

Physics

Chapter: - 1st Electric Charges and Fields



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Class: - 12th
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Unit: I

Electrostatics :- (study of stationary charges)

Chapter: 1

Electrostatic charges :-

examples of stationary charges :- (1) Rubbing of glass Rod with silk.
(2) (Rubbing of Comb in dry hairs)

Defination :- The Branch of physics, which deals with the study of (1) Charges at rest
(2) forces between static charges
(3) field and potential due to these charges.

Q What is electric charge?

Acc. to William Gilbert, charge is something possessed by material objects that makes it possible for them to exert electrical force and to Respond to electrical force.

Three Important elementary particles

- 1) electron (m_e) = 9.10940×10^{-31} kg
- 2) Proton (m_p) = 1.67262×10^{-27} kg
- 3) neutrons (m_n) = 1.67493×10^{-27} kg

The force of attraction between similar Charges is due to their masses and force of Repulsion is due to their charges.

Two kind of Charges:-

- 1) A glass rod rubbed with a piece of silk brought close to a suspended glass rod rubbed with silk repels each other.
- 2) Two ebonite Rods Rubbed with Cat's fur repel each other.
- 3) An ebonite Rod rubbed with Cat's fur attracts a glass Rod rubbed with a piece of silk. However the charged glass rods repel the Cat's fur.

Conductance

A substance which can be used to carry or conduct electric charge from one place to the other is called conductor. (Copper, iron, mercury)

Insulator

The Insulators are the materials which cannot conduct electricity i.e. they are poor conductors of electricity. (Rubber, plastic, ebonite)

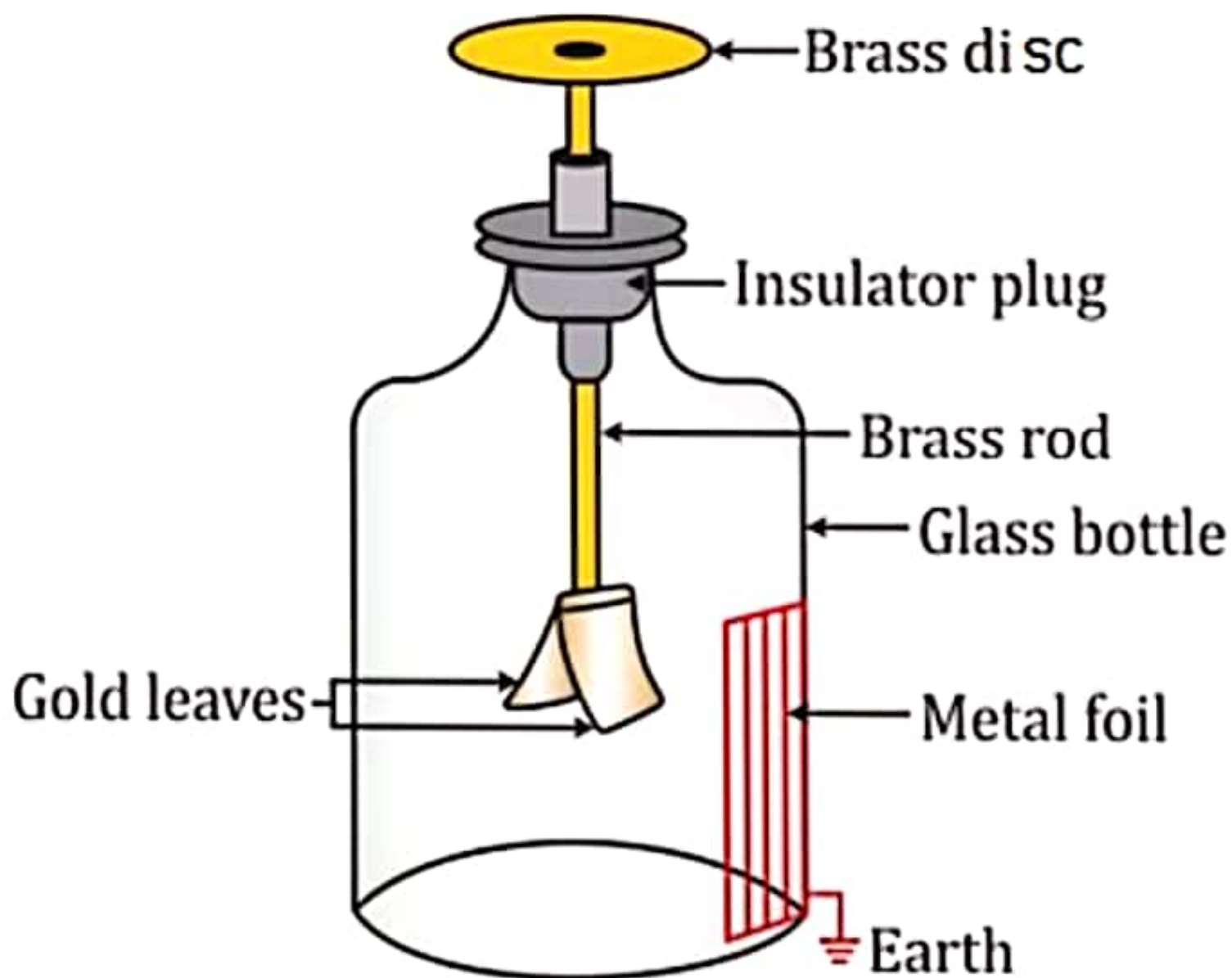
Dielectric:-

Dielectric is also a kind of Insulator. It cannot conduct electricity. But when an external field (electric) is applied, induced charges appear on the surface of the dielectric.

Hence we can define dielectrics as the insulating materials which transmit electric effects without conducting.

Gold leaf electroscope:-

A gold leaf electroscope (GLE) is an instrument which is used for detecting the presence of electric charge and its polarity. And this instrument can also be used for measuring potential difference.



Properties of electric charge:-

Q1 What do you mean by additivity of electrical charge?

Ans: Electrical charge is additive that is total charge on an extended body is the algebraic sum of charges in different region of the body. (+ve) and (-ve) charges are added like real numbers.

eg:- A body consist three charges are

$$+3q \quad -2q \quad +4q$$

$$+3q - 2q + 4q$$

$$+7q - 2q$$

$$+5q$$

Q2: Imp What is conservation of electrical charge?
(08)

Explain How you can show the electrical charge is conserved?

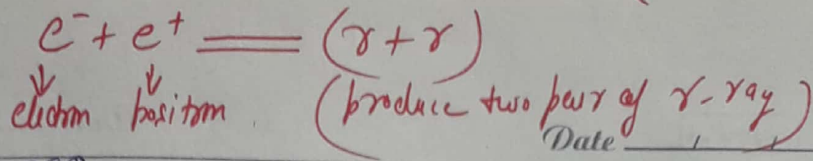
Ans: Acc. to the law of conservation of electrical charges, the net electrical charge in an isolated system remains constant.

Explanation: when a glass rod is rubbed with a piece of silk cloth. Glass rod become (+ve) charge and silk cloth become (-ve) charge. Before rubbing both glass rod and silk cloth were neutral that is total charge on the system was zero. On rubbing glass rod transfer some electron~~s~~ to the silk rod and become (+ve) charge and silk cloth become (-ve) charge. But the algebraic sum of charges on glass rod and silk cloth after rubbing continues to be zero. Hence total charge is conserved.

(08)

pair production :- γ (ray photon) $\rightarrow e^- + e^+$ (1)

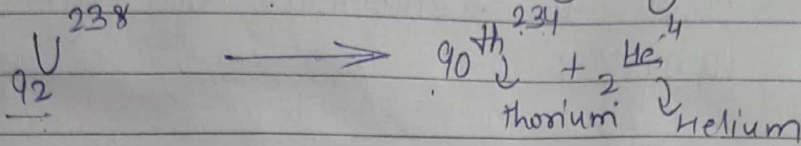
annihilation



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Nuclear transformation

When ${}_{92}^{238}\text{U}$ decay by emitting by α -particle



No. of charges before decay = 92

No. of charges after " = 90 + 2 = 92

Hence total charge is conserved

Q3: What is Quantization of electrical charge?

Ans: Quantization is the property of an electrical charge which tells that any charged body can have charge which is an integral multiple of the basic charge.

ie $q = \pm ne$

q = total charge

$n = 1, 2, 3, \dots$

eg $q = n = 3$ ie $q = \pm ne = 3 \times 1.6 \times 10^{-19} \text{ C} = \pm 4.8 \times 10^{-19} \text{ C}$

Q4: What do you mean by invariance of electric charge?

Ans: Electrical charge is independent of frame of reference. That is charge on a body does not vary but every may be its speed or speed of the observer. or Charge invariance refers to the fixed electrostatic potential of a particle, regardless of speed.

Q5: similar charges repel each other can they attract each other also.

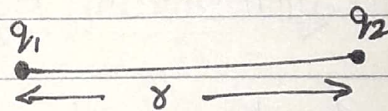
Ans: Yes, only when one charge is very large as compared to the other charge because larger charge will induced opposite charges such that Net charge on the other become opposite. Hence they attracted each other.

Q Special type of rubber which is slightly conducting is used to manufacture the tyres of aircraft why?

Ans: The Rubber used in making the tyres of aircrafts is a special type of rubber which is slightly conducting. This is done to leak any charge to the ground that develops on the tyres during take off or landing of the aircraft.

Q State and explain Coulomb's law? And write it in vector form?

Sol:- Acc. to this law the magnitude of force of attraction or repulsion b/w any two point charges at rest is directly proportional to the product of the magnitude of charges and inversely proportional to the square of the distance b/w them.



Let (r) be the distance b/w two point charges (q_1) and (q_2) then according to Coulomb's law of electrostatic force.

$$F \propto q_1 q_2 \quad \text{--- (i)}$$

$$F \propto \frac{1}{r^2} \quad \text{--- (ii)}$$

from equation (i) and (ii)

$$F \propto \frac{q_1 q_2}{r^2}$$

$$F = K \frac{q_1 q_2}{r^2}$$

$$\text{S.I. value of } k = 9 \times 10^9 \text{ Nm}^2 \text{c}^{-2} = \frac{1}{4\pi\epsilon_0}$$

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where (k) is constant of proportionality and is known as Coulomb's constant or electrostatic force constant.

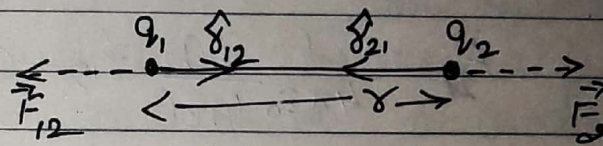
Coulomb's law in vector form :-

Q. Q

Show that Coulomb's law of electrostatic force is in accordance with Newton's third law of motion?

Sol:-

Consider two point charges (q_1 and q_2) separated by a distance (r). Coulomb's force acting on (q_1) due to (q_2)



$$\vec{F}_{12} = k \frac{q_1 q_2}{r^2} \vec{s}_{21} \quad \text{--- (i)}$$

where \vec{s}_{21} is unit vector directed from q_2 to q_1 .
Coulomb's force acting q_2 due to q_1

$$\vec{F}_{21} = k \frac{q_1 q_2}{r^2} \vec{s}_{12} \quad \text{--- (ii)}$$

where \vec{s}_{12} is unit vector directed from q_1 to q_2

$$\text{But } \vec{s}_{12} = -\vec{s}_{21}$$

equ (ii) become

$$\therefore \vec{F}_{21} = -k \frac{q_1 q_2}{r^2} \vec{s}_{21} \quad \text{--- (iii)}$$

from equ (i) and (iii)

$$\vec{F}_{12} = -\vec{F}_{21}$$

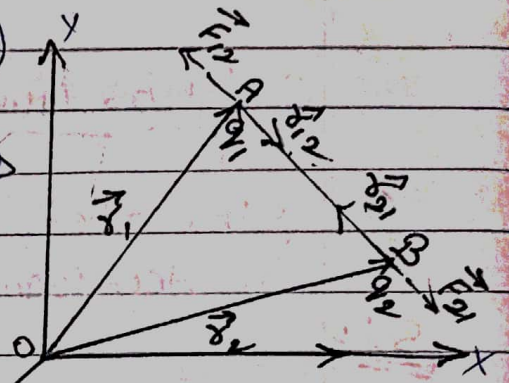
Q. Is Coulomb's law is universal? why?

Ans:-

Coulomb's law is not a universal law because it depends upon the nature of the medium in which charges are placed. moreover this law is valid for point charges at rest.

Q/ Drive an expression for force b/w charges in terms of their position vectors?

Ans:- Consider two charges (q_1) and (q_2) lying in free space. Let \vec{r}_1 and \vec{r}_2 be their position vectors. Coulomb's force acting on (q_1) due to (q_2)



$$\vec{F}_{21} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{|\vec{r}_{12}|^2} \hat{r}_{12} \quad (1)$$

Acc. to Δ law of vector addition

$$\vec{OA} + \vec{AB} = \vec{OB}$$

$$\vec{AB} = \vec{OB} - \vec{OA}$$

$$\vec{AB} = \vec{r}_2 - \vec{r}_1 = \vec{r}_{12}$$

$\hat{r} = \frac{\vec{r}}{|\vec{r}|}$ \therefore equ (1) become

$$\vec{F}_{21} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{|\vec{r}_{12}|^2} \cdot \hat{r}_{12} \quad (11)$$

But

$$|\vec{r}_{12}| = |\vec{r}_2 - \vec{r}_1|$$

and

$$\hat{r}_{12} = \frac{\vec{r}_2 - \vec{r}_1}{|\vec{r}_2 - \vec{r}_1|}$$

$$\left(\hat{r} = \frac{\vec{r}}{|\vec{r}|} \right)$$

$$\hat{r}_{12} = \frac{(\vec{r}_2 - \vec{r}_1)}{|\vec{r}_2 - \vec{r}_1|}$$

\therefore equ (11) become

$$\begin{aligned} \vec{F}_{21} &= \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{|\vec{r}_2 - \vec{r}_1|^2} \frac{(\vec{r}_2 - \vec{r}_1)}{|\vec{r}_2 - \vec{r}_1|} \\ &= \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{|\vec{r}_2 - \vec{r}_1|^3} (\vec{r}_2 - \vec{r}_1) \end{aligned}$$

$$11814 \quad \vec{F}_2 = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_{12}^2} \cdot (\vec{r}_1 - \vec{r}_2)$$

Relative Permittivity :- Relative Permittivity of a medium is defined as the ratio of the Coulomb's force (F) b/w two point charges placed in air or vacuum to the Coulomb's force (F_m) b/w the same two charges placed in the medium separated by the same distance, from each other. It is denoted by ϵ_r

$$\text{ie } \epsilon_r = \frac{F}{F_m}$$

Dielectric Constant :- Dielectric Constant of a medium is defined as the ratio of the Coulomb's force (F) between two point charges placed in the air or vacuum to the Coulomb's force (F_m) between the same two point charges placed in the medium at the same distance from each other. It is denoted by (K)

$$\text{ie } K = \frac{F}{F_m}$$

Dielectric Constant or Relative Electrical Permittivity :-

The force b/w two charges when placed in the medium

$$F_m = \frac{1}{4\pi\epsilon} \times \frac{q_1 q_2}{r^2}$$

Here ϵ (absolute electrical permittivity) in medium

The force between two charges when placed in vacuum

$$F = \frac{1}{4\pi\epsilon_0} \times \frac{q_1 q_2}{r^2} \quad \text{--- (1)}$$

$$F_0 = \frac{1}{4\pi\epsilon_0} \times \frac{q_1 q_2}{r^2} \quad \text{--- (1)}$$

dividing (2) by (1)

$$\frac{F_0}{F_m} = \frac{\frac{1}{4\pi\epsilon_0} \times \frac{q_1 q_2}{r^2}}{\frac{1}{4\pi\epsilon} \times \frac{q_1 q_2}{r^2}}$$

$$\boxed{\frac{F_0}{F_m} = \frac{\epsilon}{\epsilon_0} = \epsilon_r \text{ or } k}$$

Where (ϵ_r) is Relative electrical permittivity of the medium. It is also called dielectric constant.

Dielectric Constant :- dielectric constant of a medium is the ratio of absolute electrical permittivity of the medium to the absolute electrical permittivity of free space.
(or)

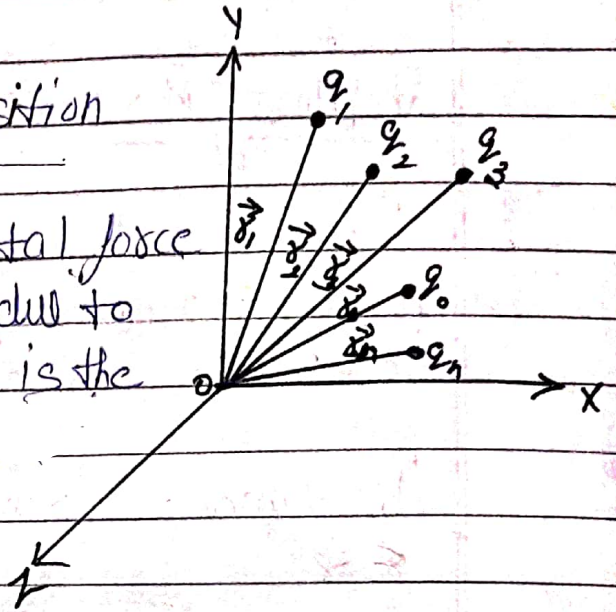
dielectric constant of a medium may be define as the ratio of force of attraction/repulsion between two point charges separated by a certain distance in air/vacuum to the force of attraction/repulsion between the same two point charge, held the same distance apart in the medium.

(electric permittivity: the ability of a substance to store electrical energy in an electric field)

Q: State and explain principle of Superposition?

Sol: Principle of Superposition

Acc. to this principle total force acting on a given charge due to No. of charges around it is the vector sum of all the forces acting on that charge due to all the charges taken one at a time.



Consider No. of charges $(q_1, q_2, q_3, \dots, q_n)$ whose position vector are $(\vec{r}_1, \vec{r}_2, \vec{r}_3, \dots, \vec{r}_n)$. Let (q_0) be charge whose position vector is (\vec{r}_0) .

Let $(\vec{F}_{01}, \vec{F}_{02}, \vec{F}_{03}, \dots, \vec{F}_{0n})$ be the forces acting on a given charge (q_0) due to charges $(q_1, q_2, q_3, \dots, q_n)$ respectively. Then total force

$$\vec{F}_0 = \vec{F}_{01} + \vec{F}_{02} + \vec{F}_{03} + \dots + \vec{F}_{0n} \quad (1)$$

$$\vec{F}_0 = \sum_{i=1}^n \vec{F}_{0i} \quad (i = 1, 2, 3, \dots, n)$$

Force acting on (q_0) due to (q_1)

$$\vec{F}_{01} = \frac{1}{4\pi\epsilon_0} \frac{q_0 q_1 (\vec{r}_0 - \vec{r}_1)}{|\vec{r}_0 - \vec{r}_1|^3} = \frac{1}{4\pi\epsilon_0} \frac{q_0 q_1}{r_{01}^2} \hat{r}_{01}$$

$$\text{Similarly } \vec{F}_{02} = \frac{1}{4\pi\epsilon_0} \frac{q_0 q_2 (\vec{r}_0 - \vec{r}_2)}{(\vec{r}_0 - \vec{r}_2)^3}$$

$$\vec{F}_{0n} = \frac{1}{4\pi\epsilon_0} \frac{q_0 q_n (\vec{r}_0 - \vec{r}_n)}{(\vec{r}_0 - \vec{r}_n)^3}$$

∴ equ (1) become

$$\vec{E}_0 = \frac{1}{4\pi\epsilon_0} \frac{q_0 q_1 (\vec{r}_0 - \vec{r}_1)}{|\vec{r}_0 - \vec{r}_1|^3} + \frac{1}{4\pi\epsilon_0} \frac{q_0 q_2 (\vec{r}_0 - \vec{r}_2)}{|\vec{r}_0 - \vec{r}_2|^3} + \frac{1}{4\pi\epsilon_0} \frac{q_0 q_n (\vec{r}_0 - \vec{r}_n)}{|\vec{r}_0 - \vec{r}_n|^3}$$

$$\vec{F} = \frac{q_0}{4\pi\epsilon_0} \sum_{i=1}^n \frac{q_i (\vec{r}_0 - \vec{r}_i)}{|\vec{r}_0 - \vec{r}_i|^3}$$

Q:- Calculate the charge carried by 12.5×10^{18} electrons.

Sol:- $q = ne = 12.5 \times 10^{18} \times 1.6 \times 10^{-19}$
 $= 2.0 \text{ C}$

Q:- How many electrons will have a total charge of one Coulomb?

Sol:- $n = \frac{q}{e} = \frac{1}{1.6 \times 10^{-19}} = 6.25 \times 10^{18}$

Q:- What is the permittivity of a medium whose dielectric constant is one?

Sol:- $\epsilon = \epsilon_0 \epsilon_r = \epsilon_0 k = 8.85 \times 10^{-12} \times 1$
 $= 8.85 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$

Q:- Define one Coulomb of charge.
(08)

Define S.I. unit of charge.

Sol:- one Coulomb is that charge which repels equal and similar charge placed at a distance of 1m from it in vacuum or air with a force of $9 \times 10^9 \text{ N}$

Continuous charge distribution

There are three type of charge distribution :-

(i) Linear Charge density :- $\lambda = \frac{\Delta Q}{\Delta l}$

It is the quantity of charge per unit length

(ii) Surface Charge density :- $\sigma = \frac{\Delta Q}{\Delta S}$

The quantity of charge per unit surface

(iii) Volume Charge density :- $\rho = \frac{\Delta Q}{\Delta V}$

The quantity of charge per unit volume.

Q: What is quantization of charge?

Ans: Existence of charges in discrete packets rather than in continuous amount is known as quantization of charge.

Q: What is the unit of electrical permittivity of free space?

Ans: $C^2 N^{-1} m^{-2}$

Q: What is the value of $\frac{1}{4\pi\epsilon_0}$ in SI?

Sol: $9 \times 10^9 N m^2 C^{-2}$.

Chapter = 2

Electrical charge and field :-

Q: Define electrical field, uniform electrical field, Non-uniform electrical field? Also define electrical field intensity, its dimension and what are its unit?

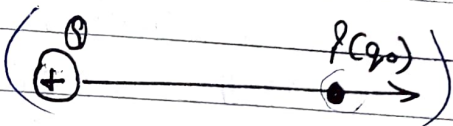
Sol: Electrical field :- The region or space around a charged body within which its influence can be felt is called electric field.
(or)

Electrical field exist in region if electric force is applied on a charged object placed in that region.

uniform electrical field :- A electrical field which has

Same strength and direction at every point in a region is called uniform electric field.

Non-uniform electrical field :- The electrical field which has diff. strength and direction at diff. points in region is called Non-uniform electrical field.

Electric field intensity :- 

Electric field intensity at any point in the electric field is defined as the force experienced by an unit (+ve) charge placed at that point

Let test charge (q_0) be placed at a point (P) in the electrical field of a source charge (Q). If (\vec{F}) is the force experienced by the test charge (q_0) in the electric field. Then electric field intensity of the source charge at point (P)

$$\vec{E} = \frac{\vec{F}}{q_0}$$

$$\vec{F} = q_0 \vec{E}$$

Dimensional formula :-

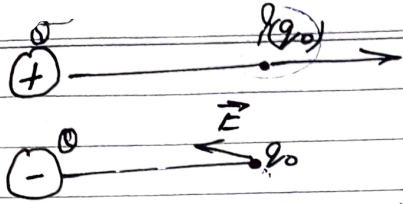
$$\vec{E} = \frac{\vec{F}}{q_0}$$

$$= \frac{[MLT^{-2}]}{[AT]}$$

$$[AT]$$

$$= [MLT^{-3}A^{-1}]$$

S.I. unit of electric field intensity :- Nc^{-1}



- Note:-
- (i) The electrical field intensity at a point due to (+ve) source charge directed toward this charge.
 - (ii) If the charge (q_0) is (+ve) the direction of the force (\vec{F}) acting on the charge will be same as that of the electrical field (\vec{E}).
 - (iii) If charge (q_0) is (-ve) then the direction of the force (\vec{F}) acting on the charge will be opposite to the direction of the electric field.

Q.1:-

Derive an expression of electric field intensity due to a point charge -

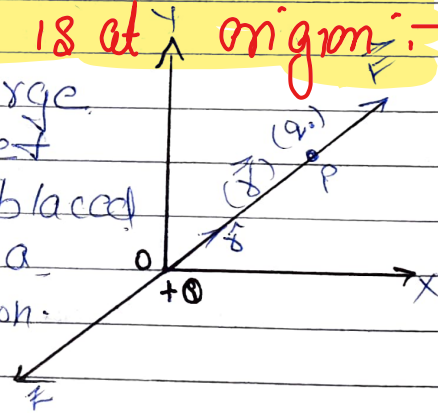
- (i) when source charge is at origin
- (ii) when the source charge is away from the origin.

(i) when source charge is at origin:-

Sol:- (i) Consider a fixed point charge

(+Q) at the origin (O). Let (q_0) be the test charge placed in the free space at a distance (r) from origin.

In the field of charge (+Q):



force experienced by (q_0) due to (+Q)

$$\vec{F} = \frac{1}{4\pi\epsilon_0} \frac{Qq_0}{r^2} \hat{r} \quad \text{--- (1)}$$

where \hat{r} is a unit vector directed along (OP)

Now the electric field intensity at point (P) due to point charge (Q)

$$\vec{E} = \frac{\vec{F}}{q_0}$$

∴ from eqn (1)

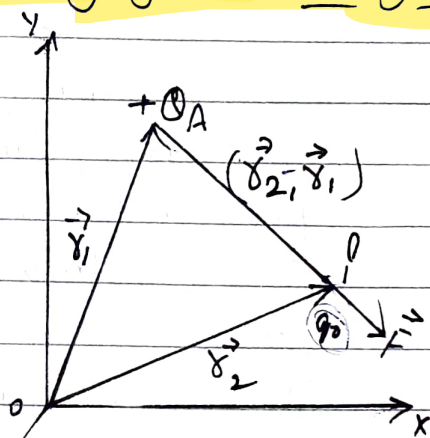
$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{Qq_0}{r^2} \times \frac{1}{q_0} \hat{r}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} \hat{r}$$

$$\vec{E} = |\vec{E}| \hat{r} = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} \hat{r}$$

(ii) When the source charge is away from the origin:

Let a source charge (Q) be placed at (A) and a test charge (q₀) be placed in its electric field.



Let $\vec{OA} = \vec{r}_1$, $\vec{OP} = \vec{r}_2$
Coulomb force acting on (q₀) due to source charge (Q)

$$\vec{F} = \frac{1}{4\pi\epsilon_0} \frac{Qq_0}{|\vec{AP}|^2} \hat{r}_{AP} \quad (i)$$

$$\vec{AP} = \vec{r}_2 - \vec{r}_1$$

$$|\vec{AP}| = |\vec{r}_2 - \vec{r}_1|$$

Now

$$\vec{AP} = |\vec{AP}| \hat{r}_{AP}$$

$$\hat{r}_{AP} = \frac{\vec{AP}}{|\vec{AP}|}$$

$$\vec{r}_{AP} = \frac{(\vec{r}_2 - \vec{r}_1)}{|\vec{r}_2 - \vec{r}_1|}$$

Put the value of \vec{r}_{AP} in equation (1)

$$\vec{F} = \frac{1}{4\pi\epsilon_0} \frac{Q_1 Q_0}{|\vec{r}_2 - \vec{r}_1|^2} \cdot \frac{(\vec{r}_2 - \vec{r}_1)}{|\vec{r}_2 - \vec{r}_1|}$$

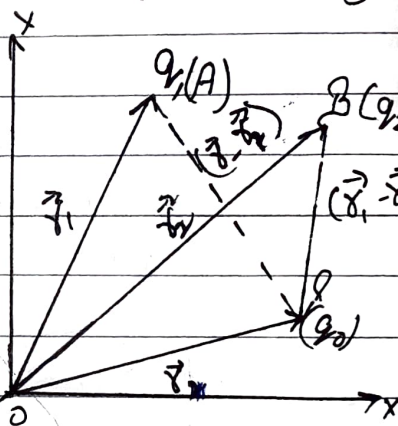
$$\vec{F} = \frac{1}{4\pi\epsilon_0} \frac{Q_1 Q_0 (\vec{r}_2 - \vec{r}_1)}{|\vec{r}_2 - \vec{r}_1|^3}$$

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• Drive an expression for electric field intensity at any point due to two point charges

1101 :-

Consider two point charges (q_1) and (q_2) placed at (A) and (B) having position vector (\vec{r}_1) and (\vec{r}_2) . Let a test charge (q_0) be placed at point (P) having position vector (\vec{r}) . The force acting on q_0 due to (q_1) and (q_2) .



$$\vec{F}_1 = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_0}{|\vec{r} - \vec{r}_1|^3} (\vec{r} - \vec{r}_1)$$

$$\vec{F}_2 = \frac{1}{4\pi\epsilon_0} \frac{q_2 q_0}{|\vec{r} - \vec{r}_2|^3} (\vec{r} - \vec{r}_2)$$

$$\vec{F} = \vec{F}_1 + \vec{F}_2$$

$$\vec{F} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_0}{|\vec{r} - \vec{r}_1|^3} (\vec{r} - \vec{r}_1) + \frac{1}{4\pi\epsilon_0} \frac{q_2 q_0}{|\vec{r} - \vec{r}_2|^3} (\vec{r} - \vec{r}_2)$$

$$\vec{F} = \frac{q_0}{4\pi\epsilon_0} \left[\frac{q_1}{|\vec{r}_0 - \vec{r}_1|^3} (\vec{r}_0 - \vec{r}_1) + \frac{q_2}{|\vec{r}_0 - \vec{r}_2|^3} (\vec{r}_0 - \vec{r}_2) \right] \quad (1)$$

Electric field intensity at q_0 due to q_1 and q_2

$$\vec{E} = \frac{\vec{F}}{q_0}$$

put equ. (1)

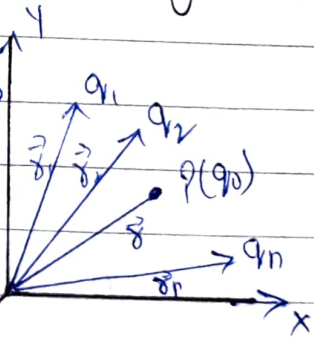
$$\vec{E} = \frac{q_0}{4\pi\epsilon_0} \left[\frac{q_1}{|\vec{r}_0 - \vec{r}_1|^3} (\vec{r}_0 - \vec{r}_1) + \frac{q_2}{|\vec{r}_0 - \vec{r}_2|^3} (\vec{r}_0 - \vec{r}_2) \right] \times \frac{1}{q_0}$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \left[\frac{q_1}{|\vec{r}_0 - \vec{r}_1|^3} (\vec{r}_0 - \vec{r}_1) + \frac{q_2}{|\vec{r}_0 - \vec{r}_2|^3} (\vec{r}_0 - \vec{r}_2) \right]$$

Q. Derive an expression for electric field intensity due to a group of charges.

What do you mean by principle of superposition

Suppose we have n point charges (q_1, q_2, \dots, q_n) and their position vectors are ($\vec{r}_1, \vec{r}_2, \dots, \vec{r}_n$). Let (q_0) be the test charge at point (P) whose total electric field (\vec{E}) due to (n) charges can be determine. Let the position vector of (q_0) is (\vec{r})



\therefore Electric field due to point charges (q_1, q_2, \dots, q_n)

$$\vec{E}_1 = \frac{1}{4\pi\epsilon_0} \frac{q_1}{|\vec{r} - \vec{r}_1|^3} (\vec{r} - \vec{r}_1) \quad (i)$$

$$\vec{E}_2 = \frac{1}{4\pi\epsilon_0} \frac{q_2}{|\vec{r}-\vec{r}_2|^3} (\vec{r}-\vec{r}_2) \quad \text{--- (ii)}$$

$$\vec{E}_n = \frac{1}{4\pi\epsilon_0} \frac{q_n (\vec{r}-\vec{r}_n)}{|\vec{r}-\vec{r}_n|^3} \quad \text{--- (iii)}$$

Acc. to the principle of superposition the net electric field strength at a point (P) due to a group of charges is equal to the vector sum of all the electric field strengths produced due to individual charges at that point

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \dots + \vec{E}_n$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q_1 (\vec{r}-\vec{r}_1)}{|\vec{r}-\vec{r}_1|^3} + \frac{1}{4\pi\epsilon_0} \frac{q_2 (\vec{r}-\vec{r}_2)}{|\vec{r}-\vec{r}_2|^3} + \dots + \frac{1}{4\pi\epsilon_0} \frac{q_n (\vec{r}-\vec{r}_n)}{|\vec{r}-\vec{r}_n|^3}$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^n \frac{q_i (\vec{r}-\vec{r}_i)}{|\vec{r}-\vec{r}_i|^3}$$

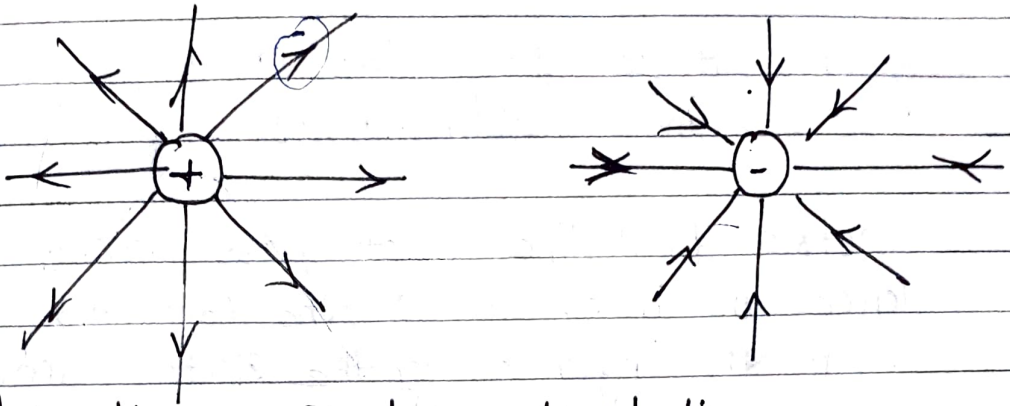
Special Case:- If point (P) lies at the origin i.e. ($\vec{r}=0$) then electric field intensity

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^n \frac{q_i \vec{r}_i}{|\vec{r}_i|^3}$$

Q Define electric field lines and what are its properties

Ans Electric field lines in an electrical field can be

defined as the path (straight or curved) along which a unit positive charge tends to move if it is totally free to do so.



Properties of Electric field line :-

- (i) The electric field lines are directed away from a positive charge and directed toward a (-ve) charge.
- (ii) A unit (+ve) test charge placed in the electric field tends to follow a path along the field line.
- (iii) Electric field lines are imaginary lines yet they have greater physical significance in describing the electrical behaviour system of charges.
- (iv) The tangent to an electrical field line at any point gives the direction of electric field at that point.
- (v) Two electric lines of force cannot cross each other. If two electric lines of force cross each other, then at the point of intersection, there will be two tangents. It means that there are two values of electric field at that point which is not possible, so two electric lines of force cannot cross each other.

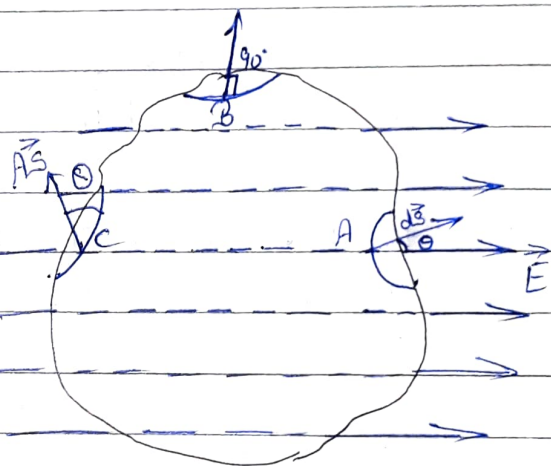
(vi) The electric field lines are crowded where the electric field is stronger and the lines spread out where the electric field is weaker.

(vii) Electrostatic field line cannot have sudden makes and sudden breaks.

(viii) Electric field line do not pass through a conductor. Hence the interior of the conductor is free from the influence of the electric current.

Q:- Define electric flux (ϕ)?

Sol:- Electric flux :- Electric flux linked with any surface is defined as the total no. of electric field lines that pass through that surface.



Let a surface area (S) be in an electric field (\vec{E}) and ($d\vec{S}$) be small area element. The electric flux of the field (\vec{E}) over an area element

$$d\phi = \vec{E} \cdot d\vec{S}$$

$$d\phi = E ds \cos\theta \quad (1)$$

Total flux over surface (S)

$$\phi = \int_S E ds \cos\theta$$

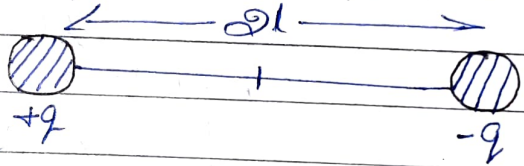
- (i) Area element (ds) at (A) has positive electric flux. linked with it because here θ less than 90° .
 - (ii) Area element (ds) at (B) has zero electric flux. linked with it because here θ is zero.
 - (iii) Area element (ds) at (C) has negative electric flux. linked with it because here θ is greater than 90° .
- Q. What is the significance of electric flux?

Sol. Electric flux represents the No. of electric line passing normally through the given surface in the electric field.

Q. Define electric dipole? And electric dipole moment. What is the significance of electric dipole?

Sol.

A pair of two equal and opposite charges separated by some distance is called an electric dipole.



eg:- Two charges (+q) and (-q) separated by a small distance (2l) constitute an dipole.

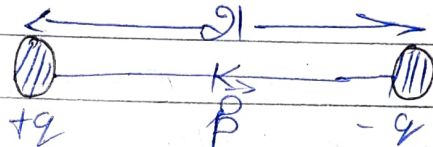
Significance of electrical dipole: Electrical dipole are very important for study for the electrical behaviours of matter. matter consist of molecule which are electrically neutral

The polar molecule have permanent dipole moment

But in random direction so there net dipole movement is zero. When electrical field applied to them they align along the direction of electric field to give net dipole **moment**. This process is called polarization. Therefore electrical dipole has **greater** physical significance in electrical properties and process of matter.

Q Define electric dipole moment (\vec{p}) ?

Sol Electric dipole moment (\vec{p}) :- Electric dipole moment



of an electric dipole is defined as the product of the magnitude of either charge and the dipole length.

$$\vec{p} = q(2\vec{l})$$

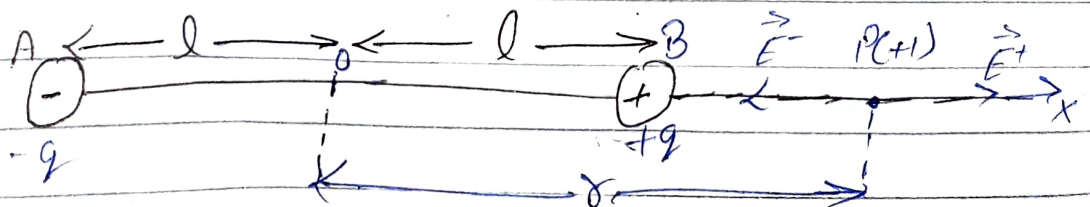
Magnitude of \vec{p}

$$p = q \times 2l$$

Electrical dipole is a vector quantity. The direction of the dipole movement is (-ve) to (+ve)

Q - Drive an expression for electric field intensity at point on the axial line of an electrical dipole? and what happen when observation point is at very-2 large distance as compared to the dipole length?

Ans :-



Axial line:- It is the line joining the centre of two charges forming an electric dipole.

Consider an electric dipole consisting of $(+q)$ and $(-q)$ charges and dipole length is $2l$.

Consider a point (P) at the distance (x) from the centre (O) of the dipole on the axial line of the dipole. Let a unit (+ve) charge placed at point B .

Now

Electric Field intensity at (P) due to $(+q)$

$$\vec{E}_+ = \frac{1}{4\pi\epsilon_0} \frac{q}{BP^2}$$

$$\vec{E}_+ = \frac{1}{4\pi\epsilon_0} \frac{q}{(x-l)^2} \quad (\text{along } (+) \text{ x-axis})$$

Electric Field intensity at (P) due to $(-q)$

$$\vec{E}_- = \frac{1}{4\pi\epsilon_0} \frac{q}{AP^2}$$

$$\vec{E}_- = -\frac{1}{4\pi\epsilon_0} \frac{q}{(x+l)^2} \quad (\text{along } (-) \text{ x-axis})$$

$$\vec{E}_- = -\frac{1}{4\pi\epsilon_0} \frac{q}{(x+l)^2} \quad (\text{along } (+) \text{ x-axis})$$

Acc. to superposition principle

$$\vec{E} = \vec{E}_+ + \vec{E}_-$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{(x-l)^2} - \frac{1}{4\pi\epsilon_0} \frac{q}{(x+l)^2}$$

$$\vec{E} = \frac{q}{4\pi\epsilon_0} \left[\frac{1}{(x-l)^2} - \frac{1}{(x+l)^2} \right]$$

$$= \frac{q}{4\pi\epsilon_0} \left[\frac{(x+l)^2 - (x-l)^2}{(x-l)^2 (x+l)^2} \right]$$

$$\vec{E} = \frac{q}{4\pi\epsilon_0} \left[\frac{r^2 + l^2 + 2lx - (r^2 + l^2 - 2xl)}{(r^2 - l^2)^2} \right]$$

$$= \frac{q}{4\pi\epsilon_0} \left[\frac{r^2 + l^2 + 2lx - r^2 - l^2 + 2xl}{(r^2 - l^2)^2} \right]$$

$$\vec{E} = \frac{q}{4\pi\epsilon_0} \left[\frac{4xl}{(r^2 - l^2)^2} \right]$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \left[\frac{q \times 2l \times 2x}{(r^2 - l^2)^2} \right]$$

$$\vec{E} = \frac{p \times 2x}{4\pi\epsilon_0 (r^2 - l^2)^2}$$

$$\vec{E} = \frac{2x p}{4\pi\epsilon_0 (r^2 - l^2)^2}$$

$$\vec{E} = \frac{2x \vec{p}}{4\pi\epsilon_0 (r^2 - l^2)^2}$$

Special case :-

If $r \gg l$ i.e. The observation point is at very large distance as compared to the dipole length then l^2 can be neglected.

$$\therefore \vec{E} = \frac{2x \vec{p}}{4\pi\epsilon_0 r^3}$$

$$\vec{E} = \frac{2 \vec{p}}{4\pi\epsilon_0 r^3}$$

Q What is the total charge on an electric dipole?

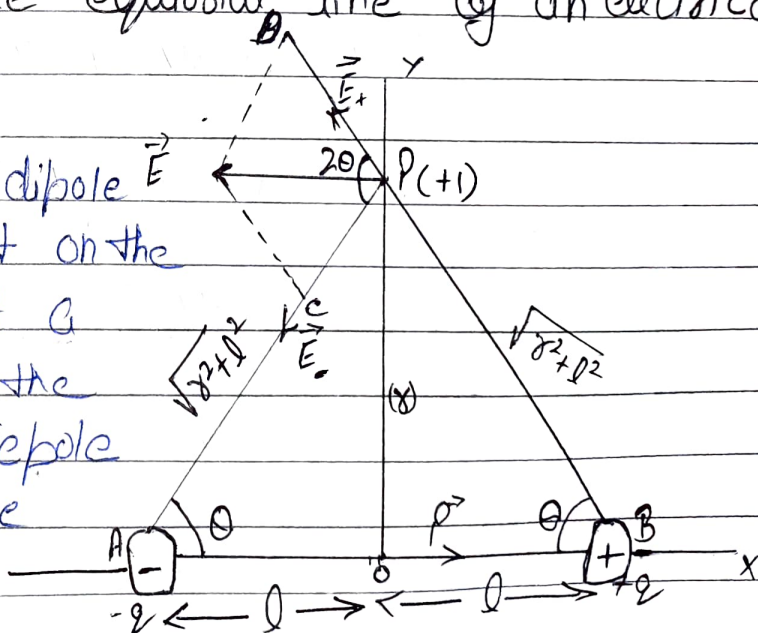
Ans: A Dipole consists a pair of equal and opposite charges i.e. (+q) and (-q) so that total charge on electrical dipole is zero.

Q What is the role of electric field in microwave cooking?

Ans: Food that contains water can be cooked better in microwave because water molecules are electric dipoles which oscillate due to the electric field of microwaves generated in the oven. The bonded molecules of water are broken because of the oscillations and therefore transferring their energy into thermal energy.

Q: Derive an expression for electric field intensity at a point on the equatorial line of an electrical dipole?

Ans: Consider an electric dipole (AB) let P be the point on the equatorial line at a distance (r) from the center of the dipole. Let a unit (+ve) charge be placed at point (P).



\therefore Electric field intensity at point (P) due to (+q) charge

$$\vec{E}_+ = \frac{1}{4\pi\epsilon_0} \frac{q}{(BP)^2} \text{ along } BP$$

$$\vec{E}_+ = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2+l^2} \text{ along } (PD) \text{ --- (i)}$$

Electric field intensity at (P) due to (-q) charge

$$\vec{E}_- = \frac{1}{4\pi\epsilon_0} \frac{q}{(PA)^2} \text{ along } PC$$

$$= \frac{1}{4\pi\epsilon_0} \frac{q}{r^2+l^2} \text{ along } PC \text{ --- (ii)}$$

from equ (i) and (ii)

$$\vec{E}_+ = \vec{E}_- = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2+l^2} \text{ --- (iii)}$$

\vec{E}_+ and \vec{E}_- are inclined at an angle of 2θ
So their resultant can be determine by using
triangle law of vector addition.

$$E = \sqrt{E_+^2 + E_-^2 + 2E_+E_- \cos 2\theta}$$

$$E = \sqrt{E_+^2 + E_+^2 + 2E_+E_- \cos 2\theta}$$

$$E = \sqrt{2E_+^2 + 2E_+^2 \cos 2\theta}$$

$$E = \sqrt{2E_+^2 (1 + \cos 2\theta)}$$

$$E = \sqrt{2E_+^2 \cdot 2\cos^2\theta} \quad \left[\because 1 + \cos 2\theta = 2\cos^2\theta \right]$$

$$E = 2E_+ \cos\theta$$

$$E = 2 \frac{1}{4\pi\epsilon_0} \frac{q}{(r^2+l^2)} \cos\theta \text{ --- (iv)}$$

from ΔAOP

$$\cos\theta = \frac{l}{\sqrt{r^2+l^2}}$$

Put value of $\cos\theta$ in eqn

$$E = 2 \times \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2+l^2} \cdot \frac{l}{(r^2+l^2)^{1/2}}$$

$$E = \frac{2}{4\pi\epsilon_0} \cdot \frac{q \cdot l}{(r^2+l^2)^{3/2}}$$

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{2 \times q \cdot l}{(r^2+l^2)^{3/2}}$$

But $q \times 2l = p$

$$E = \frac{p}{4\pi\epsilon_0 (r^2+l^2)^{3/2}} \quad \text{along } (-) \text{ x-axis}$$

$$\vec{E} = \frac{-p}{4\pi\epsilon_0 (r^2+l^2)^{3/2}} \quad \text{along } (-) \text{ x-axis}$$

Special Case :-

If $l \ll r$ The electric field intensity along $(-) \text{ x-axis}$

$$\vec{E} = \frac{-p}{4\pi\epsilon_0 r^3}$$

$$\boxed{\vec{E} \propto \frac{1}{r^3}}$$

Note (1) The direction of electric field intensity (\vec{E}) at a point on the axial line of the dipole is towards dipole axis from $(-ve)$ to $(+ve)$.

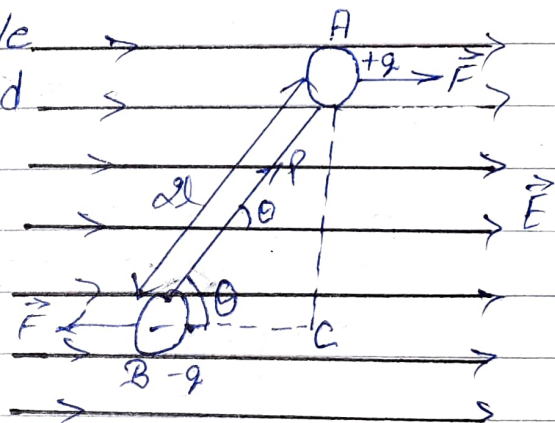
The direction of electric field intensity at a point on the equatorial line of the dipole is opposite to the dipole axis to (+ve) to (-ve) charge.

For a short dipole electric field intensity at a point on the axial line is double than at a point on the equatorial line of an dipole.

$$E_{\text{axial}} = 2 E_{\text{equatorial}}$$

Q:- Derive and expression for the torque experienced by an electric dipole placed in uniform electric field. And hence define the electric dipole moment.

Ans:- Consider an electrical dipole in a uniform electric field (\vec{E}) such that the dipole moment (\vec{P}) makes an angle (θ) with the electric field (\vec{E}). Charges ($+q$) and ($-q$) constituting the dipole experience equal and opposite forces given by $q\vec{E}$, $-q\vec{E}$ resp. due to electric field (\vec{E}).



$$\therefore F_{\text{net}} = qE - qE = 0$$

These two equal and opposite forces acting on the dipole constitute a couple. This couple tends to rotate the dipole in the clockwise

direction. Hence align the dipole along the direction of electric field.

Now

$\tau = \text{Magnitude of force} \times \perp \text{ distance b/w two forces}$

$$\tau = qE \times AC \quad \text{--- (1)}$$

from right angle triangle ABC

$$\sin \theta = \frac{AC}{2l}$$

$$AC = 2l \sin \theta$$

Put the value of AC in equation (1),

$$\tau = qE \times 2l \sin \theta$$

$$\tau = pE \sin \theta$$

$$\vec{\tau} = \vec{p} \times \vec{E}$$

$$[\because q \cdot 2l = p]$$

Another definition of electric dipole :-

We know that

$$\tau = pE \sin \theta$$

$$p = \frac{\tau}{E \sin \theta}$$

$$E \sin \theta$$

$$\Downarrow \quad E \cdot 1 = \theta = 90^\circ$$

$$[p = \tau]$$

Dipole moment is numerically equal to the Torque acting on the dipole if it is placed at right angle to the unit uniform electric field.

Special case :- (1) $\Downarrow \theta = 0$, then $\tau = pE \sin \theta$
 $\tau = 0$

\therefore when dipole moment is \parallel to electric field

No Torque acts on the dipole.

(ii) If $\theta = 180^\circ$ then $\tau = pE \sin 180^\circ = 0$
 \therefore when dipole is anti-parallel to the electric field no torque acts on the dipole.

(iii) If $\theta = 90^\circ$ then $\tau = pE \sin 90^\circ = pE$ (Maximum)
 \therefore when dipole moment is perpendicular to the electric field (\vec{E}) maximum torque acting on the dipole.

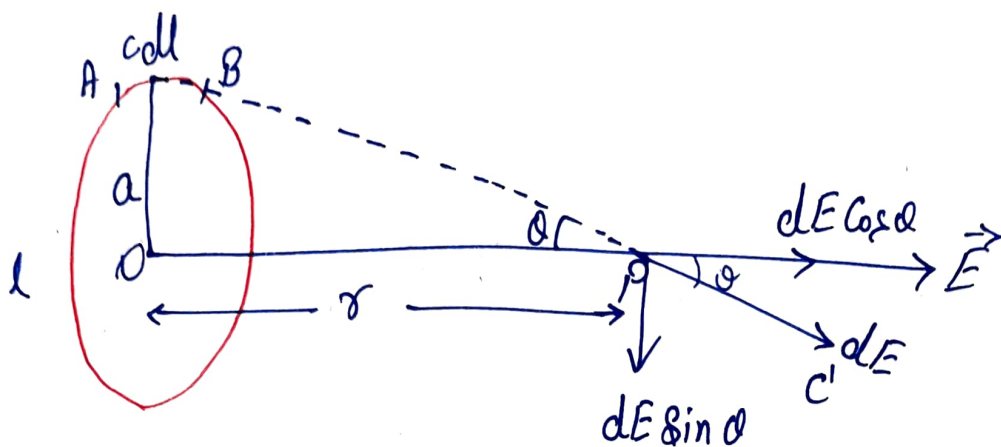
Q Why is rotation of electric field is natural and very help full.

Ans: The concept of electric field is very convenient to find out the effect of charge at some distance from it. The force of a charge on another is also due to electromagnetic waves produced because of accelerated motion of that charge. The concept of field explain the time delay and transportation of energy on this account.

Q Electric field line can hence appear as closed loop. Do you agree or not?

Ans: Electrostatic field lines starts from positive charge and ends at negative charge. So they don't appear as closed loop.

Electric field Intensity at any point on the axis of a uniformly charged Ring:-



Consider a circular loop of wire of negligible thickness, Radius (a) and centre (O) held perpendicular to the plane of the paper.

Let the loop carry charge ($+Q$) distributed uniformly over the circumference.

Consider small element dl (AB) of the loop.

Now charge on the $AB = dq = \lambda dl$

$$\text{Here } \lambda = \frac{Q}{l}$$

$$dq = \frac{Q}{2\pi a} dl \quad \text{--- (1)}$$

$$[\because l = 2\pi a]$$

$$[\because l = \text{Circumference of circle}]$$

Now electric field Intensity at point (P) due to (AB)

$$|d\vec{E}| = \frac{1}{4\pi\epsilon_0} \frac{dq}{(r)^2} \quad \text{--- along } (Pc) \text{ at angle } \theta$$

Here from ΔOCP
using I.G.T.

$$(H)^2 = (P)^2 + (B)^2$$

$$(CP)^2 = a^2 + l^2$$

$$\boxed{|CP| = \sqrt{a^2 + l^2}} \quad \text{--- (ii)}$$

$$|d\vec{E}| = \frac{1}{4\pi\epsilon_0} \frac{dq}{(a^2 + l^2)} \quad \text{--- (iii)}$$

Here $d\vec{E}$ Resolve into two Components

$d\vec{E} \cos\theta$ along the electric field

$d\vec{E} \sin\theta$ \perp to the electric field

Here $dE \sin\theta = 0$ (because \perp electric field become zero)

$$|\vec{E}| = \sum dE \cos\theta \quad \text{--- (iv)}$$

$$\text{Here } \cos\theta = \frac{B}{H} = \frac{a}{\sqrt{a^2 + l^2}} \quad \text{--- (v)}$$

using the value of equation (iii) and (v) in (iv)

$$\begin{aligned} |\vec{E}| &= \sum \frac{1}{4\pi\epsilon_0} \frac{dq}{a^2 + l^2} \frac{a}{\sqrt{a^2 + l^2}} \\ &= \sum \frac{1}{4\pi\epsilon_0} \frac{q}{2\pi a} \frac{a}{(a^2 + l^2)^{3/2}} dl \end{aligned}$$

$$\vec{E} = \frac{q \, dl}{4\pi\epsilon_0 \, 2\pi a \, (x^2 + a^2)^{3/2}} \quad \sum_{\text{whole loop}} dl$$

$$\vec{E} = \frac{q \, x}{4\pi\epsilon_0 \, 2\pi a \, (x^2 + a^2)^{3/2}} \quad (2\pi a)$$

$$|\vec{E}| = \frac{q \, x}{4\pi\epsilon_0 \, (x^2 + a^2)^{3/2}}$$

The direction of \vec{E} is along \hat{i}_x , the axis of the loop.

Special Cases: - (i) when (P) lies at the centre of the loop
[$x=0$]

$$E = \frac{q \, (0)}{4\pi\epsilon_0 \, (0 + a^2)^{3/2}}$$

$$\boxed{\vec{E} = 0}$$

(ii) when $x \gg a$ (a^2 neglecting)

$$E = \frac{q \, x}{4\pi\epsilon_0 \, x^3} = \frac{q}{4\pi\epsilon_0 \, x^2}, \text{ along } \hat{i}_x$$

$$\boxed{|\vec{E}| = \frac{q}{4\pi\epsilon_0 \, x^2} \text{ along } \hat{i}_x}$$

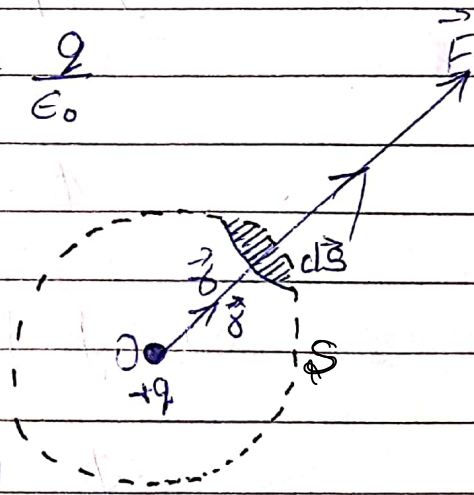
Q State and explain Gauss's theorem or Gauss law?

Ans. Gauss theorem:- Acc. to this theorem the total electric flux (ϕ) through any closed surface (S) in free space is equal to $\frac{1}{\epsilon_0}$ times the total electric charge (Q) enclosed by the surface.

$$\phi = \oint_S \vec{E} \cdot d\vec{s} = \frac{Q}{\epsilon_0}$$

Prove:-

Consider an isolated point charge (+q) at (O). Let surface (S) be that of a sphere of radius (r) around the charge (+q)



\therefore Electric field intensity due to charge (+q) at every point on the surface of sphere.

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r} \quad (i)$$

Now consider a small element of area ($d\vec{S}$)
electric flux through the small area element.

$$d\phi = \vec{E} \cdot d\vec{S}$$

$$d\phi = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r} \cdot d\vec{S}$$

$$d\phi = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r} \cdot d\vec{S} \cdot \hat{n}$$

$$d\phi = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} ds \hat{r} \cdot \hat{n} \quad (ii)$$

Since \hat{r} and \hat{n} are in the same direction

$$\therefore \hat{r} \cdot \hat{n} = |\hat{r}| |\hat{n}| \cos 0 = 1$$

\therefore equation (ii) become

$$d\phi = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} ds$$

\therefore The electric flux over the entire closed surface (S) is

$$\oint_S d\phi = \oint_S \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} ds$$

$$\phi = \frac{q}{4\pi\epsilon_0 r^2} \oint_S ds$$

where $\oint ds = 4\pi r^2$ surface area of sphere

$$\phi = \frac{q}{4\pi\epsilon_0 r^2} 4\pi r^2$$

$$\phi = \frac{q}{\epsilon_0}$$

$$\boxed{\phi = \oint_S \vec{E} \cdot d\vec{S} = \frac{q}{\epsilon_0}}$$

Notes: (1) The charges situated outside the closed surface are not considered to be contributing to total electric flux, over that surface.

(2) Gauss's law is valid for stationary charges as well as for rapidly moving charges. were as Coulomb's law would only for stationary charges.

Q Will Gauss be true if Coulomb's force depends upon r^{-3} instead of r^{-2} .

Sol No, if Coulomb's law is considered to be the basic law then Gauss law can be obtained from this law and vice versa. Both laws complement each other if dependence of Coulomb's force change from $\frac{1}{r^2}$ to $\frac{1}{r^3}$ Gauss law will not hold.

Q What is the importance of Gaussian surface?

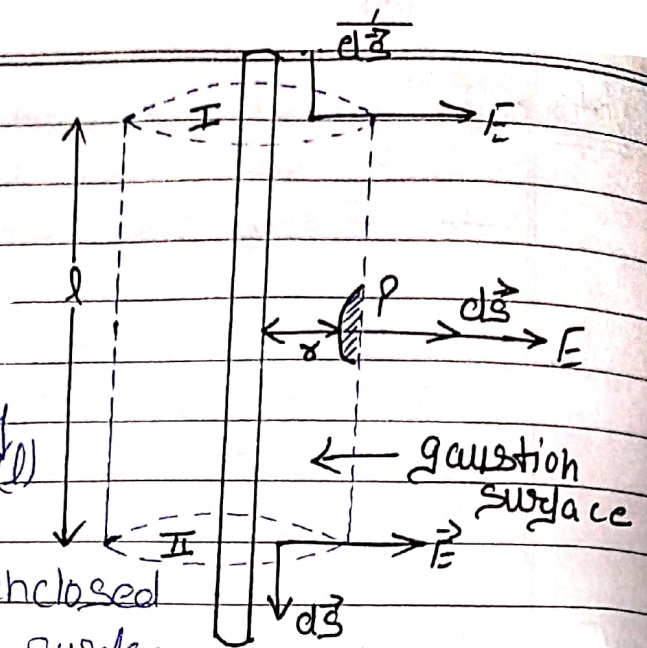
Ans: The concept of Gaussian surface helps in determining the electric field intensity due to the systematic charge distribution.

Q Derive an expression for electric field intensity due to an infinitely long straight uniformly charged wire?

Ans: Consider an infinite and very thin straight wire having uniform linear charge density (λ). To calculate the electric field intensity

\vec{E} at point (P)
distance (x) from the
uniform charged wire.

Gaussian surface of
radius (x) and length (l)
around the charged
line. The charge enclosed
by a the Gaussian surface
is $q = \lambda l$



Acc. to Gauss's theorem

$$\oint \vec{E} \cdot d\vec{s} = \frac{q}{\epsilon_0} = \frac{\lambda l}{\epsilon_0} \quad (1)$$

The cylindrical gaussian surface is divided
into three parts I, II, III

\therefore Equ (1) can be written as

$$\oint \vec{E} \cdot d\vec{s} = \int_I \vec{E} \cdot d\vec{s} + \int_{II} \vec{E} \cdot d\vec{s} + \int_{III} \vec{E} \cdot d\vec{s} = \frac{\lambda l}{\epsilon_0} \quad (2)$$

For surface I and II angle b/w \vec{E}
and \vec{s} is 90°

$$\begin{aligned} \therefore \vec{E} \cdot d\vec{s} &= E ds \cos 90^\circ \\ \vec{E} \cdot d\vec{s} &= E ds \cos 90^\circ \\ &= 0 \end{aligned}$$

\therefore The electric flux is zero across the
both ends.

The curved surface only
... eqn (II) become

$$\oint \vec{E} \cdot d\vec{s} = \int_{III} \vec{E} \cdot d\vec{s} = \frac{\lambda l}{\epsilon_0}$$

$$\text{or } \int_{III} \vec{E} \cdot d\vec{s} = \frac{\lambda l}{\epsilon_0}$$

$$\int_{III} E ds \cos 0 = \frac{\lambda l}{\epsilon_0}$$

$$\int_{III} E ds = \frac{\lambda l}{\epsilon_0}$$

$$\int \because \cos 0 = 1$$

$$\int_{III} E ds = \frac{\lambda l}{\epsilon_0}$$

$$E \times S = \frac{\lambda l}{\epsilon_0} \quad \left\{ \begin{array}{l} \text{where } S \text{ is a } C.S.A \\ \text{of cylinder} \end{array} \right.$$

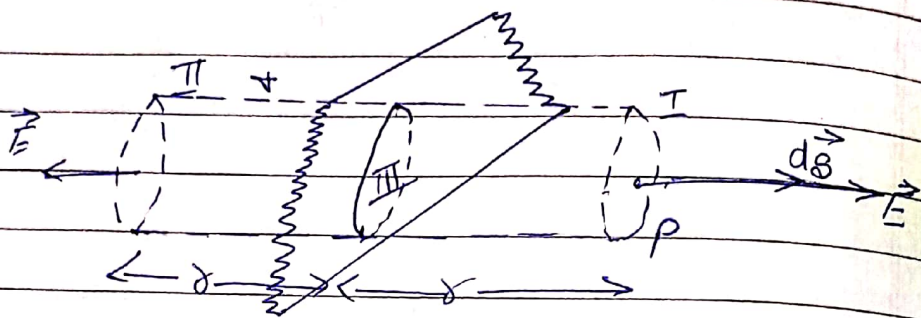
$$E \times 2\pi r l = \frac{\lambda l}{\epsilon_0}$$

$$E = \frac{\lambda l}{2\pi r l \epsilon_0} = \frac{\lambda}{2\pi r \epsilon_0}$$

$$E \propto \frac{1}{r} \quad \int \because \frac{\lambda}{2\pi \epsilon_0} = \text{Constant}$$

Q. Derive and Expression for Electric field intensity due to uniformly charged infinite plane sheet (flat sheet of charge).

Ans: Consider a thin infinite Non-Conducting plane sheet having uniform surface charge density (ie charged per unit area) σ



To calculate the electric field intensity \vec{E} at a point (P) distance (r) from the sheet draw gaussian surface in the form of a closed cylinder

Electric field \vec{E} is perpendicular to the sheet

charge enclosed by the gaussian surface ($q = \sigma S$)
Acc. to gauss theorem

$$\oint \vec{E} \cdot d\vec{S} = \frac{q}{\epsilon_0} = \frac{\sigma S}{\epsilon_0}$$

Gaussian surface is divided into three parts I, II, III i.e. ends caps and the curved surface of cylinder as shown in the fig.

\therefore equ (i) becomes

$$\oint_I \vec{E} \cdot d\vec{S} + \oint_{II} \vec{E} \cdot d\vec{S} + \oint_{III} \vec{E} \cdot d\vec{S} = \frac{\sigma S}{\epsilon_0}$$

Angle b/w \vec{E} and $d\vec{S}$ is 90° for the curved surface III.

$$\text{So } \vec{E} \cdot d\vec{S} = E ds \cos 90^\circ \\ = 0$$

$$\oint \vec{E} \cdot d\vec{S} = \int_I \vec{E} \cdot d\vec{S} + \int_{II} \vec{E} \cdot d\vec{S} = \int_I E ds \cos 0^\circ + \int_{II} E ds \cos 0^\circ$$

$$= \int_I E ds + \int_{II} E ds = ES + ES = 2ES$$

Hence equ (i) becomes

$$2ES = \frac{QS}{\epsilon_0} \Rightarrow$$

$$E = \frac{Q}{2\epsilon_0 S}$$

VV

Q

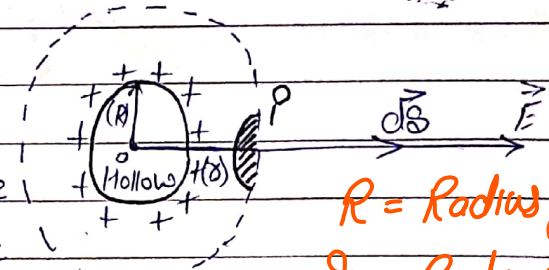
Derive and Expression for electric field intensity due to a uniformly charged thin hollow spherical shell.

- (1) At a point outside the shell.
- (2) At a point on the surface of the shell.
- (3) At a point inside the shell.

Ans: (i) At a point outside the shell.

Consider a positive charge (q) distributed uniformly on the surface of a spherical shell of radius (R)

(P) is a point outside the shell at a distance (r) from the centre of the shell ($r > R$)



$R =$ Radius of shell
 $r =$ Radius of Gaussian surface.

Gaussian surface of radius (r) with (O) as the centre

Acc. to Gauss's theorem

$$\oint \vec{E} \cdot d\vec{S} = \frac{q}{\epsilon_0}$$

$$\oint E \cdot dS \cos 0 = \frac{q}{\epsilon_0} \quad \text{--- (i)}$$

Since \vec{E} and $d\vec{S}$ are directed along the same direction, so $\theta = 0$.

$$\oint E dS \cos 0 = \frac{q}{\epsilon_0}$$

$$\oint E dS = \frac{q}{\epsilon_0} \quad \text{--- (ii)}$$

Since: (E) at all points on the Gaussian surface is same and directed radially outwards.

$$E \oint ds = \frac{q}{\epsilon_0}$$

$\oint ds =$ surface area of the Gaussian surface $= 4\pi r^2$

$$E \times 4\pi r^2 = \frac{q}{\epsilon_0}$$

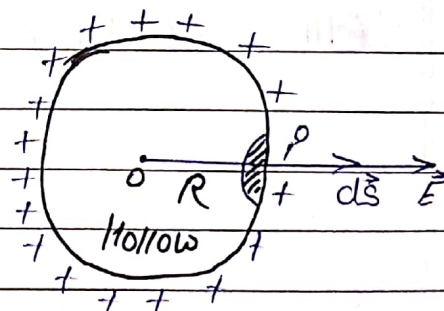
$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$$

Which is the electric field due to a point charge q at a distance (r)

$$\vec{E} = \frac{q}{4\pi\epsilon_0 r^2} \hat{r}$$

(ii) At a point on the surface of the shell :-

When point of observation (P) is at the surface of the shell,



then $r = R$ & $r \ll R$.

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{R^2}$$

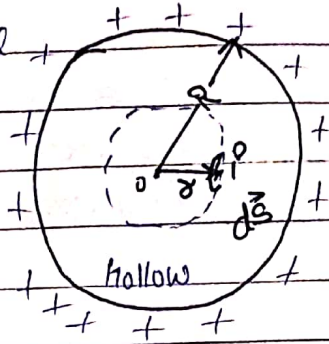
Again ($q = \sigma \times 4\pi R^2$), where $\sigma =$ Surface charge density of the charge on the shell.

$$E = \frac{1}{4\pi\epsilon_0} \frac{\sigma \times 4\pi R^2}{R^2}$$

$$E = \frac{\Gamma}{\epsilon_0}$$

(C) At a point inside the shell:-

Let (P) be the point inside the shell at a distance r' from the centre of the shell. A Gaussian sphere of radius r' with O as the centre.



Acc. to Gauss' theorem

$$\oint_S \vec{E} \cdot d\vec{s} = \frac{q}{\epsilon_0}$$

Here Gaussian surface encloses no charge $q=0$

$$\oint_S \vec{E} \cdot d\vec{s} = 0 \quad \text{or} \quad E = 0$$

Q. Deduce Coulomb's law from Gauss's theorem?

Ans - Consider two point charges

(q_1) and (q_2) separated by a distance

(r) draw a sphere of radius (r) with

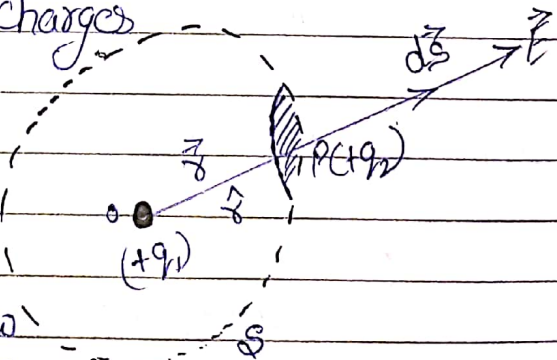
(O) at the centre, now

consider a small area

element of area ($d\vec{s}$) over the sphere the angle b/w ($d\vec{s}$) and (\vec{E}) is zero. Therefore

acc. to Gauss's theorem

$$\oint_S \vec{E} \cdot d\vec{s} = \frac{q}{\epsilon_0}$$



$$\oint E ds \cos 90^\circ = \frac{q_1}{\epsilon_0} \Rightarrow \oint E ds \cos 0 = \frac{q_1}{\epsilon_0}$$

$$\Rightarrow \oint E ds = \frac{q_1}{\epsilon_0}$$

Since (E) is uniform as it has same value at all points on the gaussian surface: $E \oint ds = \frac{q_1}{\epsilon_0}$

But $\oint ds = \text{Surface area of sphere} = 4\pi r^2$

$$E \times 4\pi r^2 = \frac{q_1}{\epsilon_0}$$

$$E = \frac{1}{4\pi\epsilon_0} \frac{q_1}{r^2} \quad \text{--- (1)}$$

This is the value electric field intensity at a point where charge q_2 is placed.

\therefore Force experienced by (q_2) is

$$F = Eq_2$$

from equation (1)

$$F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \quad \text{(Hence prove)}$$

Which is Coulomb's law.

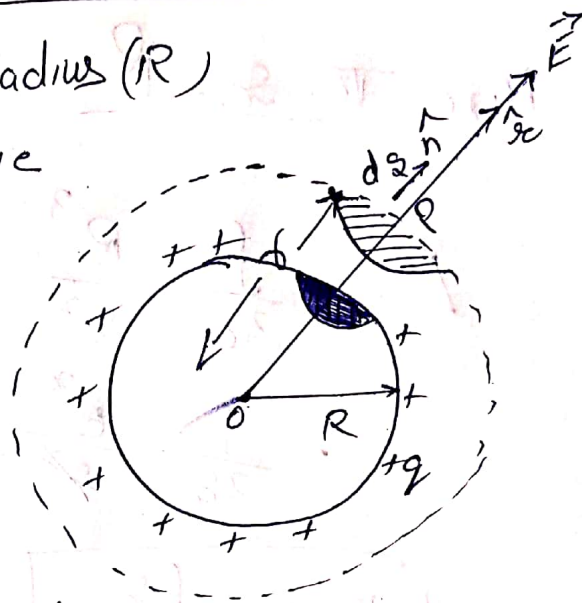
Q Electric field Intensity due to A Non-Conducting charged solid sphere:-

(a) Electric field Intensity outside the shell:-

Consider a solid sphere having Radius (R) and Centre (O) has uniform volume density (ρ)

Now we have to find out the electric field Intensity at point (P) at a distance (r) from the Centre point

Now acc. to the Gauss's law



$$\oint_S \vec{E} \cdot d\vec{s} = \oint_S \vec{E} \cdot \hat{n} ds = \frac{q}{\epsilon_0} \quad \text{--- (1)}$$

$$= \int_S E ds \cos \theta = \frac{q}{\epsilon_0}$$

[Angle b/w \hat{n} and $\vec{E} = 0$]

$$\Rightarrow \int_S E ds = \frac{q}{\epsilon_0} \Rightarrow E \int_S ds = \frac{q}{\epsilon_0}$$

$$\Rightarrow E 4\pi r^2 = \frac{q}{\epsilon_0} \Rightarrow E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \quad \text{--- (1)}$$

Now $\rho = \frac{q}{V} \Rightarrow q = \rho V \Rightarrow q = \rho \frac{4}{3}\pi R^3$

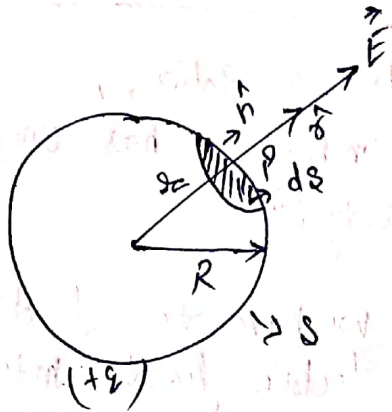
$$E = \frac{1}{4\pi\epsilon_0} \frac{\rho \frac{4}{3}\pi R^3}{r^2} \Rightarrow E = \frac{\rho R^3}{3\epsilon_0 r^2} \quad \text{--- (11)}$$

Electric field Intensity at a point on the surface of the shell :-

Here $r = R$

$$E = \frac{\rho}{3\epsilon_0} \frac{R^3}{r^2}$$

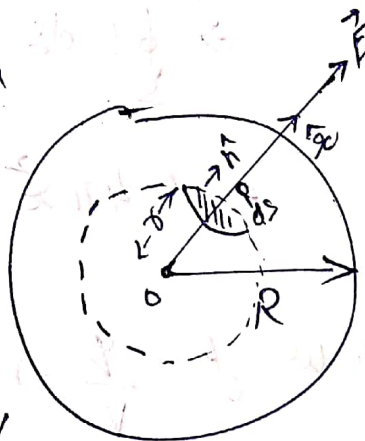
$$= \frac{\rho}{3\epsilon_0} \frac{r^3}{r^2}$$



$$|E| = \frac{\rho r}{3\epsilon_0} \quad \text{--- (iii)}$$

Electric field Intensity in the solid sphere :-

($r < R$)
 We consider r the radius of gaussian sphere is less than the solid sphere now acc to gaussian theorem



$$\rho = \oint \vec{E} \cdot d\vec{S} = \oint E \, dS \, \cos 0 = \frac{q'}{\epsilon_0}$$

$$= E \, 4\pi r^2 = \frac{q'}{\epsilon_0} \quad \text{--- (iv)} \quad [q' = \text{the charge inside the sphere}]$$

New Volume charge density

$$\rho = \frac{q'}{V} \Rightarrow \boxed{q' = \rho V}$$

Put in equation (iv)

$$E = \frac{1}{4\pi R^2} \frac{\rho V}{\epsilon_0}$$

$$E = \frac{\int \frac{4}{3}\pi r^3}{4\pi r^2 \epsilon_0}$$

$$\boxed{E = \frac{\rho r}{3\epsilon_0}}$$

Formulas:-

1) Coulomb's Law:-

$$F = \frac{1}{4\pi\epsilon_0} \frac{|q_1 q_2|}{r^2} \text{ or } F = k \frac{|q_1 q_2|}{r^2}$$

$$k = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ Nm}^2 \text{ C}^{-2}$$

2) Superposition Principle:-

$$F_1 = \frac{q_1}{4\pi\epsilon_0} \sum_{i=2}^n \frac{q_i}{r_{1i}^2} \hat{r}_{1i}$$

3) Electrostatic force due to Continuous Charge Distribution:-

a) Linear charge distribution:-

$$F = \frac{q_0}{4\pi\epsilon_0} \int_L \frac{\lambda dl}{r_0^2} \hat{r}_0$$

b) Surface charge distribution:-

$$F = \frac{q_0}{4\pi\epsilon_0} \int_S \frac{\sigma dS}{r_0^2} \hat{r}_0$$

c) Volume charge distribution:-

$$F = \frac{q_0}{4\pi\epsilon_0} \int_V \frac{\rho dv}{r_0^2} \hat{r}_0$$

④ Electric field :-

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \text{ or } E = \frac{F}{q_0}$$

⑤ Electric dipole and dipole moment :-

$$p = q(2l) \quad [2l = \text{length of dipole}]$$

⑥ Electric field due to an electric dipole

a) Axial line:

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{2p\alpha}{(x^2 - l^2)^2}$$

b) Equatorial point :-

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{\vec{p}}{(x^2 - l^2)^{3/2}}$$

⑦ Torque on an electric dipole in a uniform electric field :-

$$\tau = pE \sin\theta \Rightarrow [\vec{\tau} = \vec{p} \times \vec{E}]$$

⑧ Electric field on axial line due to charge ring:-

$$E = \frac{1}{4\pi\epsilon_0} \frac{q x}{(x^2 + a^2)^{3/2}}$$

$$\left[a = \text{radius of ring} \right]$$

⑨ Electric flux:-

$$\phi = \int_S \vec{E} \cdot d\vec{S} = \int_S E ds \cos\theta$$

⑩ Gauss's Theorem:-

$$\phi_E = \int_S \vec{E} \cdot d\vec{S} = \frac{1}{\epsilon_0} \sum_{i=1}^n q_i = \frac{q}{\epsilon_0}$$

⑪ Electric field intensity due to long straight wire:-

$$E = \frac{\lambda}{2\pi\epsilon_0 r} = \frac{2k\lambda}{r} \left[\lambda = \frac{1}{4\pi\epsilon_0} \right]$$

(12)

Electric field due to infinite charge sheet

$$E = \frac{\sigma}{2\epsilon_0}$$

(13)

Electric field due to charged Hollow Spherical Shell.

a) outside the sphere :-

$$E_{\text{out}} = \frac{\sigma R^2}{\epsilon_0 r^2}$$

b) At the surface of sphere

$$E_s = \frac{\sigma}{\epsilon_0}$$

c) Inside the sphere

$$E_{\text{in}} = 0$$

(14)

Electric Field due to Uniform Charge Solid Sphere.

a) outside the sphere :-

$$E_{\text{out}} = \frac{\rho R^3}{3\epsilon_0 r^2}$$

b) At the surface of sphere :-

$$E_s = \frac{\rho R}{3\epsilon_0}$$

c) Inside the sphere :-

$$E_{\text{in}} = \frac{\rho r}{3\epsilon_0}$$