depth}}\right)$$

In general $^1n_2 = \frac{\text{Real depth}}{\text{Apparent depth}}$.

Normal shift in the position :-

$$x = AO - AI$$

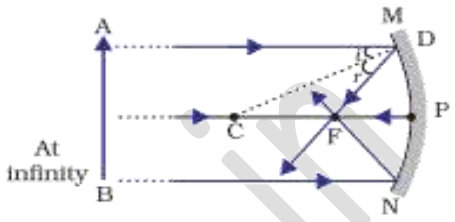
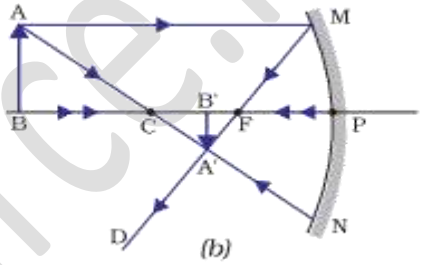
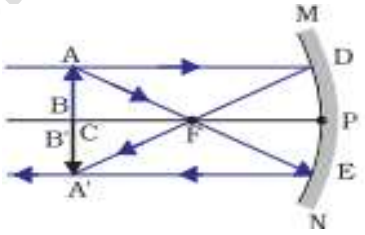
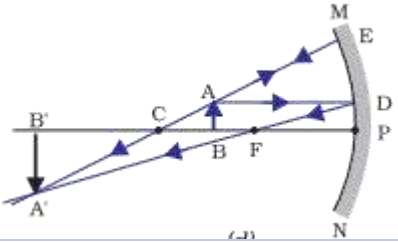
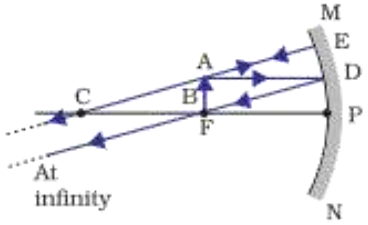
$$= AO\left(1 - \frac{AI}{AO}\right)$$

$$= AO\left(1 - \frac{1}{a_{n_w}}\right)$$

$$= AO\left(1 - \frac{1}{^1n_2}\right) \qquad [AO = t]$$

In general $\boxed{x = t\left[1 - \frac{1}{n}\right]}$

Image formation by a concave mirror for different positions of the object

Position of the Object	***Position of the Image***	***Size of the Image***	***Nature of image***	***Ray diagram***
At infinity	At the focus F	Highly diminished	Real and inverted point-sized	
Beyond C	Between F and C	Diminished	Real and inverted	
At C	At C	Same size	Real and inverted	
Between C and F	Beyond C	Enlarged	Real and inverted	
Position of the Object	***Position of the Image***	***Size of the Image***	***Nature of image***	***Ray diagram***
At F	At infinity	Highly enlarged	Real and inverted	

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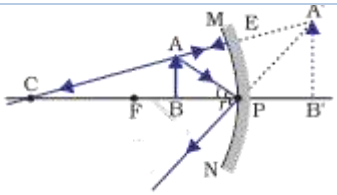
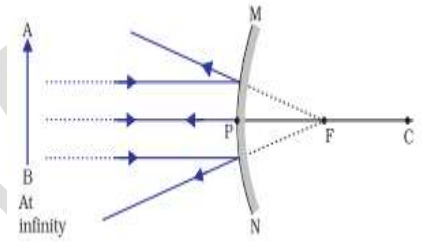
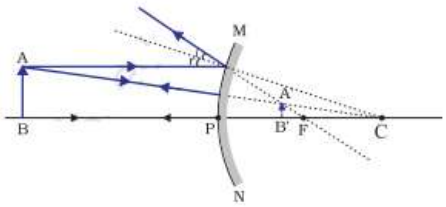
Between P and F	Behind the mirror	Enlarged	Virtual and erect	

Image formation by a convex mirror for different positions of the object

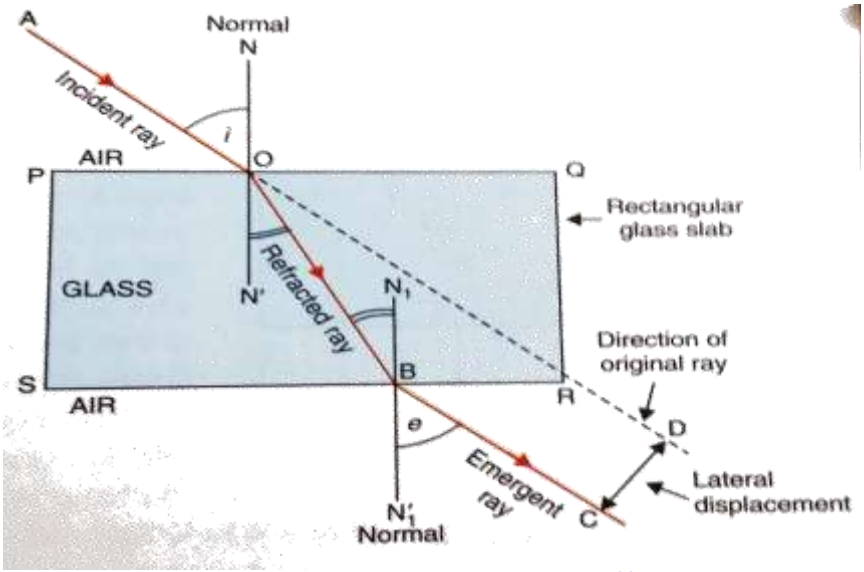
Position of the Object	***Position of the Image***	***Size of the Image***	***Nature of image***	***Ray diagram***
At infinity	At the focus F, behind the mirror	Highly diminished, point-sized	Virtual and erect	
Between infinity and the pole P of the mirror	Between P and F, behind the mirror	Diminished	Virtual and erect	

Refraction of light: -

Refraction of light is a phenomenon in optics where light waves change direction as they pass from one transparent medium to another with a different optical density.

Causes of Refraction of light: -

1) As light passes from a medium with a lower refractive index to a medium with a higher refractive index, its speed decreases, and vice versa.
2) The wavelength of light may also change during refraction, depending on the medium.



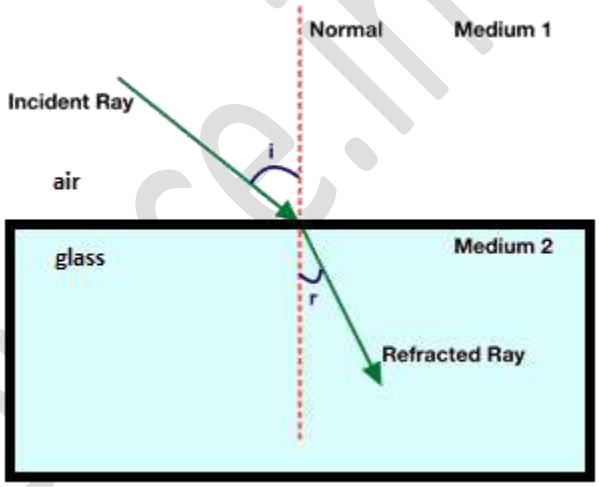
Laws of Refraction: -

1) The refracted ray, incident ray and the normal at the interface of two transparent media at the point of incidence, all lie in the same plane.
2) For the given pair of media, the ratio of the sine of the angle of incidence to the sine of angle refraction is always constant.

$$\frac{sin\ i}{sin\ r} = constant$$

Refractive index: -

when light passes obliquely from one medium to another, it changes direction in the second medium. The extent to which change in direction takes place in the given set of a medium is termed as refractive index.



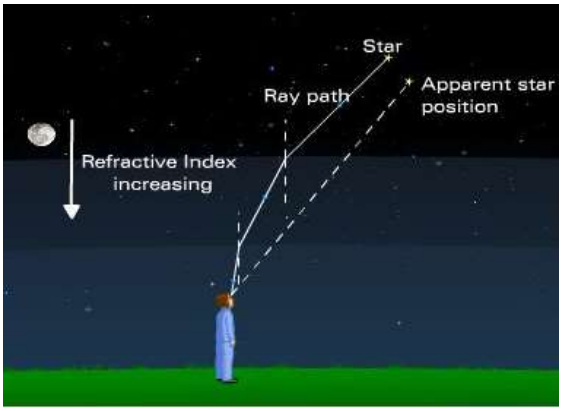
Absolute refractive index (n) of a medium can be defined as the ratio of speed of light in vacuum/air **(c)** to the speed of light in the medium **(u).**

$$\boldsymbol{n = \frac{c}{v}}$$

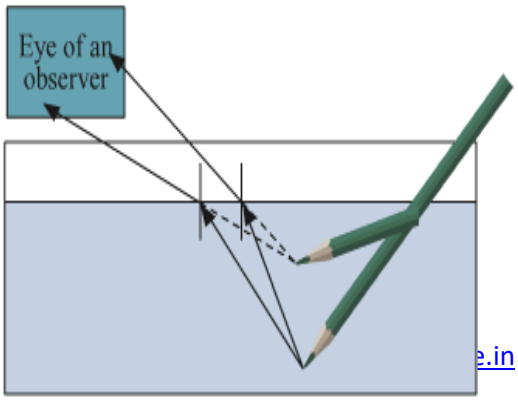
Speed of light in vacuum or air is **c = 3 X $10^2 m/s$**

Some Phenomenon of Refraction of light: -

(a) Twinkling of Stars: - The twinkling of stars is a phenomenon caused by the refraction of light as it travels through the Earth's atmosphere. As starlight enters the atmosphere, it passes through multiple layers of varying density and temperature. These fluctuations in density and temperature lead to changes in the speed of light, causing it to bend or refract.



(b) Bending of an immersed object: - The bending of an immersed object, occurs by the refraction of light, when light passes from one medium to another with a different optical density. When an object is immersed in a transparent medium (such as air, water, or glass), the speed of light changes as it moves from one medium to

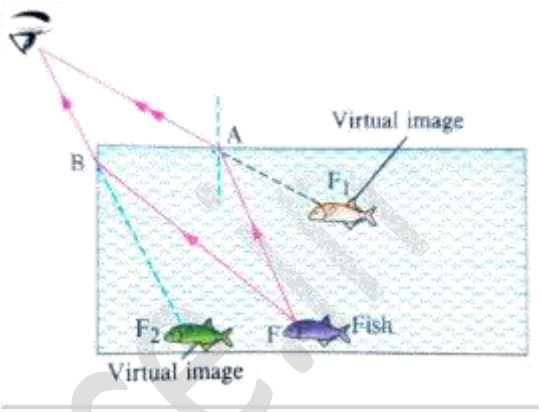


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another. The change in speed is responsible for the bending of the light rays, and consequently, the apparent shift or distortion of the immersed object.

(c) Visibility of two images of an immersed object:

- Consider a fish F under a water in the glass container. The rays of light FA and FB suffer refraction at the free surface of water and the side of the glass container respectively. After refraction, these rays appear to come from F_1 and F_2 which are the two virtual images of the fish F.



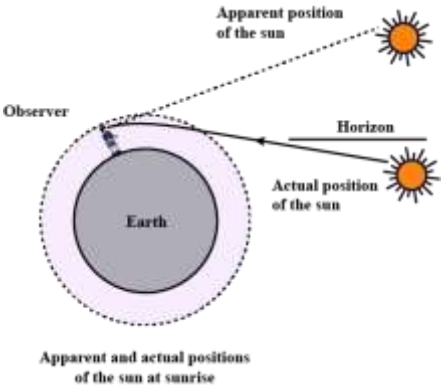
(d) Sun appears oval shaped at morning and evening (i.e., flattening of the sun): -

In the morning and evening the sun is near the horizon. The refractive index of the layers of the atmosphere decreases with height. The rays of light from the lower edges of the sun are refracted more than those from the upper edge due to passage through greater thickness of air. In other words, the rays of light from the upper and lower edge of the sun bend unequally. Due to this unequal bending of light, the image of the vertical diameter of the sun appears oval and large in size.



However, at noon when the sun is overhead, the rays of light from the sun enter the atmosphere normally and hence no bending of light takes place. Therefore, the sun appears circular at noon.

(e) Time difference during sunrise and sun set: - Sun is visible to an observer after the actual sunset (i.e., actual passing of the horizon by the sun) because of refraction of light due to denser atmosphere near the earth. Due to refraction, apparent shift in the direction of sun is about $1/2^0$ leading to delay of about 2 minutes in actual and apparent sunsets. Earth completes one rotation (i.e., 36^0) about its axis in 24 hours. Therefore, time taken for 360° shift 24 hr.



Apparent and actual positions of the sun at sunrise

Hence time taken for 1/2° shift = $\frac{24}{360} X \frac{1}{2}$

$$= \frac{1}{30} h = \frac{1}{30} X 60 min.$$

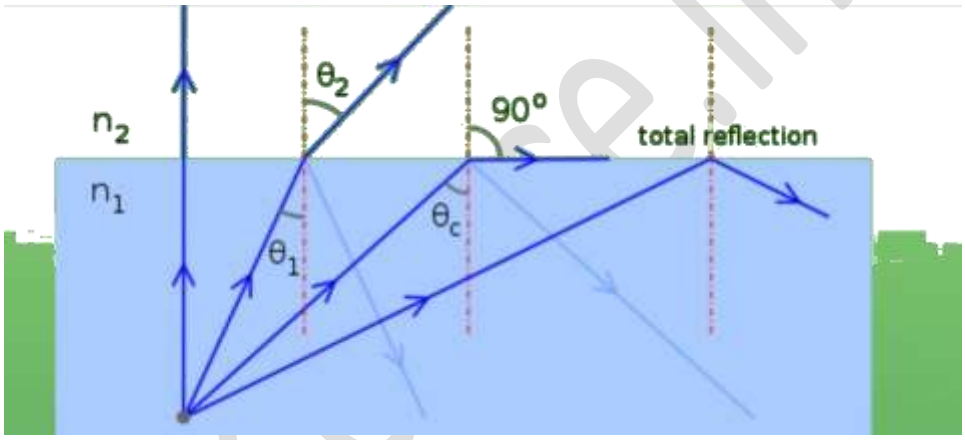
= 2 min.

For same reason the sun appears to rise about 20 min. before the actual rise.

Total internal reflection: -

Total Internal Reflection (TIR) is a phenomenon that occurs when a light ray traveling through a dense medium and strike to a boundary with a less dense medium at an angle greater than the critical angle. In this situation, instead of refracting the light is completely reflected back into the same denser medium.

Critical angle: - The critical angle is the angle of incidence at which the refracted angle becomes 90 degrees with the normal.



The conditions for total internal reflection: -

1) The light should be travel from dense medium to rarer medium.
2) The angle of incident must be greater than the critical angle.

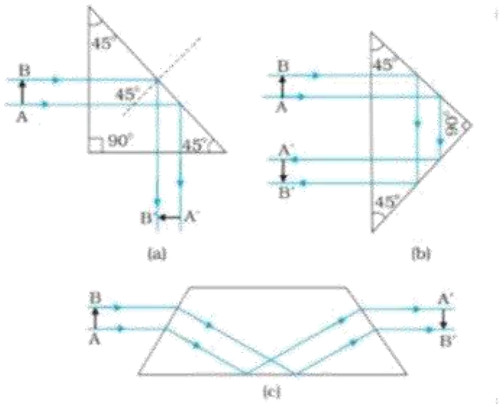
The formula for calculating the critical angle (ϑc) is given by:

$$\textbf{Sin}(\vartheta c) = \frac{n_2}{n_1}$$

- ϑc is the critical angle,
- n_1 is the refractive index of the denser medium (where light is coming from),
- n_2 is the refractive index of the less dense medium (where light is going into)

Application of total internal Reflection: -

(i) Porro prism: - *A right angled isosceles prism, called Porro-Prism, can be used in periscope or binocular.*



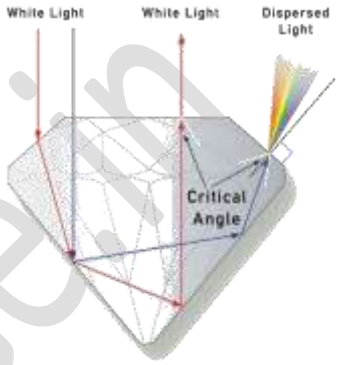
The refractive index of glass is 1.5. Therefore, the critical angle is given by

sin C = 1/1.5 or **$=\sin^{-1}(0.6667) = 41.8°$.** When the ray of light falls on the face of a right-angled prism at angle greater than 41.8° which is known as

critical angle, it will suffer total internal reflection. Right angled prisms used to bend the light through 90° and 180°.

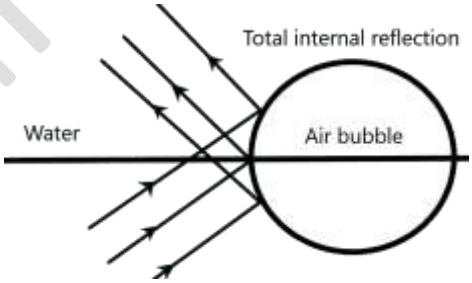
(ii) Sparkling Diamond: - The refractive index of diamond is 2.5.

$$\theta = \sin^{-1}(1/2.5) = 23.58 \approx 24^{o}$$



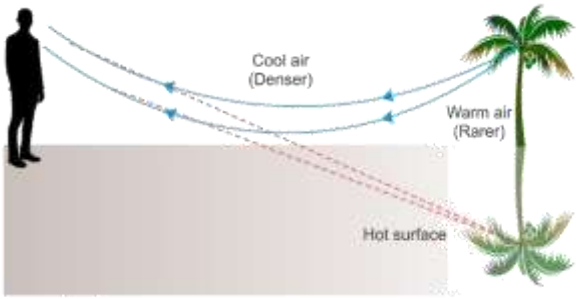
Here the critical angle as 24° for diamond air interface. A natural diamond, as it is, hardly shows brilliance. It is the special cutting by a jeweler which makes it shine. The faces of a diamond are cut in such a way that whenever light falls on any of the faces, the angle of incidence is greater than the critical angle i.e., > 24°. So, when light falls on the diamond, it suffers repeated *total internal reflections*. The light which finally emerges out from few places in certain directions makes the diamond sparkling.

(iii) Shining Air bubble in Water: - The critical angle for water- air interface is 48°45'. When light propagating from water (denser medium) is incident on the surface of air bubble (rarer medium) at an angle greater than 48°45', the total internal reflection takes place. Hence the air bubble in water shines j brilliantly.



(iv) Mirage: - Mirage is an optical illusion of water observed generally in deserts when the inverted image of an object (e.g., a tree) is observed along with the object itself on a hot day.

On a hot day, the ground heats up, making the air close to it less dense. This alters the refractive index of the air, causing light from distant objects (like trees) to bend and create an illusion on the ground. This effect, known as a mirage, occurs because of total internal reflection in the hot air near the Earth's surface. The same illusion can be observed on a hot road while traveling, where distant spots may appear wet, giving the false impression of water.

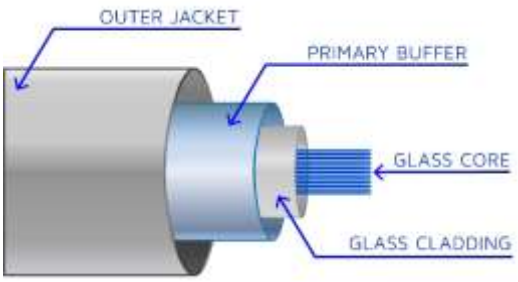


(v) Optical Fibres - An optical fibre is an extremely thin, long strand made of high-quality glass or quartz. It has a thin layer of material, called **cladding**, with a refractive index

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lower than that of the fibre core (for example, if the core index is 1.7, the cladding index might be 1.5).

When light is directed at one end of the optical fibre, it is refracted into the fibre core. The refracted light then encounters the interface between the core and the cladding at an angle greater than the critical angle. This phenomenon allows the light to undergo total internal reflection within the fibre, enabling efficient transmission of signals through the optical fibre.



Uses of Optical fibers: -

1. **Telecommunications:** Optical fibers are extensively used for long-distance communication, such as in telephone networks, internet connections, and cable television. They provide high-speed data transmission and can carry large amounts of information over vast distances.

2. **Internet Connectivity:** Fiber-optic cables form the backbone of high-speed internet connections, delivering faster and more reliable data transfer compared to traditional copper cables.

3. **Medical Imaging:** Optical fibers are used in medical instruments like endoscopes, allowing doctors to visualize internal organs and perform minimally invasive procedures.
4. **Data Networking:** Within data centers and local area networks (LANs), optical fibers are employed to transmit data between servers, computers, and other networking devices with high bandwidth and minimal signal loss.

5. **Military and Aerospace Applications:** Optical fibers are used in various military and aerospace applications for communication, sensing, and data transmission due to their lightweight nature and resistance to electromagnetic interference.

6. **Broadcasting:** Fiber optics are used in broadcasting for transmitting audio and video signals over long distances with high quality and minimal signal degradation.

7. **Industrial Applications:** Optical fibers are used in industrial settings for sensing and monitoring applications, such as detecting temperature, pressure, and strain in harsh environments.

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8. **Power Transmission:** Fiber optics can be used for monitoring power grids and transmitting data related to power distribution, helping in the efficient management and maintenance of electrical systems.



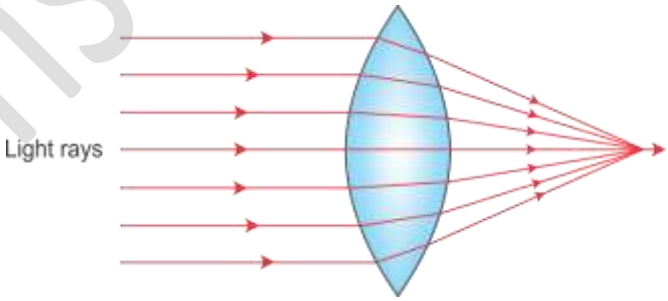
9. **Sensors:** Optical fibers are employed in various sensor applications, including temperature, pressure, and humidity sensors, as well as in distributed sensing systems for structural health monitoring.

10. **Underwater Communication:** Due to their immunity to electromagnetic interference, optical fibers are used for underwater communication, where traditional copper cables may face challenges.

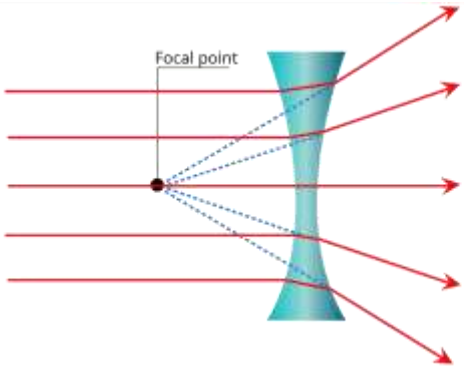
11. **Security Systems:** Optical fibers play a role in secure data transmission for applications like surveillance and security systems, as they are difficult to tap without detection.

Lens

a) Converging lens: - (convex lens)



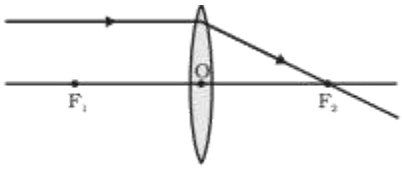
b) Diverging lens: - (concave lens)



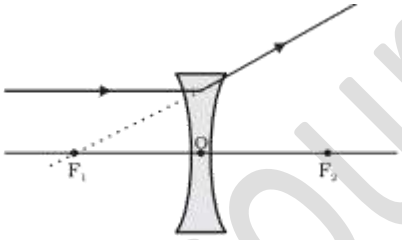
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Image Formation in lenses Using Ray Diagram

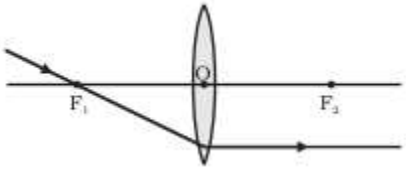
a) A ray of light from the object, parallel to the principal axis, after refraction from a convex lens, passes through the principal focus on the other side of the lens.



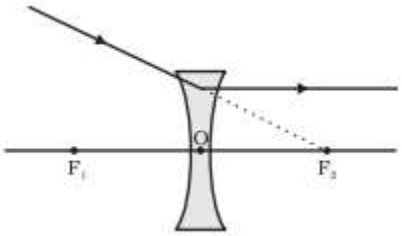
In case of a concave lens, the ray appears to diverge from the principal focus located on the same side of the lens.



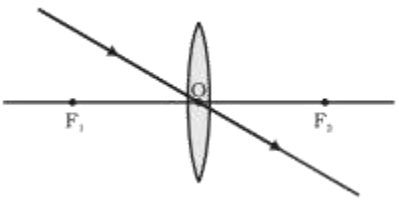
b) A ray of light passing through a principal focus, after refraction from a convex lens, will emerge parallel to the principal axis.

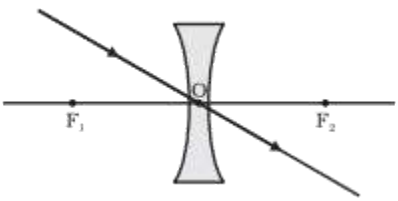


A ray of light appearing to meet at the principal focus of a concave lens, after refraction, will emerge parallel to the principal axis.



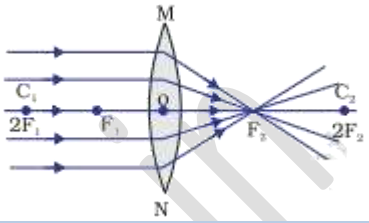
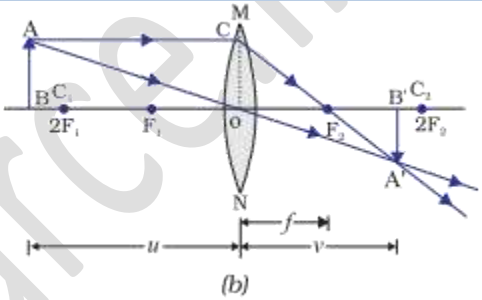
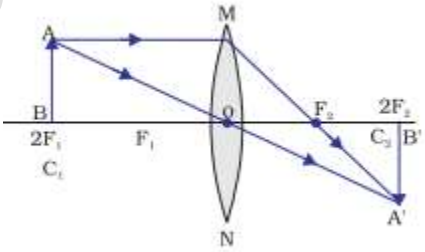
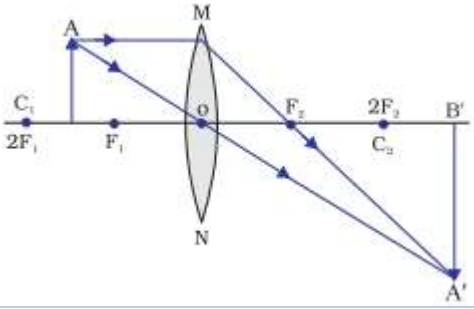
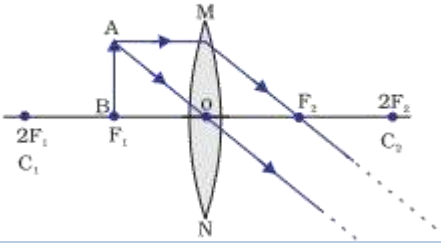
c) A ray of light passing through the optical centre of a lens will emerge without any deviation.





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Image formation by a convex lens for different positions of the object

Position of the Object	*Position of the Image*	*Size of the Image*	*Nature of image*	*Ray diagram*
At infinity	At focus F_2	Highly diminished, point-sized	Real and inverted	
Beyond $2F_1$	Between F_2 and $2F_2$	Diminished	Real and inverted	
At $2F_1$	At $2F_2$	Same size	Real and inverted	
Between F_1 and $2F_1$	Beyond $2F_2$	Enlarged	Real and inverted	
At focus F_1	At infinity	Infinitely large or highly enlarged	Real and inverted	

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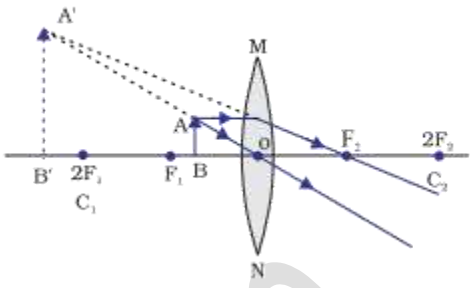
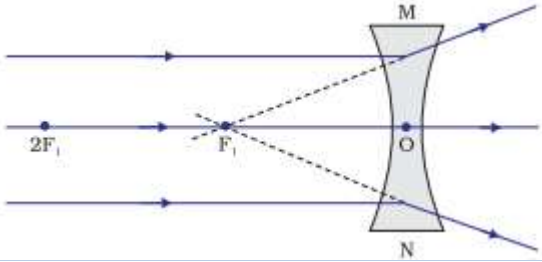
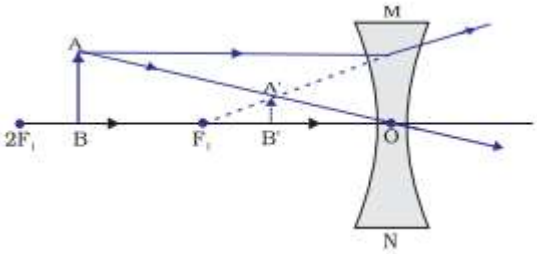
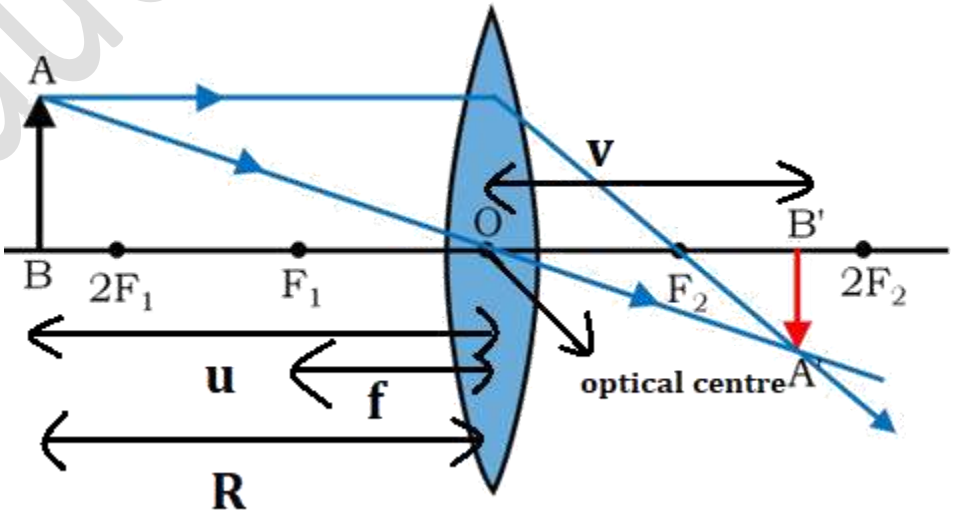
Between focus F_1 and optical centre O	On the same side of the lens as the object	Enlarged	Virtual and erect	

Image formation by a concave lens for different positions of the object

Position of the Object	*Position of the Image*	*Size of the Image*	*Nature of image*	*Ray diagram*
At infinity	At focus F_1	Highly diminished, point-sized	Virtual and erect	
Between infinity and optical centre O of the lens	Between focus F_1 and optical centre O	Diminished	Virtual and erect	

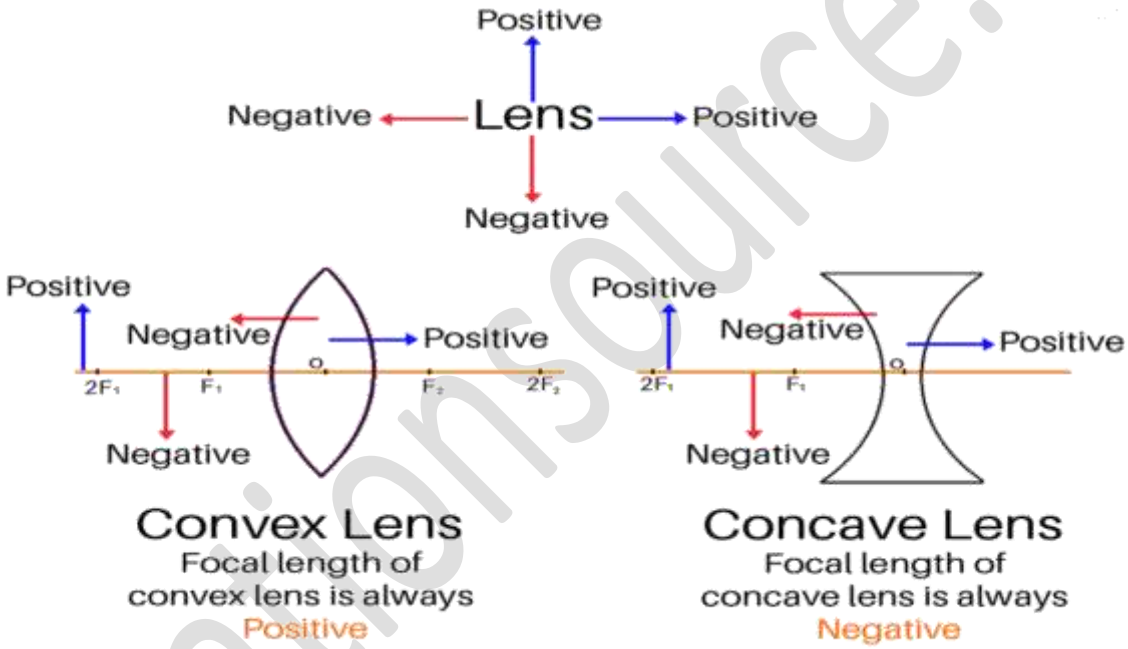
Lens Formula



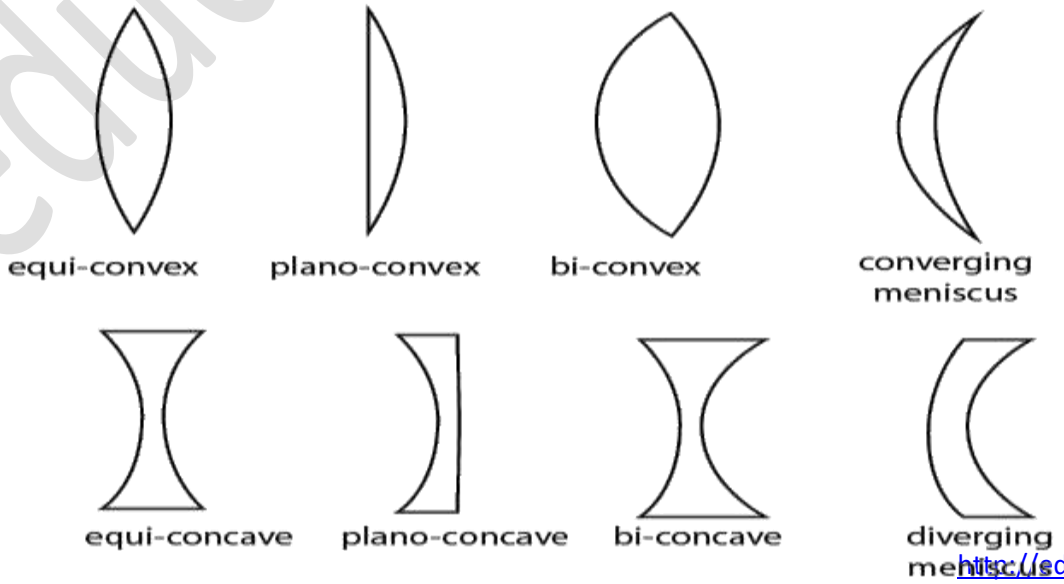
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a) F_1 and F_2 = focal length of the lens
b) **$2F_1(C_1)$ and $2F_2(C_2)$ =** Centre of Curvature of the lens
c) f = focal length of the lens.
d) R = Radius of Curvature of the lens.
e) O = Optical Centre of the lens.
f) u = Distance of the object from the optical centre.
g) v = Distance of the image from the optical centre.

Sign conversation of lens: -



Different type of lenses: -

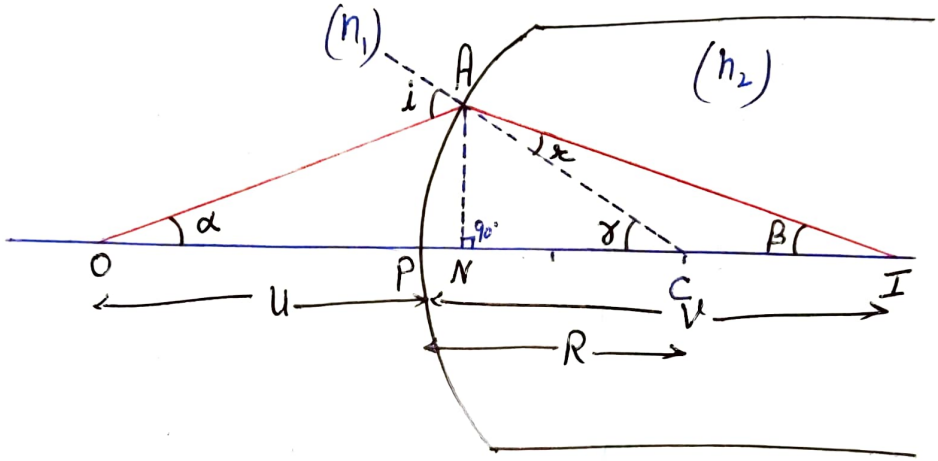


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Spherical Refracting Surfaces :-

a) Convex spherical surface when object is in rarer medium and image formed is Real :-

$(h_2 > h_1)$



Let us Consider a spherical body of Refractive Index (h_2)

h_1 = Refractive Index of air i.e $(h_2 > h_1)$

O = object is the object

I = Image of the object

C = Centre of curvature.

Step I :- Determination of i and r :-

Let α, β and γ are the angles formed by Incident ray, Refracted ray and normal with the principle axis.

from ΔAOC (Sum of opposite two angles)

$$\angle i = \angle\alpha + \angle\gamma \quad \text{—— (I)}$$

from ΔAIC

$$\angle\gamma = \angle r + \angle\beta \Rightarrow \angle r = \angle\gamma - \angle\beta \quad \text{—— (II)}$$

α, β and γ are small.

using angle $= \frac{\text{arc}}{\text{Radius}}$

$$\theta = \frac{\text{arc}}{\text{Radius}}$$

$$\alpha = \frac{AN}{ON}, \quad \beta = \frac{AN}{IN}, \quad \gamma = \frac{AN}{NC}$$

$$i = \frac{AN}{ON} + \frac{AN}{NC} \approx \frac{AN}{P_0} + \frac{AN}{P_C} \quad \text{(III)}$$

$$r = \frac{AN}{NC} - \frac{AN}{NI} \approx \frac{AN}{PC} - \frac{AN}{P_I} \quad \text{(IV)}$$

N lie very close to P
ie
$N_0 \approx P_0$
$NC \approx PC$
$NI \approx P_I$

Step II :- Now according to Snell's law

$$\frac{\sin i}{\sin r} = \frac{n_2}{n_1}$$

$$n_1 \sin i = n_2 \sin r$$

Here angles are very small ie $\sin i \simeq i$

$$n_1 \angle i = n_2 \angle r$$

from equation (III) and (IV)

$$n_1 \left[\frac{AN}{P_0} + \frac{AN}{PC} \right] = n_2 \left[\frac{AN}{P_C} - \frac{AN}{P_I} \right]$$

$$h_1\left[\frac{1}{P_O}+\frac{1}{P_C}\right]=h_2\left[\frac{1}{P_C}-\frac{1}{P_I}\right]$$

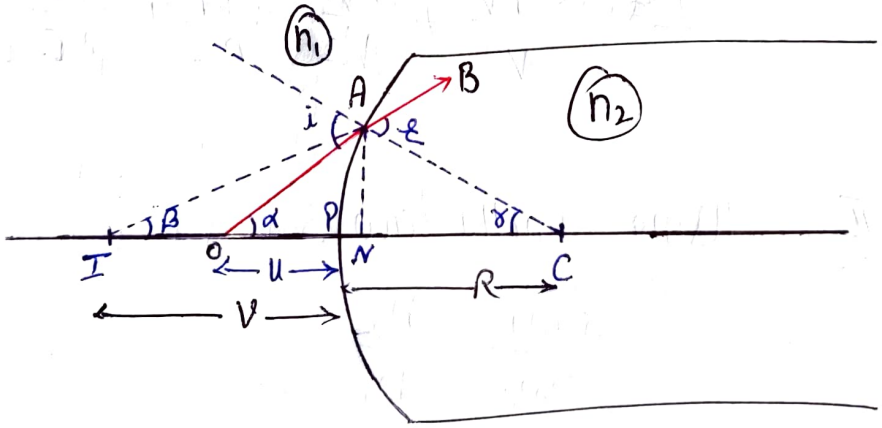
$$\frac{h_1}{P_0}+\frac{h_2}{P_I}=\frac{h_2}{P_C}-\frac{h_1}{P_C}$$

Step: 3 Applying New Cartesian sign Conventions

$P_O = -u, \quad P_C = R, \quad P_I = V$

$$\boxed{-\frac{h_1}{u}+\frac{h_2}{v}=\frac{h_2-h_1}{R}}$$

b) When Image formed is Virtual :-



Let α, β and γ be the angle made by incident Ray, Refracted ray and Normal respectively with principal axis.

Step I :- Determination of $\angle i$ and $\angle \varepsilon$

From ΔAOC

$$\angle i = \angle\alpha + \angle\gamma \quad \text{— (I)}$$

and from ΔAIC

$$\angle e = \angle\beta + \angle\gamma \quad \text{— (II)}$$

Using angle $= \frac{\text{arc}}{\text{Radius}}$ $\left(\theta = \frac{l}{r}\right)$

$$\alpha = \frac{AN}{NO}, \quad \beta = \frac{AN}{NZ} \text{ and } \gamma = \frac{AN}{NC}$$

Using (I) and (II)

$$\angle i = \frac{AN}{NO} + \frac{AN}{NC} \approx \frac{AN}{PO} + \frac{AN}{PC}$$

$$\angle e = \frac{AN}{NZ} + \frac{AN}{NC} \approx \frac{AN}{PI} + \frac{AN}{PC}$$

N is very close to P
$NO = PO$
$NC = PC$
$NI = PI$

Step II :- Using Snell's law

$$\frac{\sin i}{\sin e} = \frac{n_2}{n_1}$$

$$n_1(\sin i) = n_2(\sin e)$$

Here angles are very close

$$n_1 \angle i = n_2 \angle e \quad \text{— (III)}$$

From equation (I), (II) and (III)

$$n_1\left(\frac{AN}{P_O}+\frac{AN}{P_C}\right)=n_2\left(\frac{AN}{P_I}+\frac{AN}{P_C}\right)$$

$$n_1\left[\frac{1}{P_O}+\frac{1}{P_C}\right]=n_2\left[\frac{1}{P_I}+\frac{1}{P_C}\right]$$

$$\frac{n_1}{P_O}-\frac{n_2}{P_I}=\frac{n_2}{P_C}-\frac{n_1}{P_C}$$

$$\boxed{\frac{n_1}{P_O}-\frac{n_2}{P_I}=\frac{n_2-n_1}{P_C}}$$

Using sign Convention.

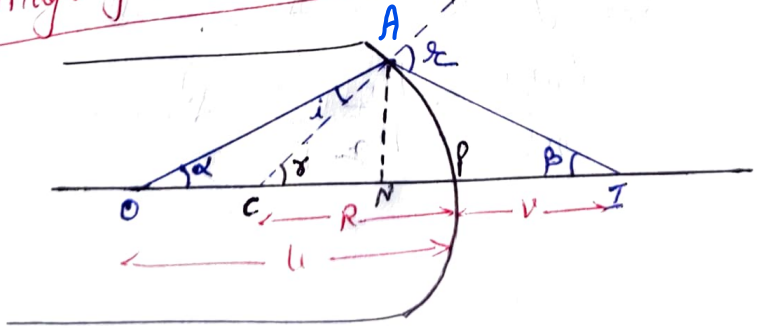
$P_O = -u$, $P_I = -v$, $P_C = R$

$$\frac{n_1}{-u}-\frac{n_2}{(-v)}=\frac{n_2-n_1}{R}$$

$$\boxed{\frac{n_1}{-u}+\frac{n_2}{v}=\frac{n_2-n_1}{R}}$$

c) Refraction at Convex Spherical Surface when object lies in Dens Medium :-

When Image formed is Real :-



O = object , I = Image $(h_2 > h_1)$

Step I :- Determination of $\angle i$ and $\angle e$

α, β and γ be the angles formed by Incident object, Image and Normal with principal axis.

from Δ AOC

$$\angle\gamma = \angle i + \angle\alpha \Rightarrow \angle i = \angle\gamma - \angle\alpha \quad \text{—— (I)}$$

from Δ AIC

$$\angle e = \angle\gamma + \angle\beta \quad \text{—— (II)}$$

Since angles are very small so that

$$\text{Angle} = \frac{\text{arc}}{\text{Radius}} \Rightarrow [\theta = l/r]$$

$$\angle\alpha = \frac{AN}{ON}, \quad \beta = \frac{AN}{IN}, \quad \gamma = \frac{AN}{NC}$$

Using above equation (I) and (II)

$$\angle i = \frac{AN}{NC} - \frac{AN}{ON} \approx \frac{AN}{PC} - \frac{AN}{OP}$$

$$\angle e = \frac{AN}{NC} + \frac{AN}{IN} \approx \frac{AN}{PC} + \frac{AN}{PI}$$

Here N very close to P
ie $NC = PC$
$NO = PO$
$NI = PI$

Step II :- According to Snell's law :-

$$\frac{\sin i}{\sin e} = \frac{n_1}{n_2}$$

$$n_2 \sin i = n_1 \sin r$$

Here angles are very small

ie ($\sin i \sim i$ and $\sin r = r$)

$$n_2 i = n_1 r$$

$$n_2\left[\frac{AN}{PC} - \frac{AN}{OP}\right] = n_1\left[\frac{AN}{PC} + \frac{AN}{PI}\right]$$

$$n_2\left[\frac{1}{PC} - \frac{1}{OP}\right] = n_1\left[\frac{1}{PC} + \frac{1}{PI}\right]$$

$$\frac{n_2}{PC} - \frac{n_2}{OP} = \frac{n_1}{PC} + \frac{n_1}{PI}$$

$$-\frac{n_1}{PI} - \frac{n_2}{OP} = \frac{n_1}{PC} - \frac{n_2}{PC}$$

$$-\frac{n_1}{PI} - \frac{n_2}{OP} = \frac{n_1 - n_2}{PC}$$

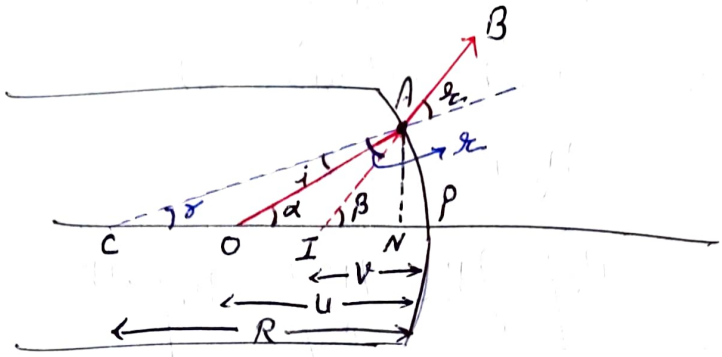
Using Sign Convention $PI = v$, $OP = -u$

$PC = -R$

$$-\frac{n_1}{v} - \frac{n_2}{-u} = \frac{n_1 - n_2}{-R}$$

$$\boxed{\frac{n_2}{u} - \frac{n_1}{v} = \frac{n_2 - n_1}{R}}$$

d) When virtual Image is formed :-



Step I : Determination of $\angle i$ and $\angle r$

Angle α = object.

β = Image

γ = Normal with principal axis.

From Δ AOC

$$\angle\alpha = \angle i + \angle\gamma$$

$$\angle i = \angle\alpha - \angle\gamma \quad \text{— (I)}$$

From Δ AIC

$$\angle\beta = \angle r + \angle\gamma$$

$$\angle r = \angle\beta - \angle\gamma \quad \text{— (II)}$$

Using $\text{angle} = \frac{\text{arc}}{\text{Radius}} \Rightarrow \left[\theta = \frac{l}{r}\right]$

$$\alpha = \frac{AN}{NO}, \quad \beta = \frac{AN}{NI}, \quad \gamma = \frac{AN}{NC}$$

Using (I) and (II)

$$\angle i = \frac{AN}{NO} - \frac{AN}{NC} \sim \frac{AN}{PO} - \frac{AN}{PC}$$

$$\angle \alpha = \frac{AN}{NI} - \frac{AN}{NC} \sim \frac{AN}{PI} - \frac{AN}{PC}$$

Step II :- Using Snell's law :-

$$\frac{\sin i}{\sin r} = \frac{n_1}{n_2}$$

$$n_2 \sin i = n_1 \sin r$$

Angles are very small ie $\sin i \sim i$ and $\sin r \sim r$

$$n_2 i = n_1 r$$

$$n_2\left[\frac{AN}{PO} - \frac{AN}{PC}\right] = n_1\left[\frac{AN}{PI} - \frac{AN}{PC}\right]$$

$$\frac{n_2}{PO} - \frac{n_2}{PC} = \frac{n_1}{PI} - \frac{n_1}{PC}$$

$$\frac{n_2}{PO} - \frac{n_1}{PI} = -\frac{n_1}{PC} + \frac{n_2}{PC}$$

$$\frac{n_2}{PO} - \frac{n_1}{PI} = \frac{-n_1 + n_2}{PC}$$

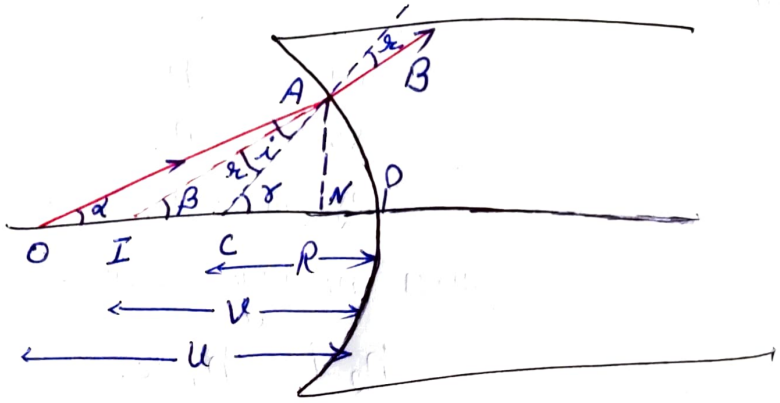
Using Sign Convention :

$$PO = -u, \quad PI = -v, \quad PC = -R$$

$$\frac{n_2}{-u} - \frac{n_1}{-v} = -\frac{n_1 + n_2}{-R}$$

$$\boxed{\frac{n_2}{-u} + \frac{n_1}{v} = \frac{n_1 - n_2}{R}}$$

Refraction at Concave Spherical Surface when object lie in Rarer Medium :-



Step I :- determination of $\angle i$ and $\angle e$

Here α, β and γ be the angles made by the incidin ray, refracted ray and Normal.

from ΔAOC

$$\angle\gamma = \angle i + \angle\alpha$$

$$\angle i = \angle\gamma - \angle\alpha \quad \text{---(I)}$$

from ΔAIC

$$\angle\gamma = \angle e + \angle\beta$$

$$\angle e = \angle\gamma - \angle\beta \quad \text{---(II)}$$

Using angle $= \frac{\text{arc}}{\text{Radius}} = \left(\frac{l}{r} = \theta\right)$

$$\alpha = \frac{AN}{ON}, \quad \beta = \frac{AN}{IN}, \quad \gamma = \frac{AN}{NC}$$

Using in equation (I) and (II)

$$\angle i = \frac{AN}{NC} - \frac{AN}{ON} \sim \frac{AN}{PC} - \frac{AN}{P_0}$$

$$\angle r = \frac{AN}{NC} - \frac{AN}{NZ} \sim \frac{AN}{PC} - \frac{AN}{PZ} \quad \text{(III)}$$

Here (Negligible)
$\therefore NC = PC$
$ON = P_0$
$NZ = PZ$

Step II : Using Snell's law

$$\frac{\sin i}{\sin r} = \frac{n_1}{n_2}$$

$$n_2 \sin i = n_1 \sin r$$

Here $\sin i \sim i$ and $\sin r = r$

$$n_2 \angle i = n_1 \angle r$$

From equation (III)

$$n_2\left[\frac{AN}{PC} - \frac{AN}{P_0}\right] = n_1\left[\frac{AN}{PC} - \frac{AN}{PZ}\right]$$

$$\frac{n_2}{PC} - \frac{n_2}{P_0} = \frac{n_1}{PC} - \frac{n_1}{PZ}$$

$$-\frac{n_2}{P_0} + \frac{n_1}{PZ} = \frac{n_1}{PC} - \frac{n_2}{PC}$$

$$-\frac{n_2}{P_0} + \frac{n_1}{PZ} = \frac{n_1 - n_2}{PC}$$

Using Sign Convention

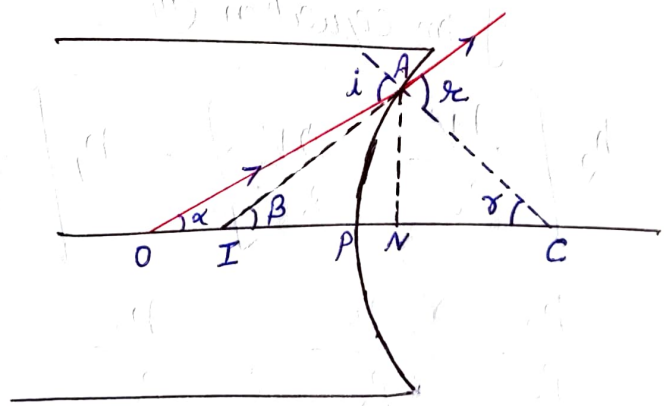
$P_O = -u, \quad P_I = -v, \quad P_C = -R$

$$\frac{-n_2}{-u} + \frac{n_1}{-v} = \frac{n_1 - n_2}{-R}$$

$$\left[\frac{n_2}{u} - \frac{n_1}{v} = \frac{n_2 - n_1}{R}\right]$$

$$\left[\frac{-n_2}{u} + \frac{n_1}{v} = \frac{n_1 - n_2}{R}\right]$$

Refraction at Concave Spherical Surface when object lies in denser Medium :-



Step I :- determination of $\angle i$ and $\angle e$

from ΔAOC

$$\angle i = \angle\alpha + \angle\gamma \quad \text{— (I)}$$

from ΔAIC

$$\angle e = \angle\beta + \gamma \quad \text{— (II)}$$

Here α, β and γ are very small.

i.e $\theta = \frac{l}{r}$

$$\alpha = \frac{AN}{ON}, \quad \beta = \frac{AN}{IN} \quad \text{and} \quad \gamma = \frac{AN}{NC}$$

Using in (I) and (II)

$$\angle i = \frac{AN}{ON} + \frac{AN}{NC} \sim \frac{AN}{OP} + \frac{AN}{PC}$$

$$\angle r = \frac{AN}{IN} + \frac{AN}{NC} \sim \frac{AN}{IP} + \frac{AN}{PC} \qquad \text{(III)}$$

Step II Using Snell's law –

$$\frac{\sin i}{\sin r} = \frac{n_1}{n_2}$$

$$n_2 \sin i = n_1 \sin r$$

Here angles are very small i.e $\sin i \approx i$ or $\sin r \sim r$

$$n_2(i) = n_1(r)$$

from equation (III)

$$n_2\left[\frac{AN}{OP} + \frac{AN}{PC}\right] = n_1\left[\frac{AN}{IP} + \frac{AN}{PC}\right]$$

$$\frac{n_2}{OP} + \frac{n_2}{PC} = \frac{n_1}{IP} + \frac{n_1}{PC}$$

$$\frac{n_2}{OP'} - \frac{n_1}{IP} = \frac{n_1 - n_2}{PC}$$

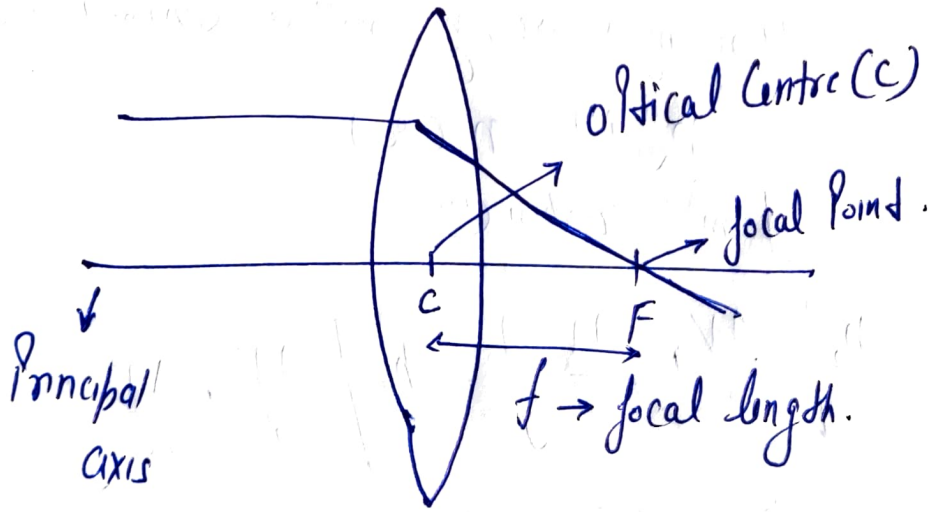
Using Cartesian Sign Convention

$$OP = -u, \quad IP' = -v, \quad PC = R$$

$$\frac{n_2}{-u} - \frac{n_1}{(-v)} = \frac{n_1 - n_2}{R}$$

$$\left[-\frac{n_2}{u} + \frac{n_1}{v} = \frac{n_1 - n_2}{R} \right]$$

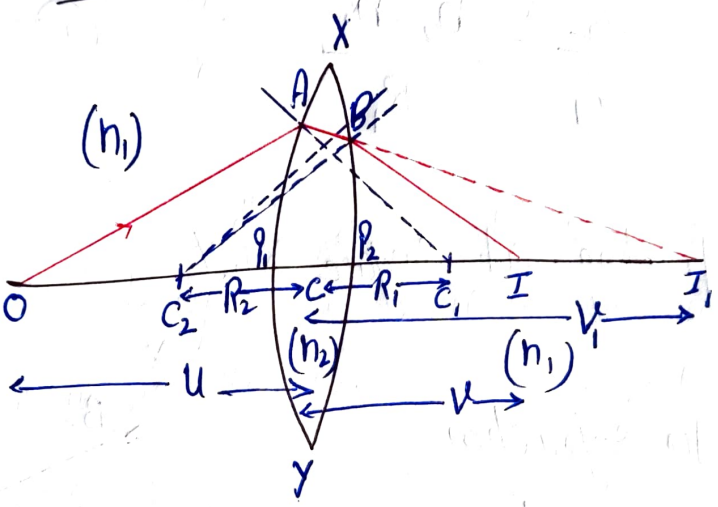
Lens :- A lens is a piece of transparent material bounded by two refracting surfaces out of which at least one is curved.



Lens Maker's formula:-

The formula giving Relation between the focal length(f) of the lens, refractive Index of the material of the lens (n) and the radii of curvature of its surface (R_1 and R_2) is known as lens maker's formula

Derivation for convex lens:-



Let us consider a convex lens made of a material of absolute refractive Index(n_2)

n_1 = refractive Index of air (ie $n_2 > n_1$)

C_1 and C_2 Centre of curvature

R_1 and R_2 Radius of curvature.

O = object

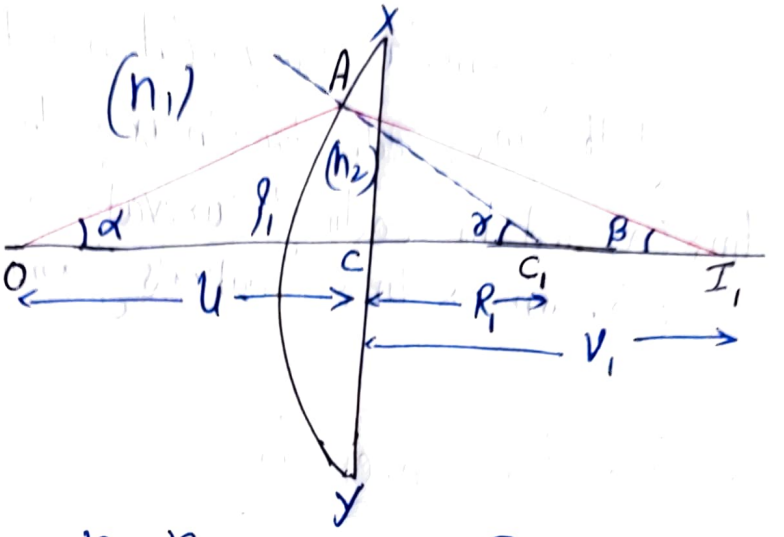
I_1 = Image formed by phase XP_1Y

I = Image formed by phase XP_2Y.

Step I: Refracting at surface XP, Y

I_1 is Image In dense medium and Real.

Since object in Rarer medium.

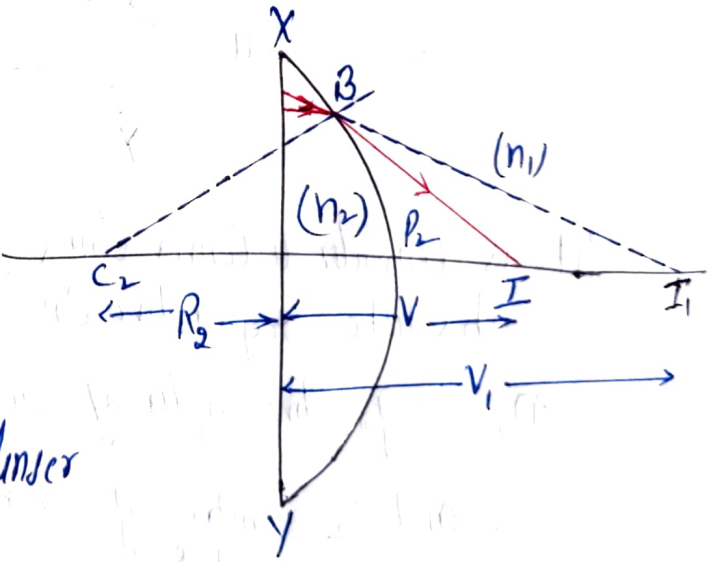


$$-\frac{n_1}{u} + \frac{n_2}{V_1} = \frac{n_2 - n_1}{R_1} \quad \text{——— (I)}$$

Step II Refracting at surface XP$_2$Y

Now after refraction from the second surface XP$_2$Y the final image is formed.

Now when object in denser medium.



$$-\frac{n_2}{V_1} + \frac{n_1}{v} = \frac{n_1 - n_2}{R_2} \quad \text{——— (II)}$$

Adding equation (I) and (II)

$$-\frac{n_1}{u} + \frac{n_2}{v_1} - \frac{n_2}{v_1} + \frac{n_1}{v} = \frac{n_2 - n_1}{R_1} + \frac{(n_1 - n_2)}{R_2}$$

$$-\frac{n_1}{u} + \frac{n_1}{v} = \frac{n_2 - n_1}{R_1} - \frac{(n_2 - n_1)}{R_2}$$

$$-\frac{n_1}{u} + \frac{n_1}{v} = (n_2 - n_1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$n_1\left(-\frac{1}{u} + \frac{1}{v}\right) = (n_2 - n_1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$-\frac{1}{u} + \frac{1}{v} = \left(\frac{n_2}{n_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Here $\frac{n_2}{n_1} = n$ (Relative refractive index)

$$\therefore \quad -\frac{1}{u} + \frac{1}{v} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

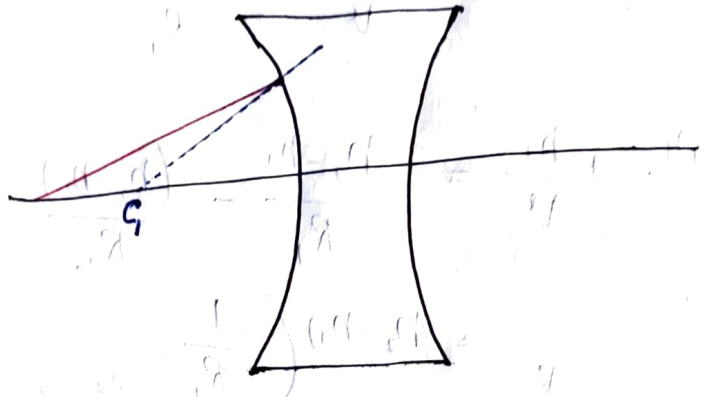
if object is at infinity ie $u = \infty$ image at focal point

$$-\frac{1}{\infty} + \frac{1}{v} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad [\because v = f]$$

Lens maker formula

Lens maker's formula for a concave lens :-



Numerical :- The Radius of curvature of the faces of a double convex lens are 10cm and 15cm if focal of the lens is 12 cm, find the refractive index of the material of the lens.

Solution :- Here $R_1 = 10cm, R_2 = 15cm$

$f = 12cm$, $n = ?$

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{12} = (n-1)\left(\frac{1}{10} + \frac{1}{15}\right)$$

$$\frac{1}{12} = (n-1)\left(\frac{3+2}{30}\right)$$

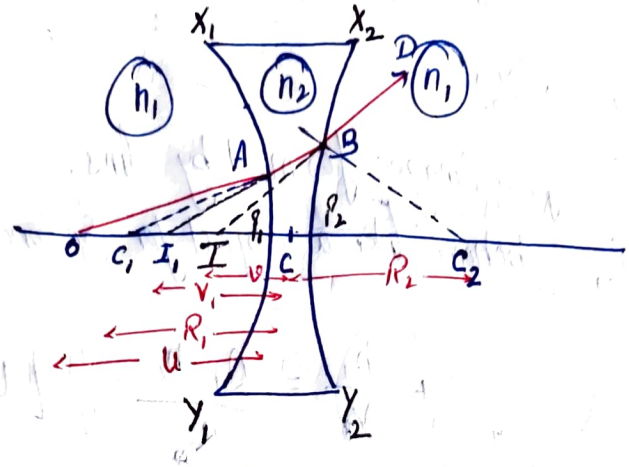
$$\frac{1}{12} = (n-1)\left(\frac{5}{30}\right)$$

$$(n-1) = \frac{30}{5} \times \frac{1}{12}$$

$$n = \frac{30}{60} + 1$$

$$\boxed{n = \frac{3}{2} = 1.5}$$

Lens maker's formula for a concave lens :-



Let us consider a concave lens $X_1 I_1 Y_1$ and $X_2 I_2 Y_2$ two spherical surfaces having Radius of curvature R_1 and R_2.

Step I :- Refraction at $X_1 I_1 Y_1$

O = object
R_1 = Radius of curvature
C_1 = Centre of curvature.
I_1 = Image formed by first surface.

Since object in rarer and image in denser medium

$$-\frac{n_1}{u} + \frac{n_2}{v} = \frac{n_2 - n_1}{R}$$

$u = u_1$, $v = v_1$ and $R = R_1$

$$-\frac{n_1}{u} + \frac{n_2}{v_1} = \frac{n_2 - n_1}{R_1} \quad \text{— (1)}$$

Step II : Refraction at $X_2 P_2 Y_2$

Ray AB inside the lens will suffer another refraction at surface $X_2 P_2 Y_2$

I = final image formed by lens.

I_1 = behave like a object for second surface.

$$-\frac{n_2}{u} + \frac{n_1}{v} = \frac{n_1 - n_2}{R} \qquad [u = v_1, v = v, R = R_2]$$

$$-\frac{n_2}{v_1} + \frac{n_1}{v_0} = -\frac{(n_2 - n_1)}{R_2} \quad \text{——— (II)}$$

Step III : Adding (I) and (II)

$$-\frac{n_1}{u} + \frac{n_2}{v_1} = \frac{n_2}{v_1} + \frac{n_1}{v_0} = \frac{n_2 - n_1}{R_1} + \frac{n_2 - n_1}{R_2}$$

$$-\frac{n_1}{u} + \frac{n_1}{v} = (n_2 - n_1)$$

$$\frac{1}{v} - \frac{1}{u} = \left(\frac{n_2 - n_1}{n_1}\right)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$\frac{1}{v} - \frac{1}{u} = \left(\frac{n_2}{n_1} - 1\right)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

Here $\frac{n_2}{n_1} = n$

$$\frac{1}{v} - \frac{1}{u} = (n-1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

for ($u = \infty$) the image is formed at principal focus

$\therefore v = f$

$$\frac{1}{f} - \frac{1}{\infty} = (n-1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$\boxed{\frac{1}{f} = (n-1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]}$$

Numerical: A double Concave lens of glass of refractive index 1.6 has Radii of curvature of 40 cm and 60 cm. Calculate its focal length in air.

Sol: Given $n = 1.6$, $R_1 = -40$ cm and $R_2 = 60$ cm

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$\frac{1}{f} = (1.6-1)\left(\frac{1}{-40} - \frac{1}{60}\right)$$

$$\frac{1}{f} = (0.6)\left(\frac{-3-2}{120}\right)$$

$$\frac{1}{f} = \frac{6}{10}\left[\frac{-5}{120}\right]$$

$$\frac{1}{f} = -\frac{1}{40}$$

$$\boxed{f = -40\text{ cm}}$$

Numerical:- A biconvex lens has a focal length $\frac{2}{3}$ times the Radius of curvature of either surface. Calculate the refractive index of lens material.

Sol:-

$$f = \frac{2}{3}R_1 \Rightarrow R = \frac{3}{2}f$$

$$R_1 = R, \quad R_2 = -R$$

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f} = (n-1)\left(\frac{1}{R} + \frac{1}{R}\right)$$

$$\frac{1}{f} = (n-1)\left(\frac{2}{R}\right)$$

$$(n-1) = \frac{1}{f} \times \frac{R}{2}$$

$$(n-1) = \frac{1}{f} \times \frac{1}{2} \times \frac{3}{2} f$$

$$(n-1) = \frac{3}{4}$$

$$n = \frac{3}{4} + 1$$

$$n = \frac{7}{4} = 1.75$$

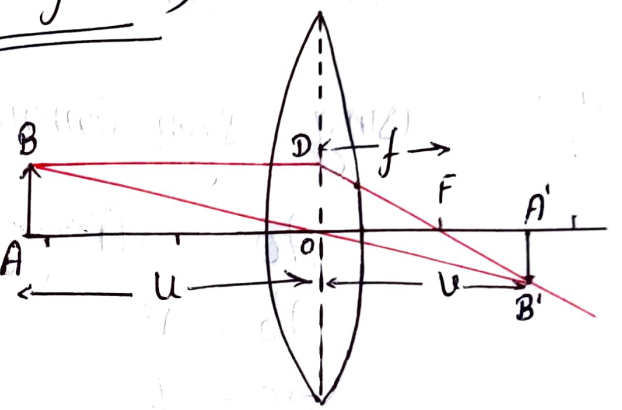
$$\boxed{n = 1.75}$$

Thin Lens formula :-

The Relation between the distance of object (u) and image (v) from the lens and the focal length (f) of the lens is known as thin lens formula.

(1) Convex lens (Real Image is formed)

Consider a Convex lens focal length (f)



Let AB = object
u = distance of object
A'B' = Image
v = distance of Image from optical centre.

Step I

From ΔABO and $\Delta A'B'O$

$$\frac{AB}{A'B'} = \frac{AO}{A'O} \quad \text{---} \quad (1)$$

From ΔDOF and $A'B'F$

$$\frac{DO}{A'B'} = \frac{OF}{A'F} \qquad [DO = AB]$$

$$\frac{AB}{A'B'} = \frac{OF}{A'F} \quad \text{---} \quad (II)$$

From (I) and (II)

$$\frac{AO}{A'O} = \frac{OF}{A'F}$$

Here $A'F = OA' - OF$

$$\frac{AO}{A'O} = \frac{OF}{OA' - OF} \quad \text{---(III)}$$

Step II :- Using Sign Convention

$AO = -u$

$A'O = v$

$OF = f$

$$\frac{-u}{v} = \frac{f}{v-f}$$

$$-u(v-f) = vf$$

$$-uv + uf = vf$$

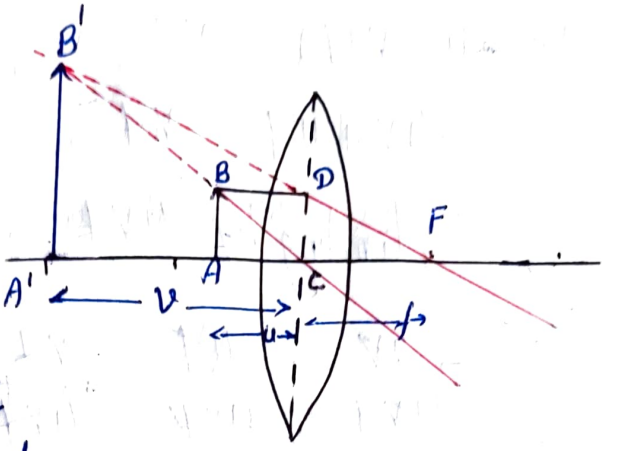
dividing by uvf on both side

$$-\frac{uv}{uvf} + \frac{uf}{uvf} = \frac{vf}{uvf}$$

$$-\frac{1}{f} + \frac{1}{v} = \frac{1}{u}$$

$$\boxed{-\frac{1}{u} + \frac{1}{v} = \frac{1}{f}} \rightarrow \text{Thin lens formula}$$

⇒ Convex lens (Virtual Image formed) :-



Consider a Convex lens of focal length (f)

$A'B'$ = Image

AB = object

v = distance of image

u = distance of object.

Step I:- from ΔABC and $A'B'C$

$$\frac{AB}{A'B'} = \frac{AC}{A'C} \quad \text{——(I)}$$

from ΔCDF and $\Delta A'B'F$

$$\frac{DC}{A'B'} = \frac{CF}{A'F} \quad \text{——(II)} \Rightarrow [DC = AB]$$

from equation (I) and (II)

$$\frac{AC}{A'C} = \frac{CF}{A'F} \qquad [A'F = A'C + CF]$$

$$\frac{AC}{A'C} = \frac{CF}{A'C + CF}$$

Using sign convention

$AC = -u$, $A'C = -v$, $CF = f$

$$\frac{+u}{+v} \times \frac{f}{-v+f}$$

$$u(-v+f) = vf$$

$$-uv + uf = vf$$

dividing by uvf

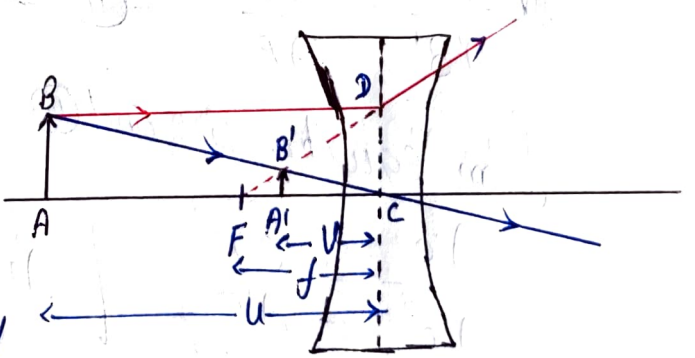
$$-\frac{uv}{uvf} + \frac{uf}{uvf} = \frac{vf}{uvf}$$

$$-\frac{1}{f} + \frac{1}{v} = \frac{1}{u}$$

$$\boxed{-\frac{1}{u} + \frac{1}{v} = \frac{1}{f}}$$ lens formula.

(III) Concave lens :-

Let us consider the Concave lens of focal length (f) and object AB.



The image formed by the lens is virtual $A'B'$

AB = object

u = distance of object

v = distance of image from lens.

Step I :-

From ΔABC and $\Delta A'B'C$

$$\frac{AB}{A'B'} = \frac{Ac}{A'c} \quad \text{---} (1)$$

Similarly ΔDCF and $\Delta A'B'F$

$$\frac{Dc}{A'B'} = \frac{CF}{A'F}$$

Here $Dc = AB$

$$\frac{AB}{A'B'} = \frac{CF}{A'F} \quad \text{---} (11)$$

From (1) and (11)

$$\frac{AC}{A'c} = \frac{CF}{A'F} \qquad [A'F = Fc - A'c]$$

$$\frac{Ac}{A'c} = \frac{CF}{Fc - A'c}$$

Using sign convention

$Ac = -u$, $A'c = -v$, $CF = -f$

$$\frac{+u}{+v} = \frac{-f}{-f+v}$$

$$\frac{u}{v} = \frac{-f}{-f+v}$$

$$u(-f+v) = -fv$$

$$-fu + uv = -fv$$

dividing by uvf

$$\frac{-fu}{uvf} + \frac{uv}{uvf} = \frac{-fv}{uvf}$$

$$-\frac{1}{v} + \frac{1}{f} = -\frac{1}{u}$$

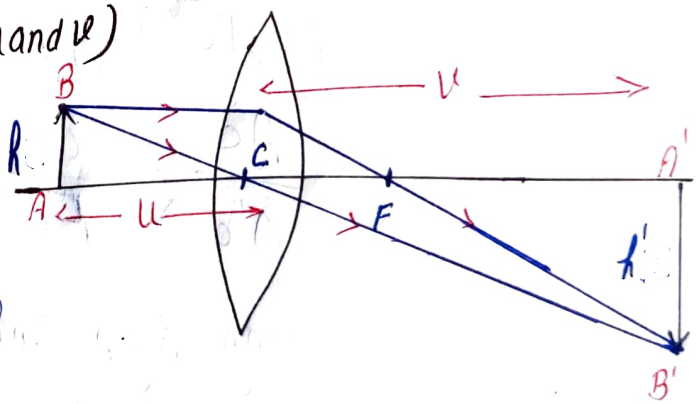
$$\boxed{-\frac{1}{u} + \frac{1}{v} = -\frac{1}{f}} \rightarrow \text{Lens formula}$$

Linear magnification :-

Linear magnification is defined as the Ratio of the size of the image to the size of object.

(1) Linear magnification (in terms u and v)

a) Convex lens, Real image :-



from $\Delta A'B'C$ and ΔABC

$$\frac{A'B'}{AB} = \frac{CA'}{CA} \quad \text{— (1)}$$

applying Sign Convention

$A'B' = -h$, $AB = h$

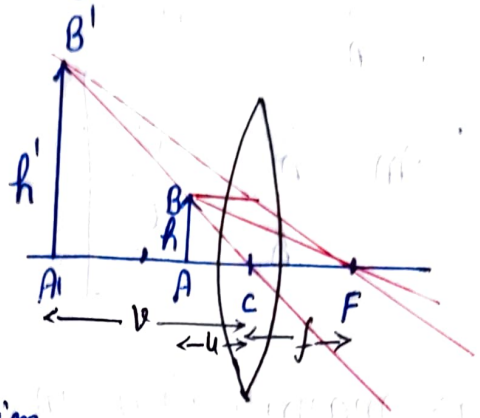
$AC = -u$, $A'C = v$

using in (1)

$$\frac{-h}{h} = \frac{v}{-u}$$

$$\boxed{m = -\frac{h}{h} = -\frac{v}{u}}$$

b) Convex lens :- (virtual image)



from triangle A'B'C and Δ ABC

$$\frac{A'B'}{AB} = \frac{A'C}{AC} \quad -(1)$$

Using Sign Convention

$A'B' = h'$, $AB = h$

$A'C = -v$, $AC = -u$

using in (1)

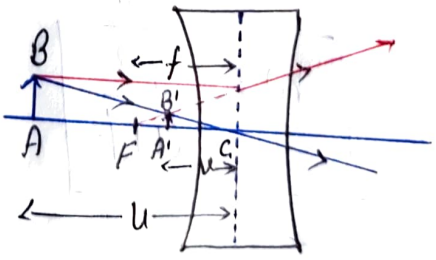
$$\frac{h'}{h} = \frac{+v}{+u}$$

$$\boxed{m = \frac{h'}{h} = \frac{v}{u}}$$

(c) Concave lens :-

from Δ A'B'C and ΔABC

$$\frac{A'B'}{AB} = \frac{A'B'}{AC} \quad -(1)$$



Using Sign Convention

$A'B' = h'$, $AB = h$

$AC = -u$, $A'C = -v$

$$\frac{h'}{h} = +\frac{v}{+u}$$

$$\boxed{m = \frac{h'}{h} = \frac{v}{u}}$$

(II) Linear magnification in terms of u and f

We know that

$$-\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

multiplying by u

$$-1 + \frac{u}{v} = \frac{u}{f}$$

$$\frac{u}{v} = \frac{u}{f} + 1$$

$$\frac{u}{v} = \frac{u+f}{f}$$

$$\frac{v}{u} = \frac{f}{u+f} \qquad \left(\frac{v}{u} = m\right)$$

$$\boxed{m = \frac{f}{u+f}}$$

iii) Linear magnification in Terms of v and f :-

$$+\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$$

multiplying v both side

$$-\frac{v}{u}+1=\frac{v}{f}$$

$$-\frac{v}{u}=\frac{v}{f}-1$$

$$\frac{v}{u}=1-\frac{v}{f}$$

$$\frac{v}{u}=\frac{f-v}{f} \qquad \left[m=\frac{v}{u}\right]$$

$$\boxed{m=\frac{f-v}{f}}$$

Power of a Lens :-

Power of the lens is define as the reciprocal of focal length of the lens expressed in meter.

$$P=\frac{1}{f(m)}=\frac{100}{f(cm)}$$

from lens maker formula

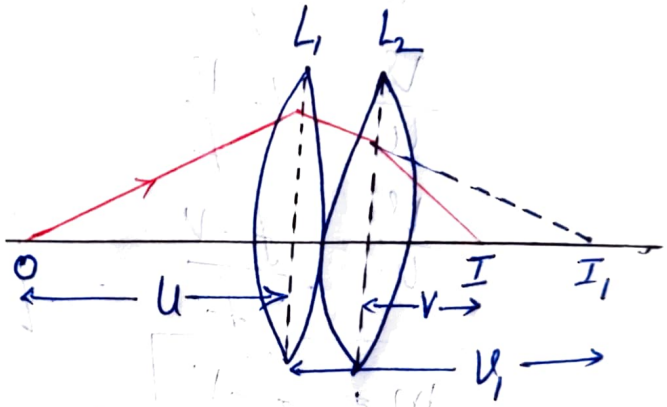
$$\frac{1}{f}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$\therefore$ Power of lens

$$P = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

S.I. unit of power is dioptre (D).

$\Rightarrow$ Thin Lenses in Contact :-



Let us Consider two (L_1, L_2) lens focal length f_1 and f_2.

O = object

u = distance of object.

I_1 = Image formed by lens L_1

V_1 = is the distance of image formed by lens (L_1)

I = final image.

V = final distance of the image.

Now lens formula for lens (L_1)

$$-\frac{1}{u} + \frac{1}{v_1} = \frac{1}{f_1} \quad \text{---} \quad (1)$$

lens formula for lens (L_2)

$$-\frac{1}{v_1} + \frac{1}{v} = \frac{1}{f_2} \quad \text{---} \quad (II)$$

adding equation (I) and (II)

$$-\frac{1}{u} + \frac{1}{v_1} - \frac{1}{v_1} + \frac{1}{v} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$-\frac{1}{u} + \frac{1}{v} = \frac{1}{f_1} + \frac{1}{f_2} \qquad \left[\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{F}\right]$$

$$\boxed{-\frac{1}{u} + \frac{1}{v} = \frac{1}{F}}$$

if (n) number of lenses Combine together

$$\boxed{\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} + \cdots + \frac{1}{f_n}}$$

Power of lens :

$$P = \frac{1}{f}$$

If two lens are in Contact

$$P = P_1 + P_2$$

If more than two are in Contact

$$\boxed{P = P_1 + P_2 + \cdots + P_n}$$

Magnification of an equivalent lens :-

magnification of two lens

$$m = m_1 \times m_2$$

If there are (n) number of lens

$$\boxed{m = m_1 \times m_2 \times \ldots \times m_n}$$

Uses of Combination of lens :-

It used in optical instruments like Telescope, microscope, Camera and binocular's.

Numerical:- ① Calculate the power of an equiconvex lens having focal length 0.5 ② What is refractive index of glass if focal length of a lens of Radii of curvature 10 cm and 15 cm is 12 cm ③ A Convex has 20 cm focal length in air. what is focal length in water ? $\mu_w = 1.33$ and $\mu_g = 1.5$.

Solution:- ① Power $P = \frac{1}{f} = \frac{1}{0.5} = \frac{10}{5} = 2$

$$P = 2$$

(11) $f = 12\,cm$, $R_1 = 10\,cm$, $R_2 = -15\,cm$, $n = ?$

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{12} = (n-1)\left(\frac{1}{10} + \frac{1}{15}\right)$$

$$\frac{1}{12} = (n-1)\left(\frac{3+2}{30}\right)$$

$$\frac{1}{12} = (n-1)\left(\frac{5}{30}\right)$$

$$(n-1) = \frac{6}{12} = \frac{1}{2}$$

$$n = \frac{1}{2} + 1 = \frac{3}{2} = 1.5$$

$\rightarrow$ **$n = 1.5$**

Refrechive index

(111) given

$f_a = 20\,cm$, $n = \frac{n_g}{n_a}$, $n_g = 1.5$, $n_{wo} = 1.33$

$$\frac{1}{f_a} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{20} = \left(\frac{n_g}{n_a} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{20} = \left(\frac{1.5}{1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{20} = (0.5)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{20 \times 0.5} = \frac{1}{20 \times 5}$$

$$\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{10}$$

Also focal length in water

$$\frac{1}{f_w} = \left(\frac{n_g}{n_w} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f_w} = \left(\frac{n_g}{n_w} - 1\right)\left(\frac{1}{10}\right)$$

$$\frac{1}{f_w} = \left(\frac{1.5}{1.33} - 1\right)\left(\frac{1}{10}\right)$$

$$\frac{1}{f_w} = \left(\frac{1.5 - 1.33}{1.33}\right)\frac{1}{10}$$

$$\frac{1}{f_w} = \left(\frac{0.17}{1.33}\right)\frac{1}{10}$$

$$f_w = \frac{1.33 \times 10}{0.17}$$

$$\boxed{f_w = 78\ cm}$$

Numerical :- Two thin lenses of focal lengths +10 cm and -5 cm are kept in Contact. What is the (a) focal length (b) Power of the Combination.

Solution :- given

a) $f_1 = 10\,cm, \; f_2 = -5\,cm$

focal length

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$\frac{1}{f} = \frac{1}{10} - \frac{1}{5}$$

$$= \frac{1-2}{10}$$

$$\frac{1}{f} = -\frac{1}{10} \Rightarrow \boxed{f = -10\,cm}$$

b) Power of Combination lens

$$P = \frac{1}{f} = \frac{1}{-10}$$

$$P = \frac{100}{-10} = -10\,D$$

$$\boxed{P = -10\,D}$$

Numerical:- A Convex lens of focal length 30 cm and Concave focal length 60 cm are placed in Combination. If the object is placed 40cm away from the Combination, find the position of the image.

Solution:-

$f_1 = 30\ cm,\ f_2 = -60\ cm,\ u = -40cm$

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$\frac{1}{F} = \frac{1}{30} - \frac{1}{60} = \frac{2-1}{60} = \frac{1}{60}$$

$$\boxed{F = 60cm}$$

Position of image (using lens formula)

$$-\frac{1}{u} + \frac{1}{v} = \frac{1}{F}$$

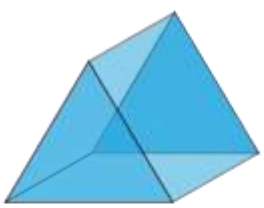
$$-\frac{1}{(-40)} + \frac{1}{v} = \frac{1}{60}$$

$$\frac{1}{v} = \frac{1}{60} - \frac{1}{40}$$

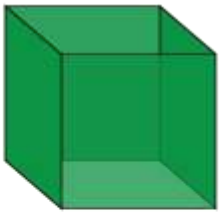
$$\frac{1}{v} = \frac{2-3}{120}$$

$$\frac{1}{v} = -\frac{1}{120} \Rightarrow \boxed{v = -120cm}$$

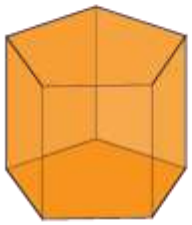
Prism: - A simple prism is a homogeneous transparent refractive medium bounded by at least two non-parallel plane surfaces inclined at some angle.



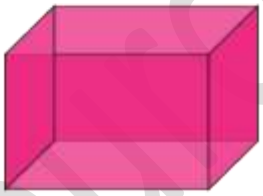
Triangular prism



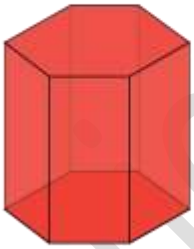
Square prism



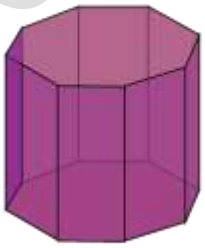
Pentagonal prism



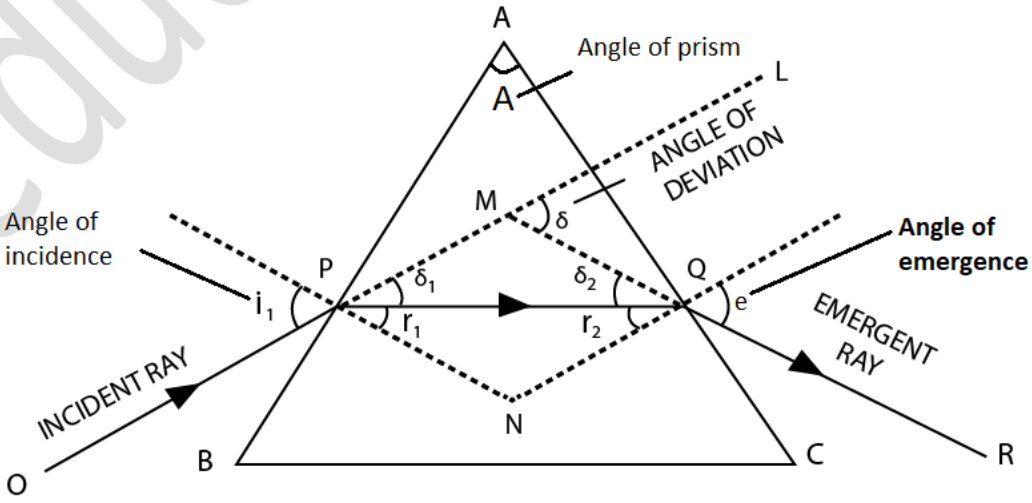
Rectangular prism



Hexagonal prism



Octagonal prism

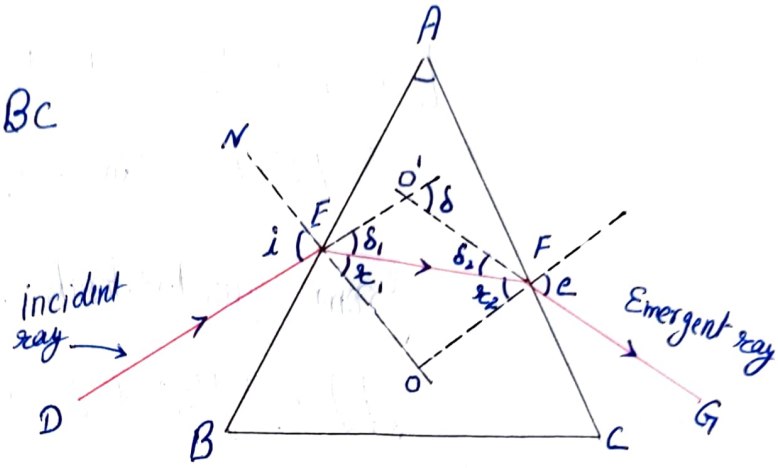


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Refraction due to prism :-

Let us consider a Prism ABC

DE = Incident ray
$\angle i$ = angle of Incident
FG = emergent ray
$\angle e$ = angle of emergent.
δ = angle of deviation.



The angle between the Incident ray and emergent ray is known as angle of deviation.

Determination of angle of deviation (δ)

Here $\angle DEN = \angle O'EO$ [vertical opposit angle]

$$\angle i = \angle \delta_1 + \angle r_1$$

$$\angle \delta_1 = \angle i - \angle r_1 \quad \text{——} (I)$$

Similarly

$$\angle \delta_2 = \angle e - \angle r_2 \quad \text{——} (II)$$

from triangle O'EF

$$\delta = \delta_1 + \delta_2 \quad \text{(Total angle of deviation)}$$

$$\delta = \angle i - \angle r_1 + \angle e - \angle r_2$$

$$\delta = (\angle i + \angle e) - (\angle r_1 + \angle r_2) \quad \text{——} (III)$$

from ΔEOF (using angle sum property)

$$\angle FEO + \angle FOE + \angle EFO = 180$$

$$\angle r_1 + \angle FOE + \angle r_2 = 180$$

$$\angle FOE = 180 - (\angle r_1 + \angle r_2) \quad \text{— (IV)}$$

from quadrilateral AEOF

$$\angle A + \angle AEO + \angle EOF + \angle OFA = 360$$

$$\angle A + 90 + \angle FOE + 90 = 360$$

$$\angle A + 180 + 180 - (\angle r_1 + \angle r_2) = 360$$

$$\angle A + 360 - (\angle r_1 + \angle r_2) = 360$$

$$\angle A = (\angle r_1 + \angle r_2) \quad \text{— (V)}$$

from equation (III) and (V)

$$\boxed{\delta = (i + e) - A}$$

$$\boxed{\delta + A = i + e}$$

Thus, sum of angle of deviation and the angle of prism is equal to the sum of the incident angle the angle of emergence.

⇒ Deviation produced by a prism of small angle i.e. very thin prism :-

Consider a small angle prism (Thin prism)

According to Snell's law

$$n = \frac{\sin i}{\sin r_1}$$

Here angle of incident very small

ie $\sin i \approx i$

IIrly $\sin r_1 = r_1$

(rarer to denser) $\Rightarrow n = \frac{i}{r_1}$ or $\boxed{i = n r_1}$ — (I)

Now for denser to rarer medium

$$\frac{1}{n} = \frac{\sin r_2}{\sin e} = \frac{r_2}{e}$$

$$n = \frac{e}{r_2} \Rightarrow \boxed{e = n r_2}$$ — (II)

Now from angle of deviation.

$$\delta = (i + e) - A$$ — (III)

Using (I) and (II) in (III)

$$\delta = (n r_1 + n r_2) - A$$

$$\delta = n(r_1 + r_2) - A$$

We know that

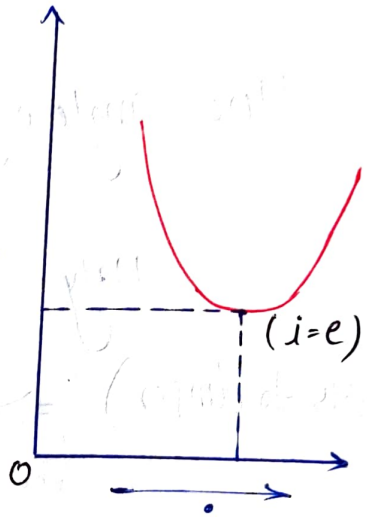
$$r_1 + r_2 = A$$

$$\delta = nA - A$$

$$\boxed{\delta = (n-1)A}$$

Prism formula :-

If the angle of incident is increased gradually, then the angle of deviation decreases, attains a minimum value (δ_m) and then again increases



When angle of deviation is minimum, the prism is said to be placed in the minimum deviation position. There is only one angle of incident for which angle of deviation is minimum.

ie $(i = e)$ and $(r_1 = r_2) = r$, $\delta = \delta_{min}$

We know that $r_1 + r_2 = A$

ie $r + r = A$

$2r = A \Rightarrow \boxed{r = A/2}$

Now $\delta + A = i + e$

$\delta_{min} + A = i + i$

$$\delta_m + A = 2i$$

$$\left[i = \frac{\delta_m + A}{2} \right]$$

from Snell's law

$$n = \frac{\sin i}{\sin r}$$

$$n = \frac{\sin\left(\frac{\delta_{min} + A}{2}\right)}{\sin(A/2)}$$

→ Prism formula

Note:- A Ray of light will not emerge out of a prism for any angle of incidence if $(A > 2C)$

C = critical angle.

ie $\frac{\sin 90}{\sin r_1} = n \quad [\angle i = 90]$

$$n = \frac{1}{\sin r_1}$$

$$\sin r_1 = \frac{1}{n} = \sin C$$

$$\boxed{r_1 = C}$$

Numerical :- A ray of light passes through a glass prism, such that the angle of incidence is equal to angle of emergence. If angle of emergence is 3/4 times the angle of the prism, Calculate the angle of elvication when angle of prism is 30.

Sol given that

$$\angle i = \angle e \quad \text{and} \quad \angle e = \frac{3}{4} A$$

$$A = 30 \quad \text{ie} \quad \angle e = \frac{90}{4}$$

$$\delta + A = i + e$$

$$\delta + 30 = \angle e + \angle e$$

$$\delta = 2e - 30$$

$$\delta = 2\left(\frac{90}{4}\right) - 30$$

$$\boxed{\delta = 15^\circ}$$

Numerical A thin prism of 2° angle gives a deviation of 1° what is the value of refractive index of the material of the prism.

Solution :- given that

$$A = 2^\circ, \quad \delta = 1^\circ$$

$$\delta = (n-1)A$$

$$(n-1) = \frac{\delta}{A} = \frac{1}{2}$$

$$n = \frac{1}{2} + 1$$

$$\boxed{n = \frac{3}{2}} = 1.5$$

Numerical: Calculate the refractive index of the material of an equilateral prism for which the angle of minimum deviation is 60.

Solution:

given $\delta_{min} = 60$.

equilateral prism i.e. $\theta = A = 60$

$$n = \frac{\sin[(A+\delta_m)/2]}{\sin(A/2)}$$

$$= \frac{\sin[(60+60)/2]}{\sin(60/2)}$$

$$= \frac{\sin(60)}{\sin(30)} = \frac{\sqrt{3}/2}{1/2}$$

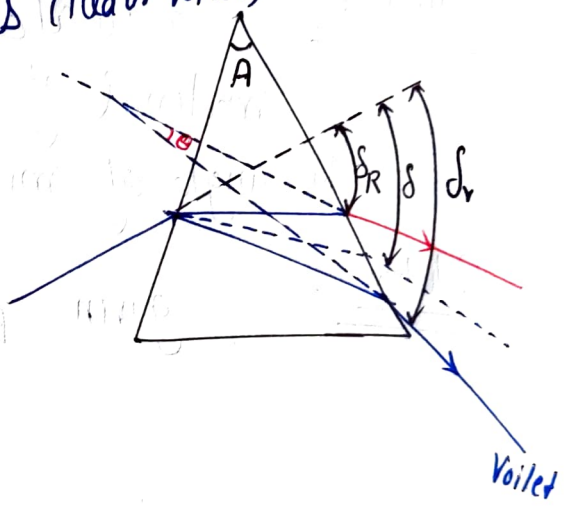
$$\boxed{n = \sqrt{3}}$$

Dispersion of Light Through Prism :-

The phenominon of splitting white light into its constituind colours is called dispersion of light.

Angular Dispersion and Dispersive Power :-

Angular dispersion is defined as the difference in the deviation suffered by the two extreme colours (Red or Voiled) in passing through a prism. (θ)



ie $\theta = \delta_v - \delta_r$ — (I)

for Red Colour

$\delta_r = (n_r - 1)A$

for voiled Colour

$\delta_v = (n_v - 1)A$

using in equation (I)

$\theta = (n_v - 1)A - (n_r - 1)A$

$= (n_v - 1 - n_r + 1)A$

$$\boxed{\theta = (n_v - n_r)A} \quad \text{— (II)}$$

Dispersive Power : It the Ratio of the angular dispersion to the mean deviation produced by the prism.

It is denoted by (ω)

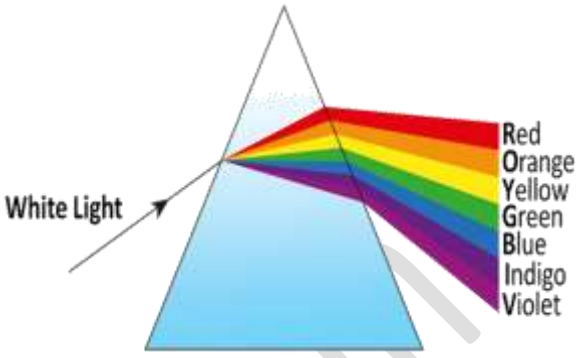
$$\boxed{\omega = \frac{\theta}{\delta} = \frac{\delta_v - \delta_r}{\delta}}$$

from (II)

$$\omega = \frac{(n_v - n_r)A}{(n-1)A} = \frac{(n_v - n_r)}{(n-1)}$$

Dispersion of light though prism: -

The phenomenon of splitting of white light into its constituent colour when passes thought the transparent prism.



Cause of Dispersion of Light

The separation of different colors present in white light is because of different deviations suffered by them, when they pass through a prism because refractive index of a medium is different for different colour.

According to the Cauchy's formula: -

$$n(\lambda) = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4}$$

This relation shows that the refractive index of the material depends upon the wave length of light. $\boldsymbol{n\ \alpha\ \frac{1}{\lambda}}$

Here the refractive index of the material is inversely proportional to the wavelength. The wavelength of different colour is different i.e., red has maximum and violet has minimum wave length $(\boldsymbol{\lambda_r > \lambda_v})$ therefore, n for red is minimum and violet has maximum $(\boldsymbol{n_r < n_v})$.

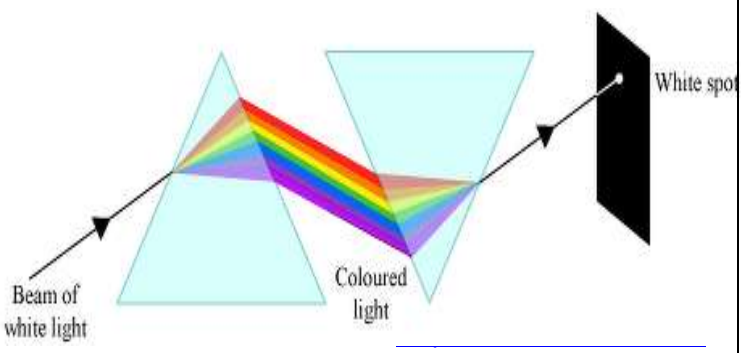
Now deviation produced by the small angled prism is:

$$\boldsymbol{\delta = (n-1)A}$$
$$\boldsymbol{\delta_r = (n_r - 1)A}$$
$$\boldsymbol{\delta_v = (n_r - 1)A}$$
$$\boldsymbol{\delta_r < \delta_v}$$

This shows that the **red colour deviates least and the violet colour deviates the most.**

Issac Newton's classic experiment on dispersion: -

Sir Isaac Newton conducted a classic experiment on dispersion. He passed a beam of white light through a triangular glass prism and concluded that the light from the Sun form a spectrum of different colours.



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This Shows that: -

(a) Dispersive power depends only on the nature of the material of the prism.
(b) Dispersive power is independent of the angle of prism.
(c) Dispersive power is a dimensionless quantity and has no unit.

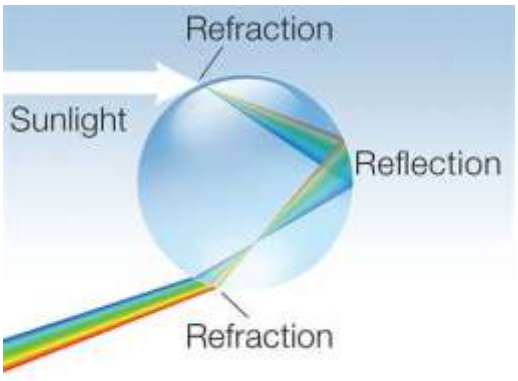
Natural phenomenon related to Sunlight: -

(a)Rainbow: -

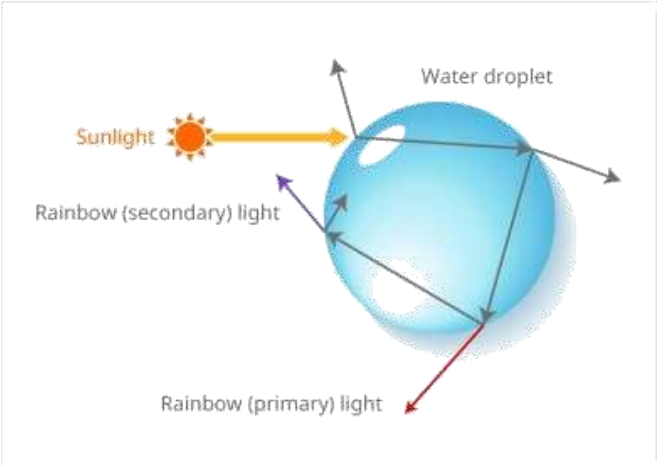
Rainbow is formed due to the dispersion of light suffering refraction and reflection in the droplets present in the atmosphere. Thus, rainbow is the combined effect of refraction, reflection and dispersion of light.

There are two type of rainbows-

1. **Primary Rainbow:**
   - Formed when sunlight falls on a water droplet suspended in the air and under goes two refractions and one reflection.
   - The droplet acts as a prism, causing the sunlight to undergo refraction, dispersion, and reflection.
   - Refracted light is dispersed into its constituent colors, with red deviated the least and violet deviated the most.
   - Reflected light undergoes another refraction as it exits the droplet.
   - The observer sees a concentrated color from each droplet, creating a spectrum.
   - The outer edge of the primary rainbow is red, and the inner edge is violet.

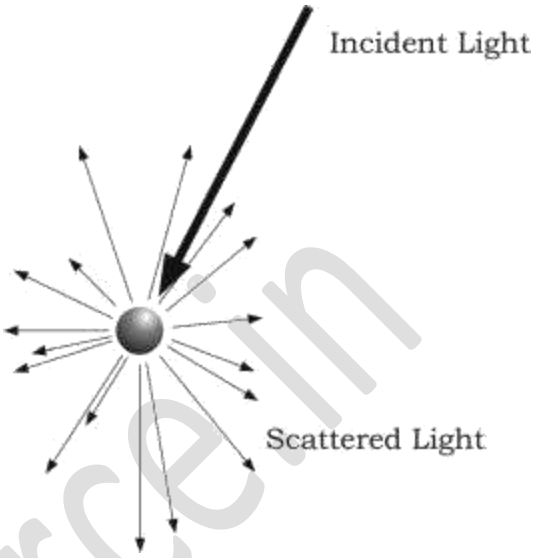


2. **Secondary Rainbow:**
   - Formed due to two refractions and two total internal reflections of light incident on the droplets.
   - The secondary rainbow appears as a colored band with violet on the outside and red on the inside.
   - It is fainter than the primary rainbow, and the colors are inverted.



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(b) Scattering of Light



The phenomenon of dispersing light in all directions by atoms and molecules is referred to as the scattering of light. When sunlight penetrates the Earth's atmosphere, it is absorbed by the small atoms and molecules present in the atmospheric gases. Subsequently, these atoms and molecules re-emit light in various directions. The entity responsible for scattering light is termed a scatterer.

The extent of light scattering is contingent upon the size of the scatterers within the atmosphere. According to Rayleigh, the intensity of the scattered light is inversely proportional to the fourth power of the wavelength of the incident light.

(c) Blue Color of the Sky:

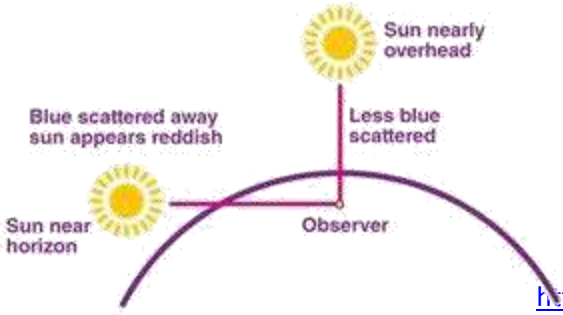


When white sunlight enters the Earth's atmosphere, it consists of various colors. The path of blue light is much shorter compared to that of red light because the wavelength of blue light is much smaller. Consequently, the intensity of scattered blue light is significantly higher than that of red light($I \alpha \frac{1}{\lambda^4}$. As a result, the predominant color observed when the sky is clear is blue. This phenomenon contributes to the characteristic blue appearance of the sky.

(d) Reddish Sun at Sunset or Sunrise:



During sunset or sunrise, the sun and its surroundings often exhibit a red hue due to the phenomenon of light scattering. When sunlight travels during these periods, it covers a longer distance through the Earth's atmosphere before reaching our eyes. The blue color in the sunlight is significantly scattered by particles in the atmosphere, leaving the light enriched with



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red tones. Consequently, the sun appears reddish at sunset or sunrise because the longer path of sunlight results in less scattering of red light compared to blue light.

(e) White Color of Clouds:

Clouds typically appear white as they are composed of dust particles and water droplets. The size of these particles is considerably large compared to the wavelength of sunlight. Consequently, there is minimal scattering of light. The incident light from the sun passes through the clouds with little deviation, resulting in the transmission of all colors of light. As a cumulative effect, we perceive the transmitted light as almost white. Therefore, clouds generally exhibit a white appearance.

(f) Red Color of Danger Signals:

Danger signals are designed to be red in color because when light interacts with the signal, all colors, except red, are scattered to a greater extent. The red color, suffering the least scattering, remains concentrated around the signal, causing it to illuminate prominently. This property ensures that the signal is visible from significant distances, enhancing its effectiveness as a warning sign.



Optical instruments: -

(a) Microscope : - A microscope is an optical instrument to see very small object as magnified one.

There are two types of microscopes: -

1. Simple Microscope
2. Compound Microscope

(b) Telescope : - Telescope is an optical instrument to clearly observe the distant objects.

There are two types of telescopes: -

1. Refracting type telescope.
2. Reflecting type telescope.

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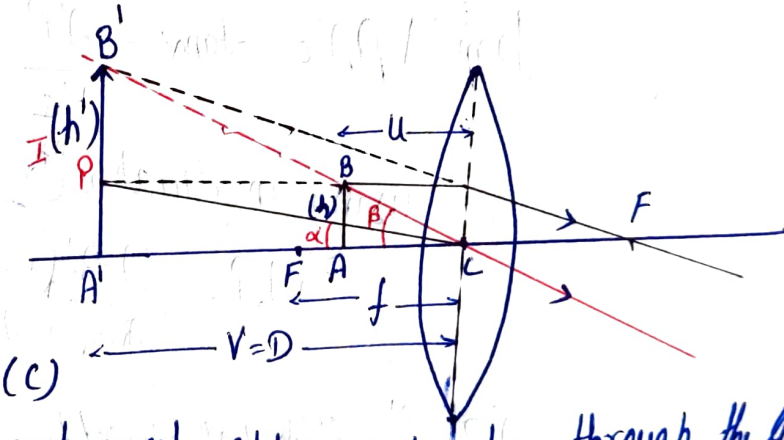
Simple Microscope :-

It Consist of a Convex lens of small focal length.
A magnifying glass is an simple microscope.

Principle :- A simple microscope is based upon the fact that an object placed between the optical Centre and the focus of a Convex lens, forms a virtual erect and magnified image on the same of the lens.

Working :-

Consider a Convex lens of focal length (f).
Let AB be an object lie in between focal (f) Point and optical Centre (C)



The ray of light do not meet after refraction through the lens. They appear to Come from Point B' so that Image A'B' is formed.

Magnification Power :- (For distinct vision)

It is defined as the Ratio of the angle subtended by the Image to the angle subtended by the object at the eye when both are placed at the least distance of distinct vision Independtly.

α = angle subtended by object

β = angle subtended by image.

Magnifying Power. M.P. = $\frac{\beta}{\alpha}$

Here angles are very small ie $\alpha = \tan\alpha$ and $\beta = \tan\alpha$

$$M.P. \frac{\tan\beta}{\tan\alpha} \quad \text{---} \quad (1)$$

from $\Delta A'B'C$ $\tan\beta = \frac{A'B'}{A'C}$

from $\Delta PA'C$ $\tan\alpha = \frac{A'P}{A'C}$

using in equation (1)

$$M.P = \frac{A'B'}{A'C} \times \frac{A'C}{A'P} = \frac{A'B'}{A'P}$$

$A'B' = h'$ (Height of object)

$A'P = h$ (Height of image)

$$M.P = \frac{h'}{h} = m$$

from the Convex lens

$$M.P. = \frac{h'}{h} = \frac{v}{u} = m$$

Here $v = -D$ and $u = -u$

$$M.P = \frac{+D}{+u} = \frac{D}{u}$$

$$\boxed{m = \frac{D}{u}} \quad \text{— (II)}$$

from lens formula

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$\frac{1}{-u} = \frac{1}{f} - \frac{1}{v}$$

$$\frac{1}{u} = \frac{1}{f} + \frac{1}{D}$$

Multiplying by D

$$\frac{D}{u} = \frac{D}{f} + 1 \quad \text{— (III)}$$

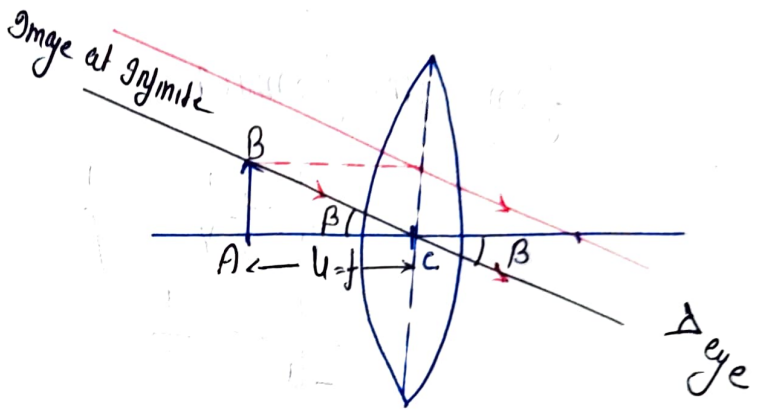
from equation (II) and (III)

$$\boxed{m = 1 + \frac{D}{f}}$$

(11) For Normal Vision (eye focussed at infinity)

The microscope is in normal adjustment when the image is formed at infinity ie ($V = \infty$)

for perfectly relaxed normal eye.



$$-\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$$

$u = -u$ and $v = \infty$

$$\frac{1}{u}+\frac{1}{\infty}=\frac{1}{f}$$

$$\frac{1}{u}=\frac{1}{f}$$

Multiplying By (D) both side

$$\frac{D}{u}=\frac{D}{f} \quad \text{---} (1)$$

Magnifying Power

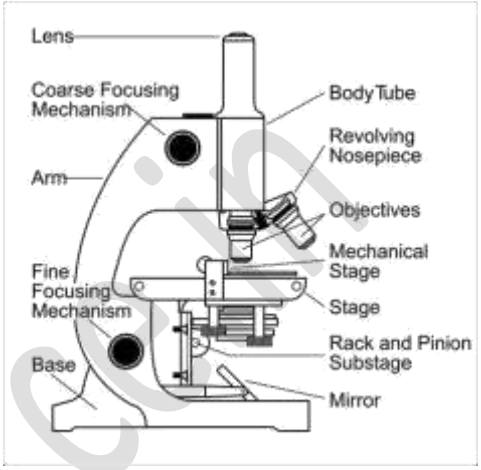
$$M.P. = \frac{\beta}{\alpha} = \frac{O/u}{O/D} = \frac{D}{u}$$

$u = f$

$$\boxed{M.P. = D/f}$$

Compound Microscope: -

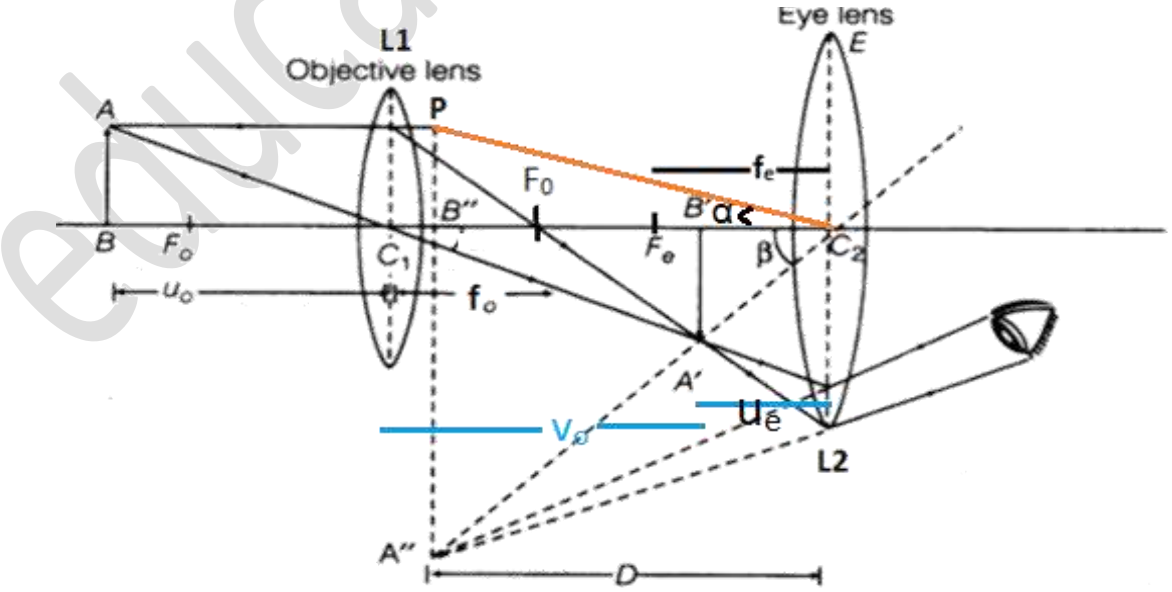
A compound microscope consists of two suitable lenses to give large magnification by compounding the magnification given by the lenses.



Construction: - A compound microscope consists of two lenses: the objective and the eyepiece. The objective lens, with a small aperture and short focal length, is positioned facing the object of interest, producing a real, inverted, and magnified image. The eyepiece, characterized by a large aperture and a focal length greater than that of the objective, compounds the magnification effect of the objective, providing an enlarged and virtual image.

Principle: - When an object is placed in front of a convex lens **O** at a distance between $\mathbf{F_o}$ and $\mathbf{2F_o}$; the real, inverted and magnified image is formed on the other side of this lens. If this image lies within the focal length of another convex lens **E** of large aperture, then the image acts as an object for this lens. The final image produced by this lens is virtual, inverted and highly magnified.

Theory: - Let AB be an object which lies between F_o, and 2Fo of lens L_1. The real, inverted and magnified image A'B' is formed on the other side of this lens L_1. The position of the lens E or L_2 is adjusted in such a way that the image A"B" of A'B' is formed on the same side at a least distance of distinct vision. This final image A"B" is virtual, inverted and highly magnified.



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Compound Microscope :-

Magnification Power :-

The magnifying power of a compound microscope is defined as the ratio of the angle subtended by the final image to the angle subtended by the object at the eye when both are at a distance of least distance of distinct vision.

α = angle subtended by the object at the eye.

β = angle subtended by the image at the eye.

$$M.P = \frac{\beta}{\alpha} \quad \text{---(I)}$$

Here α and β are small angles ie

$\alpha = \tan\alpha$ and $\beta = \tan\beta$

From $\Delta A'B'C_1$ $\tan\alpha = \frac{PB''}{C_2B''}$ | From $\Delta A''B''C_2$ $\tan\beta = \frac{A''B''}{C_2B''}$ ---(II)

From (I) and (II)

$$M.P = \frac{A''B''/C_2B''}{PB''/C_2A''} = \frac{A''B''}{PB''}$$

Since $PB'' = AB$ $\therefore MP = \frac{A''B''}{AB}$ ---(III)

Multiplying and dividing by $A'B'$

$$M.P = \frac{A''B''}{AB} \times \frac{A'B'}{A'B'}$$

$$M.P. = \frac{A''B''}{A'B'} \times \frac{A'B'}{AB}$$

magnification for objective lens

$$m_o = \frac{A'B'}{AB}$$

magnification for eye lens

$$m_e = \frac{A''B''}{A'B'}$$

Using in above equation

$$\boxed{M.P. = m_e \times m_o}$$

Determination of m_o :-

$$m_o = \frac{A'B'}{AB} = -\frac{V_o}{U_o}$$

where, U_o and V_o are distances of objects and image from objective lens.

Determination of m_e :-

$$m_e = \frac{A''B''}{A'B'} = \frac{V}{U_e} = \frac{D}{U_e} \quad \text{(IV)}$$

from lens formula

$$-\frac{1}{U} + \frac{1}{V} = \frac{1}{f_e}$$

$U = -U_e$, $V = -D$

$$-\left(\frac{1}{-U_e}\right) + \left(\frac{1}{-D}\right) = \frac{1}{f_e}$$

$$\frac{1}{u_e} - \frac{1}{D} = \frac{1}{f_e}$$

multiplying both side by D.

$$\frac{D}{u_e} - 1 = \frac{D}{f_e}$$

$$\boxed{\frac{D}{u_e} = 1 + D/f_e} \quad - (V)$$

from (IV) and (V)

$$\boxed{m_e = 1 + \frac{D}{f_e}}$$

Using the value of m_o and m_e

$$\boxed{M.P. = \frac{v_o}{-u_o}\left(1 + D/f_e\right)}$$

⇒ Length of microscope:-

$$\boxed{L = v_o + u_e}$$

⇒ Viewing extremely small objects :-

if we have to observe extremely small objects. AB is place very close to the focus of the objective lens. $u_o = f_o$

The image A'B' is formed very close to eye lens.

$v_o = L_o$

$$\boxed{M.P = -\frac{L}{f_o}\left(1 + \frac{D}{f_e}\right).}$$

Note :- If image formed at infinity

$$m_e = \frac{D}{f_e}$$

$$\because \quad -\frac{1}{u_e} + \frac{1}{v_e} = \frac{1}{f_e}$$

Here $u_e = -u_e$, $v_e = \infty$.

$$\frac{1}{u_e} + \frac{1}{\infty} = \frac{1}{f_e}$$

$$\frac{1}{u_e} = \frac{1}{f_e}$$

multiplying by D

$$\frac{D}{u_e} = \frac{D}{f_e} \qquad \left[m_e = \frac{D}{u_e} \right]$$

$$\boxed{m_e = \frac{D}{f_e}}$$

ie $$\boxed{M.P = \frac{L}{f_o}\left(\frac{D}{f_e}\right)}$$

Q You are given 3 lenses having power as $P_1 = 6D$, $P_2 = 3D$, $P_3 = 12D$. which two of these lenses will you select to construct a microscope?

Solution :- Magnifying power $M \propto \frac{1}{f_o f_e} \propto P_e P_o$

for microscope high power lens required

But objective piece is more higher than eye piece.

$\therefore$ P_3 = objective, P_1 = eye piece.

Telescope: -

Telescopes serve as optical instruments for the clear observation of distant objects and are broadly categorized into two types: refracting and reflecting.

(a) Refracting Telescopes:

1. **Astronomical Telescope:** doesn't impact observations due to the nearly spherical shape of heavenly bodies. Designed for observing celestial objects such as the moon, distant stars, and planets. The resulting image is virtual and inverted, but this inversion
2. **Terrestrial Telescope:** Used to observe objects on Earth, like trees and houses. It produces a virtual and erect final image, making it suitable for terrestrial observations.

(b) Reflecting Telescopes:

1. **Cassegrain Type:** A type of reflecting telescope design.
2. **Newtonian Type:** Another design within the reflecting telescope category.

1. Astronomical Telescope: -

Construction: - It consist two convex lenses placed in two metal tubes aligned with each other. The lens facing the distant object, called the objective lens, has a large aperture and focal length. The other lens, known as the eyepiece, has a smaller aperture and focal length and is used for observing the image.

Principle and Theory: Principle of Astronomical telescope can be discussed by considering two extreme cases: **(a) Distinct vision (near point) (b) Normal vision (eye focussed at infinity)**

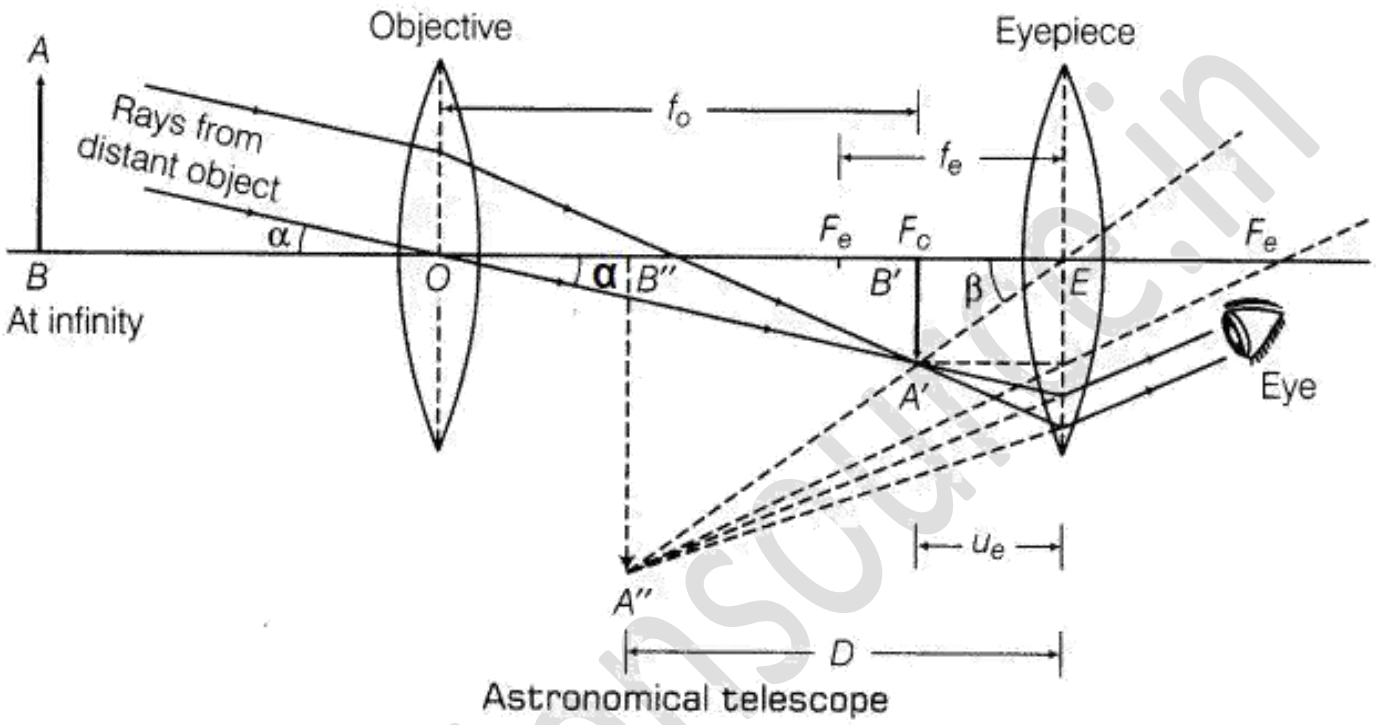
a) **Distinct vision (near point): -** When the final image is formed at a distance of distinct vision, the telescope is said to be in distinct vision adjustment.

When the objective lens (O) of the telescope is directed towards the object, a real and inverted image (A'B') is formed at the focal plane of the objective lens. The position of the eyepiece (E) is adjusted in such a way that the final image (A"B") is formed at a distance of least distance of distinct vision.

Theory: - A parallel beam of light rays from the distant object (lying at infinity) falls on the objective lens an angle **α.** After refraction through the objective lens, these rays meet at the focal plane of the less Hence A'B' is the real and inverted image formed at **F**.

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This image **A'B'** acts as an object for the eyepiece and lies between its focus and optical centre, T final image **A"B"** of the object made by the eyepiece is virtual, inverted and magnified. It is at a distance of least distance of distinct vision.



Magnifying power: -

Magnifying power of an astronomical telescope is defined as the ratio of the angle subtended by the final image to the angle subtended by the object at the eye.

α = Angle subtended by the object
β = Angle subtended by the image to the eye.

$$\text{M.P.} = \frac{\beta}{\alpha}$$

Here the angle is too small **i.e., α = tanα** and **β = tanβ**

$$\text{M.P.} = \frac{\tan\beta}{\tan\alpha}$$

from $\Delta A'B'E$

$$\tan\beta = \frac{A'B'}{B'E}$$

from $\Delta A'B'O$

$$\tan\alpha = \frac{A'B'}{B'O}$$

using in above equation

$$M.P = \frac{A'B'}{B'E} \times \frac{B'O}{A'B'} = \frac{B'O}{B'E}$$

$$M.P. = \frac{B'O}{B'E} \quad \text{---(I)}$$

Here $B'O = f_o$, $B'E = -u_e$

$$M.P = \frac{f_o}{-u_e} \quad \text{---(II)}$$

(a) For distinct vision :-

$$-\frac{1}{u} + \frac{1}{v} = \frac{1}{f_e}$$

$u = -u_e$, $v = -D$

$$-\frac{1}{-u_e} + \frac{1}{-D} = \frac{1}{f_e}$$

$$\frac{1}{u_e} - \frac{1}{D} = \frac{1}{f_e}$$

$$\frac{1}{u_e} = \frac{1}{f_e} + \frac{1}{D}$$

$$\frac{1}{u_e} = \frac{f_e}{f_e}\left(\frac{1}{f_e} + \frac{1}{D}\right)$$

$$\frac{1}{u_e} = \frac{1}{f_e}\left(1 + \frac{f_e}{D}\right) \quad \text{---(iii)}$$

Using (iii) equation in (ii)

$$\boxed{M.P. = \frac{-f_o}{f_e}\left(1 + \frac{f_e}{D}\right)}$$

Length of telescope :-

$$L = OF_o + F_o E$$

$$\boxed{L = f_o + u_e}$$

$$\boxed{L = f_o + \frac{f_e D}{D + f_e}}$$

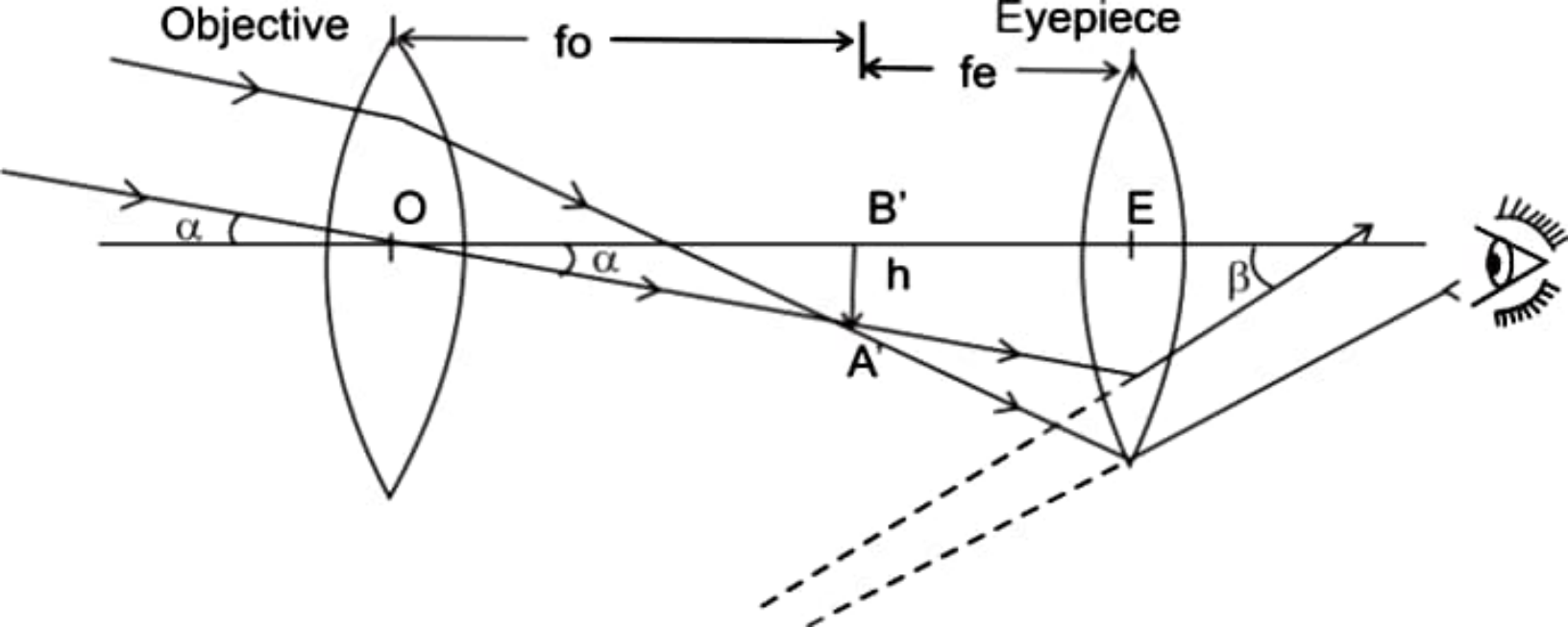
$$\left[\frac{1}{u_e} = \frac{1}{f_e} + \frac{1}{D}\right]$$

$$\frac{1}{u_e} = \frac{D + f_e}{f_e D}$$

$$\boxed{u_e = \frac{f_e D}{D + f_e}}$$

b) Normal Vision :- When final image is formed at infinity, the telescope is said to be in normal adjustment.

The position of the eyepiece is adjust in such a way that the final image of the object is virtual, inverted and highly magnified.


Objective
fo
Eyepiece
fe
α
O
B'
E
α
h
β
A'

Here $u_e = f_e$

using equation (11)

$$\boxed{M.P. = \frac{f_o}{f_e}}$$

Length of the telescope :-

$$L = OB' + B'E$$

$$\boxed{L = f_o + f_e}$$

Q A Astronomical telescope consists of two thin lens set 36 cm apart, and has a magnifying power 8. Calculate the focal length of the length. Two stars have an actual separation of one minute of arc. Find the angle of separation as seen through the telescope.

Sol :

given $M.P. = 8$ and $L = 36\,cm$

$$M.P. = \frac{f_o}{f_e} = 8 \quad \text{and} \quad f_o + f_e = 36$$

$f_o = 8f_e$ $\xrightarrow{\text{using}}$ $8f_e + f_e = 36 \Rightarrow 9f_e = 36$

$f_e = \frac{36}{9}4$

$$\boxed{f_e = 4cm}$$

$$\boxed{f_o = 8(4) = 32cm}$$

Separation seen = M.P. × actual

$= 8 \times 1' = 8'$

b) Limitations of Refracting Type Telescopes:

The refracting type telescope, which employs lenses, encounters significant drawbacks primarily due to spherical and chromatic aberrations.

1. **Spherical and Chromatic Aberrations:** Spherical and chromatic aberrations in the lenses lead to the formation of a final image that is both colored and blurred. As a result, the telescope struggles to provide a clear and detailed analysis of minute object details.

2. **Color and Blurring Effects:** The presence of aberrations causes the final image of an object to exhibit color distortions and blurriness. This hinders the telescope's ability to accurately resolve fine details in observed objects.

3. **Requirements for Detailed Observation:** Achieving effective observation of minute details in celestial bodies, such as the moon, necessitates the telescope to possess high magnifying power, resolving power, and light-gathering power. These requirements demand an objective with a substantial aperture, for instance, around 1 meter.

4. **Challenges in Large Aperture Manufacturing:** Unfortunately, the production of objective lenses with very large apertures, such as 1 meter, poses significant manufacturing challenges. The complexities involved in creating lenses of such dimensions make it a difficult and impractical task.

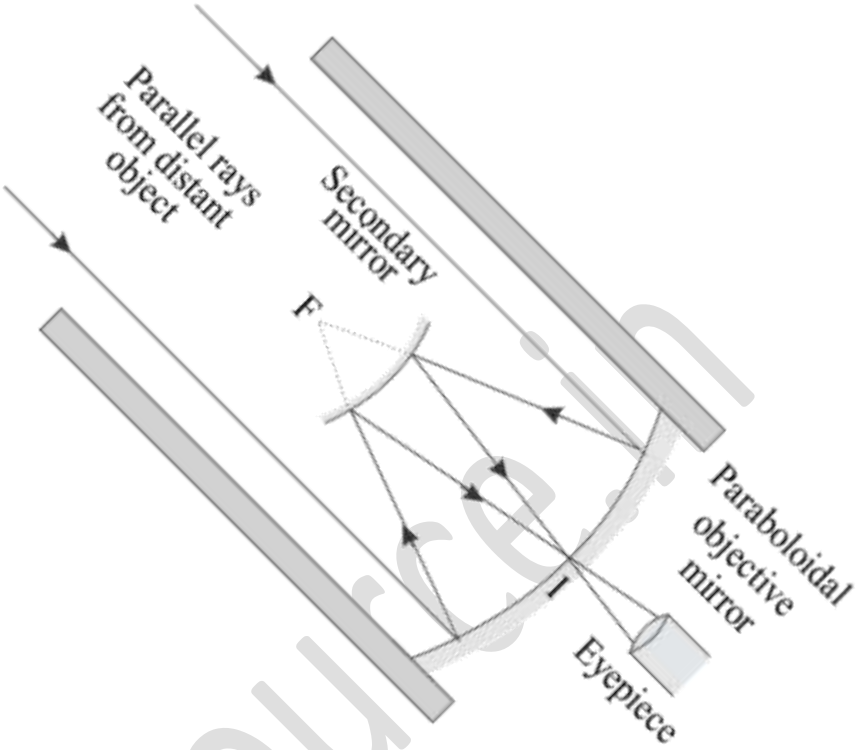
Reflecting Type Telescope:

To overcome the limitations associated with refracting type telescopes, a reflecting type telescope is employed. In this design, the objective lens is substituted with a paraboloidal mirror of substantial aperture, which eliminates issues like spherical and chromatic aberrations. Various types of reflecting telescopes exist, and one notable example is the Cassegrain Type Telescope.

Cassegrain Type Telescope: This telescope features a concave mirror **(O)** with a large aperture, accompanied by a circular hole at its center. Positioned in front of the objective mirror is a small convex mirror. Observations are made through an eyepiece **(E)** located in front of the objective hole. The process involves a parallel beam of light from a distant star striking the objective, which converges it toward the principal focus **(Fo).** The reflected light is then intercepted by the convex mirror, creating an inverted image. This final image is observed through the eyepiece.

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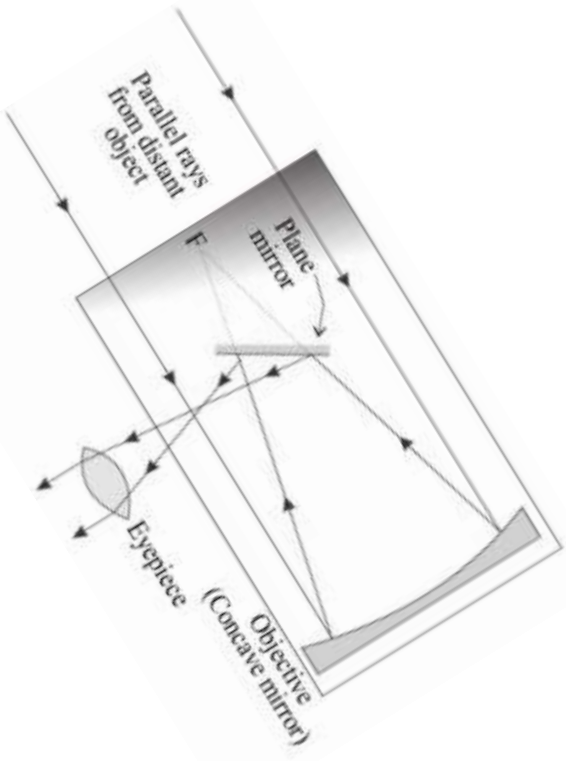
An exemplary instance of a Cassegrain Type Telescope is the 500 cm reflecting telescope at Mount Palomar, renowned as one of the world's largest telescopes. This telescope design effectively addresses the challenges associated with aberrations, offering enhanced observational capabilities for studying celestial objects.



Newtonian Telescope:

The Newtonian Telescope is another type of reflecting telescope characterized by a concave mirror of substantial aperture. In this design, a plane mirror is positioned at one end of a metallic tube, while the other end remains open. This plane mirror is set at an angle of **45°** with the tube's axis. The final image is observed through the open end of the tube.

The operational sequence involves a parallel beam of light from a distant star or object incident on the objective mirror **(O).** The reflected light falls on the plane mirror **(M),** which deviates the light rays. Consequently, the final image is formed in front of the eyepiece, acting as a magnifier.



If f_o = focal length of the concave mirror used as objective

f_e = focal length of the eye piece,

Then the magnifying power of reflecting telescope

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$$\text{M.P.} = \frac{f_o}{f_e}$$

M.P. where f_o =R/2 and R is the radius of curvature of the objective.

Advantages of Reflecting Type Telescopes:

1. **Aberration-Free Imaging:** Reflecting type telescopes utilizing parabolic mirrors are free from chromatic and spherical aberrations. This results in the formation of sharp and clear images of observed objects.

2. **Weight Considerations:** Mirrors in reflecting telescopes weigh significantly less than lenses of comparable quality. This characteristic makes the telescope design more manageable and less prone to weight-related issues.

3. **Support Structure:** The full back of a mirror in reflecting telescopes can be supported, whereas lenses typically rely on support at their rims. This structural advantage enhances stability and robustness.

4. **Brightness of Image:** Since mirrors in reflecting telescopes reflect the entire light incident on them, the images formed by these telescopes are notably bright. This brightness enhances the visibility of observed objects.

5. **Manufacturability:** Paraboloidal mirrors with large apertures, essential for effective telescopes, can be relatively easily manufactured. This makes the production process more feasible.

Disadvantages of Reflecting Type Telescopes:

1. **Frequent Adjustments:** Reflecting telescopes of this type often require frequent adjustments, making them inconvenient for general use. Regular tuning and maintenance are necessary for optimal performance.

2. **Limited Viewer Positioning:** Viewers using reflecting telescopes must either be positioned near the focal point of the main mirror or require additional mirrors to redirect and make the observation feasible. This constraint limits flexibility in viewer positioning.

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Difference between Reflecting type and Refracting type telescope :-

Characteristic	Refracting Telescope	Reflecting Telescope
Optical Components	Lenses (Objective lens and eyepiece)	Mirrors (Usually concave mirror as the objective)
Aberrations	Prone to spherical and chromatic aberrations	Generally free from chromatic aberrations, minimizing spherical aberrations
Image Orientation	Inverts the image	Can provide both upright and inverted images
Size and Weight	Lenses can be heavy, bulkier design for larger apertures	Mirrors are generally lighter, making the design more manageable
Maintenance	May require more frequent adjustments and maintenance	Generally, requires less frequent adjustments and maintenance
Viewer Position	Eyepiece allows comfortable viewing from various angles	Viewer might need to be positioned near the focal point of the main mirror or use additional mirrors
Manufacturing Challenges	Large lenses with significant apertures can be challenging and expensive	Mirrors, particularly paraboloidal mirrors, with large apertures are relatively easier to manufacture
Brightness of Image	May suffer from light loss due to absorption and reflection in the lenses	Can provide brighter images as mirrors reflect the entire incident light