MAGNETIC EFFECTS OF CURRENT AND MAGNETISM

Chapter–4: (Moving Charges and Magnetism)

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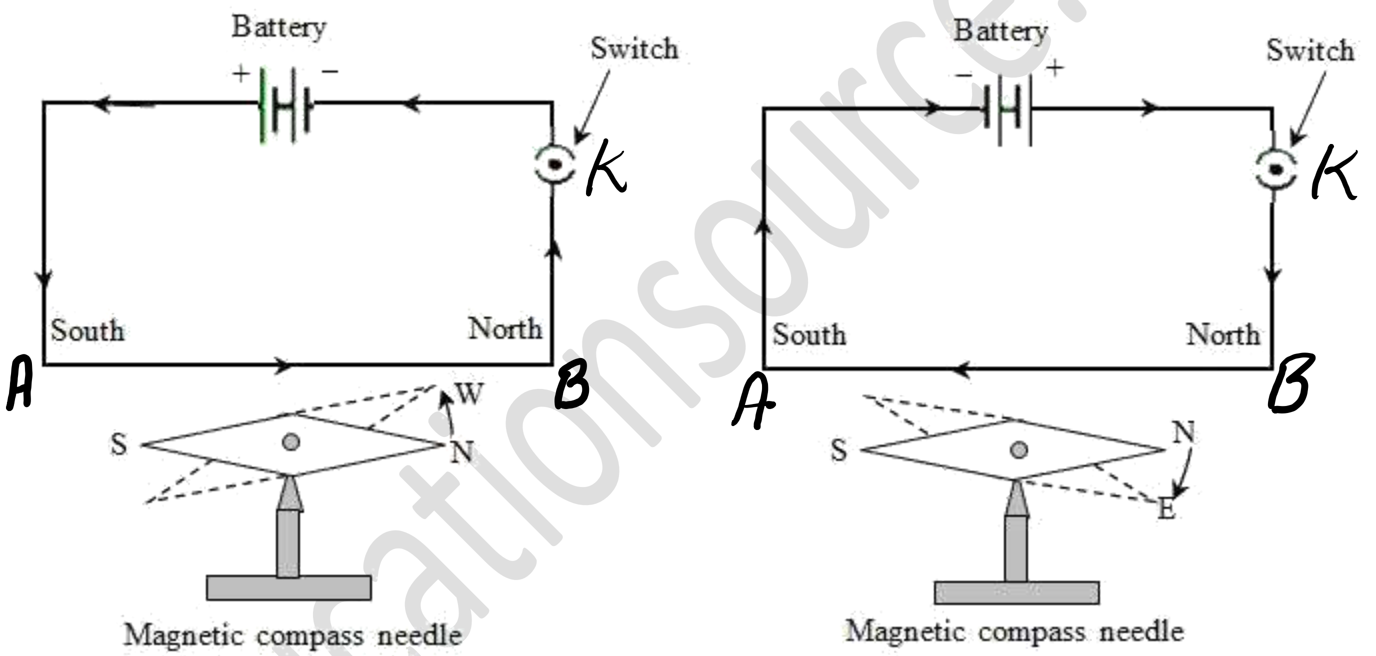
Oersted's experiment: -

Take a magnetic needle NS, which can rotate freely about a vertical axis in a horizontal plane. Hold a conducting wire AB over the magnetic needle NS parallel to it. Complete the circuit by closing the key K such that current flows from A to B.

It will be found that N-Pole of the magnetic needle gets deflected towards the west **Fig.1.** If the direction of current in the wire is reversed (i.e., from B to A), the N-pole of magnetic needle gets deflected towards east **Fig.2.** Since the magnetic needle can be deflected only by the interaction of another magnetic field, therefore, the current in the wire must be producing a magnetic field in the surrounding space. The direction of deflection of magnetic needle due to current in the wire is given by **Ampere swimming rule.**



Ampere's swimming rule: - According to this rule, if we imagine a man is swimming along the wire in the direction of current with his face always turned towards the needle, so that the current enters through his feet and leaves at his head, then the N-pole of the magnetic needle will be deflected towards his left hand.

Unit :- 3

Magnetic effect of Current and magnetism :-

Chapter :- 4

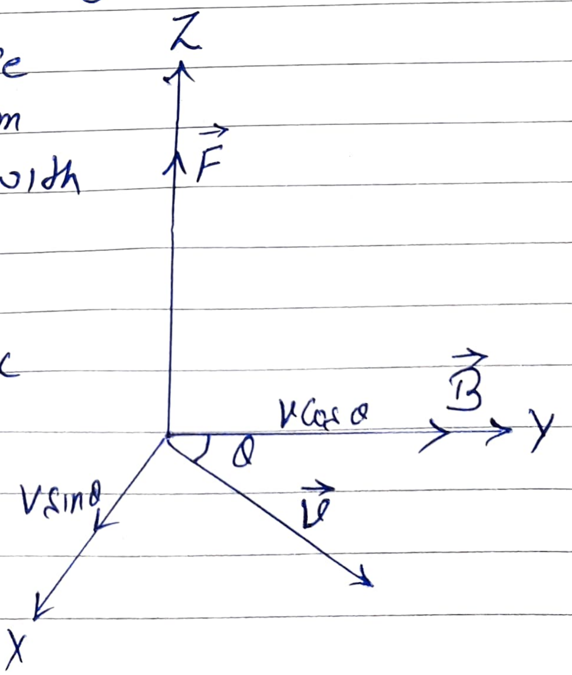
Magnetic field due to current (moving charge)

⇒ **Magnetic field :-** The magnetic field is a space around a conductor carrying current or magnet in which its magnetic effect can be felt.

The magnetic field disappers as soon as the current is switched off or charge stop moving. ie It mean that moving charge is a source of Magnetic field aswell as electric field

Magnetic field denoted by $(\vec{B})$.

Consider a (+ve) charge (q) moving in a uniform magnetic field $\vec{B}$, with a velocity $(\vec{v})$. let (θ) be the angle between the magnetic field and velocity of the particle.



Now we Can Resolve the .... velocity in two Components $v\cos\theta$ and $v\sin\theta$.

Now Charge moving in magnetic field so It experiance a force, which depends upon the factors.

(I) The magnitude of the force ($\vec{F}$) experienced by the moving Charge directly proportional to the magnitude of charge

$$F \propto q \quad \text{— (I)}$$

(II) $F \propto v\sin\theta$ — (II) ($\perp$ Component of velocity)

(III) $F \propto (B)$ — (III) (applied magnetic field)

Now Camparing above factors

$$F \propto q v \sin\theta B$$

$$F = k q v B \sin\theta$$

Here K (Constant) $= 1$

$$F = q v B \sin\theta$$

$$|\vec{F}| = q|\vec{v} \times \vec{B}|$$

$$\vec{F} = q(\vec{v} \times \vec{B}) \quad \text{— (IV)}$$

The direction of $\vec{F}$ is direction of cross product of velocity $\vec{v}$ and magnetic field $\vec{B}$.

Here The direction of force is $\perp$ to the applied magnetic field and velocity component.

(Right-hand-Screw Rule) or (Right-Hand thumb Rule).

<u>Special Case</u>:-

(i) If $\theta = 0°$ or $180°$ then $\sin\theta = 0$

$$F = qvB(0) = 0.$$

(ii) If $V = 0$ then $F = qVB\sin\theta = 0$.

[If charge particle at Rest].

(iii) If $\theta = 90$, then $\sin\theta = 1$.

$$F = qvB \text{ (Maximum value)}.$$

ie. It means, if a charge particle is moving along a line $\perp$ to the direction of magnetic field it experiences maximum force.

It can be - explain by fleming's left Hand Rule.

S.I. Unit is Tesla(T)

Dimensional formula:

$$B = \frac{F}{qv\sin\alpha}$$

$$= \frac{[MLT^{-2}]}{[AT][LT^{-1}]}$$

$$= [M L^{0} A^{-1} T^{-2}]$$

Problem :- An α-particle of mass 6.65×10^{-27} kg is travelling at right angles to a magnetic field with a speed of 6×10^{5} m/s. The strength of the magnetic field is 0.2 T. Calculate the force on the α-particle and its acceleration.

Sol: given that mass of the α particle $= 6.65 \times 10^{-27}$ kg

$v = 6 \times 10^{5}$ m/s

$B = 0.2$ T

we have to calculate

$$F = qvB\sin\alpha \quad —(1)$$

Here $q = 2e = 2 \times 1.6 \times 10^{-19}$ C

$\theta = 90°$ ($v \perp B$)

$$F = 2 \times 1.6 \times 10^{-19} \times (0.2)(6 \times 10^{5}) \times \sin 90^\circ$$

$$= (0.4 \times 1.6 \times 6) \times 10^{-14}$$

$$F = (2.4 \times 1.6) \times 10^{-14}$$

$$F = 3.84 \times 10^{-14}$$

$$\boxed{F = 3.84 \times 10^{-14} N}$$

acceleration $F = ma$

$$a = \frac{F}{m} = \frac{3.84 \times 10^{-14}}{6.65 \times 10^{-27}}$$

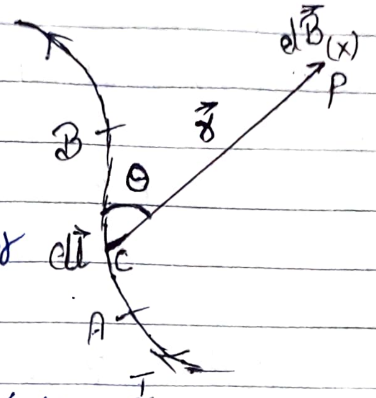
$$= \frac{3.84}{6.65} \times 10^{-14+27}$$

$$= 0.577 \times 10^{13}$$

$$\boxed{a = 5.77 \times 10^{12} m/s}$$

Q:- What is Biot-Savart law?

Sol:- Biot-Savart law is used to determine the strength of the magnetic field at any point due to a current carrying conductor.



Consider a very small element (AB) of length (dl). of a Conductor Carrying Current (I) the strength of the magnetic field ($d\vec{B}$) due to this small Current element at a point (P) distant ($\vec{r}$) from the element is found to be depending upon following quantity.

(i) $dB \propto dl$
(ii) $dB \propto I$
(iii) $dB \propto \sin\theta$
(iv) $dB \propto \frac{1}{r^2}$

$$\therefore dB \propto \frac{dl\, I \sin\theta}{r^2}$$

$$dB = K \frac{dl\, I \sin\theta}{r^2}$$

where (K) is a Constant of proportionality

$$K = \frac{\mu_0}{4\pi}$$

$$\boxed{dB = \frac{\mu_0}{4\pi} \frac{I\, dl \sin\theta}{r^2}} \quad \text{——— (1)}$$

Vector form :-

$$|d\vec{B}| = \frac{\mu_0}{4\pi} \frac{I(d\vec{l} \times \hat{r})}{r^2}$$

Here $\hat{r} = \frac{\vec{r}}{r}$

$$\boxed{|d\vec{B}| = \frac{\mu_0}{4\pi} \frac{I(d\vec{l} \times \vec{r})}{r^3}} \quad \text{--- (11)}$$

direction of $d\vec{B}$:- The direction of $d\vec{B}$ would obviously be the direction of the cross product of $(d\vec{l} \times \vec{r})$.
It is represented by the Right handed Screw Rule or Right Hand Rule.

Magnetic field induction at Point (P) due to current through entire wire is

$$\vec{B} = \int \frac{\mu_0}{4\pi} \frac{I(d\vec{l} \times \vec{r})}{r^3}$$

$$\vec{B} = \int \frac{\mu_0}{4\pi} \frac{I\, dl \sin\alpha}{r^2}$$

Biot savart's law in term of current density $\vec{J}$,

$$J = \frac{I}{A} \Rightarrow J = \frac{I\,dl}{A\,dl} = \frac{I\,dl}{dV}$$

$$J = \frac{I\,dl}{dV} \Rightarrow I\,dl = J\,dV$$

Put in equation (II)

$$\boxed{|d\vec{B}| = \frac{\mu_0}{4\pi} \frac{\vec{J} \times \vec{r}}{r^3} dV} \quad \text{—— (III)}$$

Biot savart's law in term of charge (q) and its velocity (v)

$$I\,d\vec{l} = \frac{q}{dt} \cdot d\vec{l} \Rightarrow q\frac{d\vec{l}}{dt} = q\vec{v}$$

using equation (II)

$$\boxed{d\vec{B} = \frac{\mu_0}{4\pi} \frac{q(\vec{v} \times \vec{r})}{r^3}} \quad \text{—— (IV)}$$

Biot-Savart's law in terms of magnetising force or magnetising intensity (H):-

$$d\vec{H} = \frac{d\vec{B}}{\mu_0} = \frac{1}{4\pi} \frac{I\,d\vec{l} \times \vec{r}}{r^3}$$

$$d\vec{H} = \frac{1}{4\pi} \frac{I\, d\vec{l} \times \hat{r}}{r^2}$$

$$\boxed{dH = \frac{1}{4\pi} \frac{I\, dl \sin\theta}{r^2}}$$

Some Special Cases :

1) if $(\theta = 0)$

$$dB = \frac{\mu_0}{4\pi} \frac{I\, dl \sin\theta}{r^2}$$

$$= \frac{\mu_0}{4\pi} \frac{I\, dl \sin(0)}{r^2}$$

$$\boxed{dB = 0}$$

This mean that Point (P) lies on the axis of the linear Conductor.

$\therefore$ there is no magnetic field on the thin linear Current Carrying Conductor.

7) if $\theta = 90$ (i.e the point is $\perp$ to the current element)

$$dB = \frac{\mu_0}{4\pi} \frac{I\,dl}{r^2} \sin 90$$

$$\boxed{dB = \frac{\mu_0}{4\pi} \frac{I\,dl}{r^2}}$$ (maximum value of magnetic field)

8) if $\theta = 180$

$$dB = 0$$ (minimum or zero)

Some Important points:

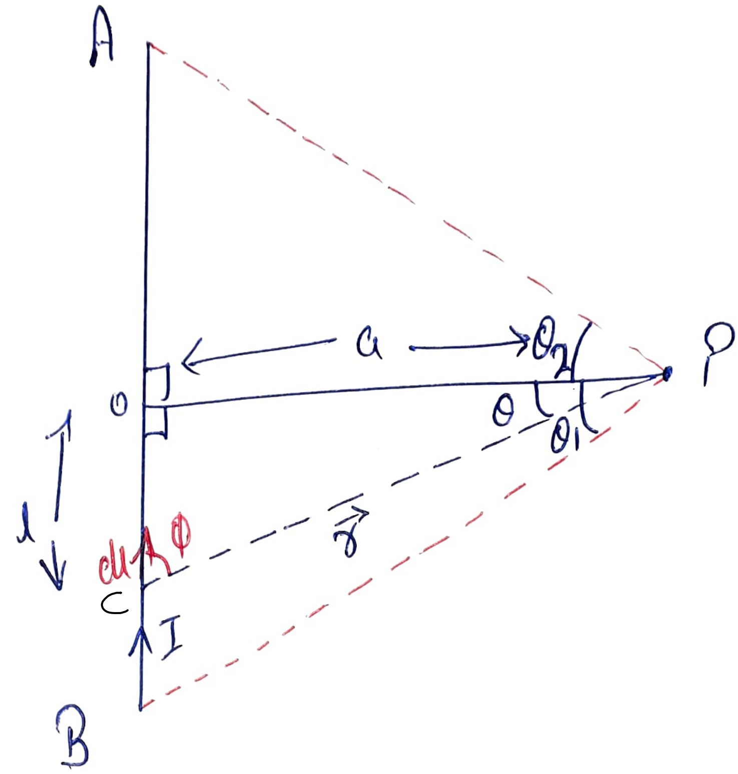
1) Biot savart's law is valid for a symmetrical current distribution.

(ii) Biot savart's law is applicable only to very small length conductor carrying current.

(iii) This law is correspondent to Coulomb's law in electrostatics.

Q Desire an expression of magnetic field due to infinity long straight wire carrying current?

Sol:



Consider a long straight wire AB carrying current I. Let (P) at a distance (a) from the wire consider a small element (dl) at a distance ($\vec{r}$) from point (P)

Now Acc. to Biot - Savart law of magnetic field.

at point (P) Due to small element of the wire is given by

$$dB = \frac{\mu_0}{4\pi} \frac{I dl \sin\phi}{r^2} \quad -(1)$$

In right angle Δ POC

$$\sin\phi = \frac{a}{r} = \cos\theta \quad -(ii)$$

$$r = \frac{a}{\cos\theta}$$

also $\tan\theta = \frac{l}{a}$

$$l = a\tan\theta$$

$$= \frac{dl}{d\theta} = \frac{d}{d\theta} a\tan\theta$$

$$dl = a\sec^2\theta \, d\theta \quad -(iii)$$

put value of r, l, $\sin\phi$ and dl in equ (1)

$$dB = \frac{\mu_0}{4\pi} \frac{I\,(a\sec^2\theta\, d\theta)\cos\theta}{a^2/\cos^2\theta}$$

$$\boxed{dB = \frac{\mu_0}{4\pi} \frac{I\cos\theta\, d\theta}{a}}$$

Magnetic field due to the whole Conductor AB Can be calculated by intigrating

[θ_2 is +ve because upward the paper] and $-\theta_1$ down ward to the paper].

$$B = \int_{-\theta_1}^{\theta_2} dB$$

$$B = \frac{\mu_0 I}{4\pi a} \int_{-\theta_1}^{\theta_2} \cos\theta \, d\theta$$

$$B = \frac{\mu_0 I}{4\pi a} [\sin\theta]_{-\theta_1}^{\theta_2}$$

$$B = \frac{\mu_0 I}{4\pi a} [\sin\theta_2 - \sin(-\theta_1)]$$

$$B = \frac{\mu_0 I}{4\pi a} [\sin\theta_2 + \sin\theta_1]$$

Case I

If the straight wire is infinte long then $\theta = 90°$

$$B = \frac{\mu_0 I}{4\pi a} [\sin 90° + \sin 90°]$$

$$B = \frac{\mu_0 I}{4\pi a} [1+1]$$

$$B = \frac{\mu_0 I}{4\pi a} [2]$$

$$B = \frac{2\mu_0 I}{4\pi a}$$

$$B = \frac{\mu_0 I}{4\pi} \frac{2I}{a}$$

Case II: when the Conductor AB is of infinite length but the point (P) lies near the end A (or B)

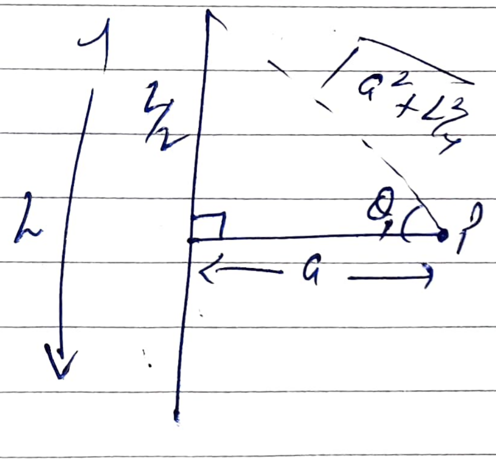
$\theta_1 = 90$ and $\theta_2 = 0$

$$B = \frac{\mu_0}{4\pi}\frac{I}{a}[\sin 90 + \sin 0]$$

$$\boxed{B = \frac{\mu_0}{4\pi}\frac{I}{a}}$$

∴ magnetic field at the centre of the conductor is twice than that near its one of the ends.

Case III: If length of conductor is finite say L and point (P) lies on right bisector of conductor:-



$$\sin\phi = \frac{L/2}{\sqrt{a^2+L^2/4}}$$

$$= \frac{L/2}{\frac{\sqrt{4a^2+L^2}}{2}}$$

$$\sin\phi = \frac{L}{\sqrt{4a^2+L^2}}$$

$$B = \frac{\mu_0 I}{4\pi a}\left[\sin\phi + \sin\phi\right]$$

$$= \frac{\mu_0 I}{4\pi a}\left[2\sin\phi\right]$$

$$= \frac{\cancel{2}\mu_0 I}{\underset{2}{\cancel{4}}\pi a}\left[\frac{L}{\sqrt{4a^2+L^2}}\right]$$

$$= \frac{\mu_0 I}{2\pi a}\left[\frac{L}{\sqrt{4a^2+L^2}}\right]$$

(iv) When Point (P) lie on the wire Conductor the $\vec{dl}$ and $\vec{r}$ for each element of the straight wire Conductor are $\parallel$ r l.

$\therefore$ $\vec{dl} \times \vec{r} = 0$

$\therefore$ $dl\, r \sin 0$ $\quad [\theta = 0]$

$\therefore$ $B = \frac{\mu_0}{4\pi} I\, dl \sin 0$

$$\boxed{B = 0}$$

Sample problem:- A long straight Conductor Carries a current of 100 A. At what distance from the Conductor is the magnetic field Caused by the Current equal to 0.5×10^{-4} T?

Sol:- given that $I = 100\,A$

$B = 0.5 \times 10^{-4}\,T$

$$B = \frac{\mu_0 2I}{4\pi a}$$

or $$a = \frac{\mu_0 2I}{4\pi B}$$

$$\mu_0 = 10^{-7}$$

$$a = \frac{10^{-7} \times 2 \times 100}{0.5 \times 10^{-4}}$$

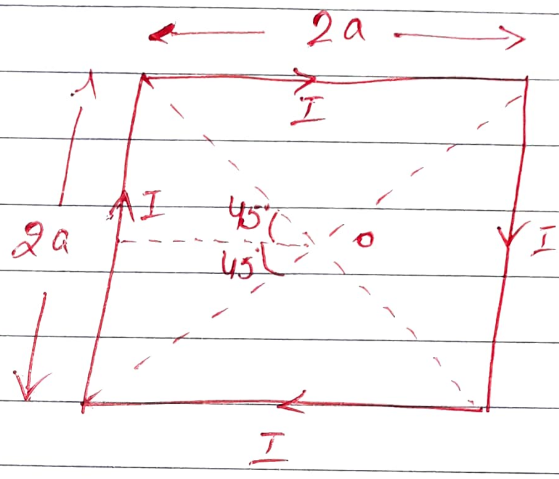
$$= \frac{2 \times 100 \times 10^{-3} \times 10^{2}}{\cancel{5}}$$

$$= 4 \times 10^{-1}\,m$$

$$\boxed{a = 0.4\,m}$$

Q Find an expression for the magnetic field induction at the Centre of a coil bent in the form of a square of side 2a, Carrying current I,

Sol: Here $\phi_1 = 45^\circ$
and $\phi_2 = 45^\circ$



There are 4 side of the square So that the total magnetic field at the Centre of the Coil.

B = No. of sides x magnetic field (Biot-Savart law)

$$B = 4 \times \frac{\mu_0 I}{4\pi a}(\sin 45 + \sin 45)$$

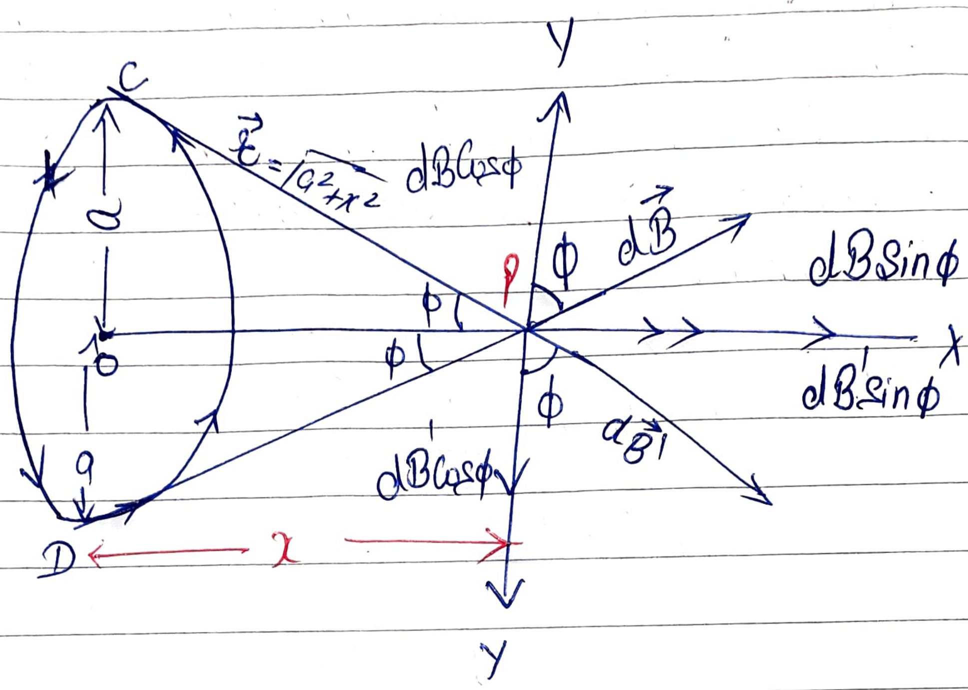
$$= \frac{4\mu_0 I}{4\pi a}\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)$$

$$= \frac{\mu_0 I}{\pi a}\left[\frac{2}{\sqrt{2}}\right]$$

$$\boxed{B = \frac{\sqrt{2}\mu_0 I}{\pi a}}$$

Q Determine the Magnetic field at a point on the axis of a Circular Coil carrying Current?

Sol:-



Consider a circular Coil of Radius (a) with center O. let the plane of the Coil be perpendicular to the plane of the paper and the Current (I) flowing in the Coil.

Suppose (P) is the any Point at a distance x from the Centre of the Coil.

ie $OP = x$

Consider two small elements of the coil each of length 'dl' at Point D and C.

Now from Δ OPC using P.G.T.

$$H^2 = P^2 + B^2$$

$$(CP)^2 = (OC)^2 + (OP)^2$$

$$r^2 = a^2 + x^2 \Rightarrow \boxed{r = \sqrt{a^2+x^2}} \text{ --- (1)}$$

let $\angle CPO = \angle DPO = \angle YPE = \angle Y'PE = \phi$

According to Biot-savart's law

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\, dl \sin 90^\circ}{r^2}$$

[$\because$ a is small $\therefore \theta = 90^\circ$]

Put the value of (r)

$$dB = \frac{\mu_0}{4\pi} \frac{I\, dl \sin 90}{(\sqrt{a^2+x^2})^2}$$

$$\boxed{dB = \frac{\mu_0}{4\pi} \frac{I\, dl}{(a^2+x^2)}} \quad \text{---} \quad (2)$$

The direction of $d\vec{B}$ is in the plane $\perp$ to $d\vec{l}$ and $\vec{r}$ $(PE \perp CP)$

Similarly, the magnitude of magnetic field Induction at Point (P) due to current element dl

$$dB' = \frac{\mu_0}{4\pi} \frac{I\, dl \sin 90^\circ}{r^2} = \frac{\mu_0}{4\pi} \frac{I\, dl}{(a^2+x^2)} \quad \text{---} \quad (3)$$

(Here $PF \perp DP$)

from equation $dB = dB' = \dfrac{\mu_0}{4\pi} \dfrac{I\, dl}{(a^2+x^2)}$ ---

Now Resolving $d\vec{B}$ and dB' in two rectangular Components.

(I) $dB \cos\phi$ acting along PY and $dB \sin\phi$ acts along PX

(II) $dB' \cos\phi$ acting along PY' and $dB' \sin\phi$ acts along PX.

Thus total magnetic field Induction at (P) due to current Carrying Coil.

$$B = \int dB \sin\phi = \int \frac{\mu_0 I dl}{4\pi (a^2+x^2)} \sin\phi$$

$$= \int \frac{\mu_0}{4\pi} \frac{I \, dl}{(a^2+x^2)} \sin\phi$$

$$= \frac{\mu_0}{4\pi} \frac{I}{a^2+x^2} \sin\phi \int dl$$

Here $\sin\phi = \frac{a}{\sqrt{a^2+x^2}}$ and $\int dl = 2\pi a$

(circumference of circle)

$$B = \frac{\mu_0}{4\pi} \frac{I}{(a^2+x^2)} \frac{a}{\sqrt{a^2+x^2}} 2\pi a$$

$$B = \frac{\mu_0}{4\pi} \frac{2\pi a^2 I}{(a^2+x^2)^{3/2}}$$

If there are n (number) of turns

$$\boxed{B = \frac{\mu_0}{4\pi} \frac{2\pi n I a^2}{(a^2+x^2)^{3/2}}}$$

Special Cases:- 1. When point (P) lies at the Centre of the Circular Coil then $x = 0$

$$B = \frac{\mu_0}{4\pi} \frac{2\pi n I a^2}{a^3}$$

$$\boxed{B = \frac{\mu_0}{4\pi} \frac{2\pi n I}{a}}$$ [(Same as perivious Que.]

2:- When Point (P) lies far away from the Centre of the Coil.

Here $x >> a$

$\therefore x^2 + a^2 = x^2$

$$B = \frac{\mu_0}{4\pi} \frac{2\pi n I a^2}{x^3} = \frac{\mu_0}{4\pi} \frac{2 n I A}{x^3}$$

$[\because A = \pi a^2)$ Area of loop

$$B = \frac{\mu_0}{4\pi} \frac{2M}{x^3}$$

$nIA = M$ = Magnetic dipole moment of the current loop.

Problem: An electric current is flowing in a circular coil of Radius (a). At what distance from the centre on the axis of the coil will the magnetic field be $1/8^{th}$ of its value at the centre?

Sol: Magnetic field induction on the axis at a distance x.

$$B_1 = \frac{\mu_0}{4\pi} \frac{2\pi n I a^2}{(a^2+x^2)^{3/2}}$$

Magnetic field at the centre of the coil

$$B_2 = \frac{\mu_0}{4\pi} \frac{2\pi n I}{a}$$

Acc. to Question

$$B_1 = \frac{B_2}{8} =$$

$$\frac{\cancel{\mu_0}}{\cancel{4\pi}} \frac{2\pi n I a^2}{(a^2+x^2)^{3/2}} = \frac{\cancel{\mu_0}}{\cancel{4\pi}} \frac{\cancel{2\pi} \cancel{n} \cancel{I}}{8a}$$

$$\frac{a^2}{(a^2+x^2)^{3/2}} = \frac{1}{8a}$$

$$8a^3 = (a^2+x^2)^{3/2}$$

$$(2a)^3 = (a^2+x^2)^{3/2}$$

$$2a = (a^2+x^2)^{1/2}$$

Squaring both side

$$4a^2 = a^2+x^2$$

$$4a^2 - a^2 = x^2$$

$$x^2 = 3a^2$$

$$\boxed{x = \sqrt{3}\,a}.$$

Comparition between Ampere's circuital law and Biot-Savart's law

similarities :- a) Both relate the magnetic field with current

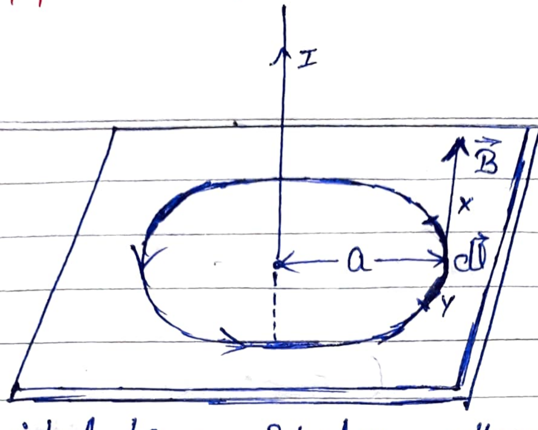
b) Both express the same physical consequences of a steady current.

difference :- a) Ampere's law is applicable for symmetrical current distribution whereas the Biot Savart's law is applicable for asymmetrical current distribution.

(b) Ampere's law is for integral of $\vec{B}$ and Biot Savart's law is for different from of $\vec{B}$

(c) Ampere's law is based on the principle of electro magnetism and Biot savart's law is based on magnetism.

Q. State and explain ampere's circuital law?



Ampere's circuital law state that the line integral of magnetic field around any closed path in free space is equal to absolute permitivity (μ_0) times the net current inclosed by the path.

ie $$\oint \vec{B}\cdot d\vec{l} = \mu_0 I$$

Proof :- Consider an infinitly long straight Conductor Carring current (I). The magnetic line of force are produced around the Conductor at concentric circuit. Therefore the magnetic field due to this current carring infinite Cunductor at a distance (a).

$$B = \frac{\mu_0}{4\pi}\frac{2I}{a} \quad \text{——} (1)$$

Consider a circuit of Radius (a). Let (xy) be the small element of length ($d\vec{l}$). ($d\vec{l}$) and ($\vec{B}$) are in the same direction because direction ($\vec{B}$) is along the tangent of the circuit.

$$\vec{B}\cdot d\vec{l} = B\,dl \cos\theta$$
$$= B\,dl \cos 0$$
$$\vec{B}\cdot d\vec{l} = B\,dl$$

$$\therefore \oint \vec{B}\cdot d\vec{l} = \oint B\cdot dl$$

from equation (1)

$$\oint \vec{B}\cdot d\vec{l} = \oint \frac{\mu_0}{4\pi} \frac{2I}{a}\, dl = \frac{\mu_0}{4\pi}\frac{2I}{a}\oint dl$$

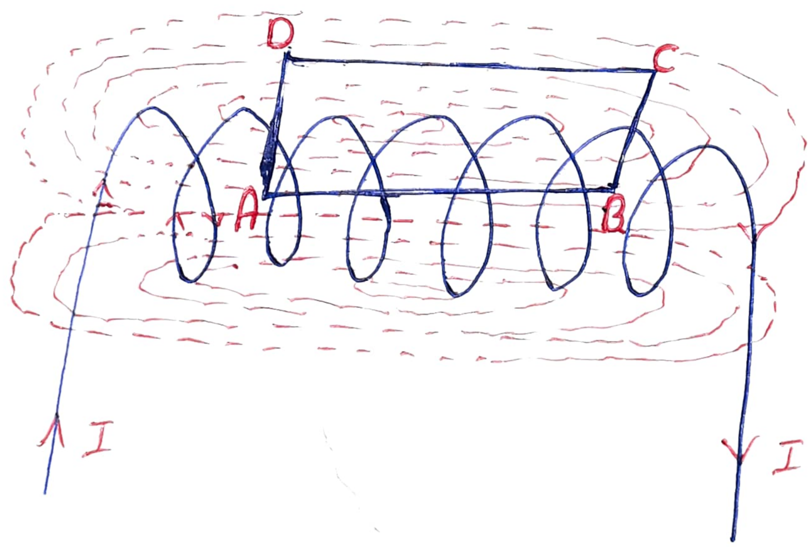
Closed $\oint dl = 2\pi a$ $\because$ ...... Circumference of Circle

$$\oint \vec{B}\cdot d\vec{l} = \frac{\mu_0}{\cancel{4\pi}} \frac{\cancel{2}I}{\cancel{a}} \cancel{2}\pi \cancel{a}$$

$$\boxed{\oint \vec{B}\cdot d\vec{l} = \mu_0 I}$$

Q:- explanation of Magnetic field due to The Solenoid? having (n) number of turns.

So



A cylinderical Coil of many tightly wound turns of insulated wire with generally diameter of the Coil smaller than its length is called a solenoid.

Consider a very long solenoid having n turns per unit length of solenoid. Let current (I) be flowing through the solenoid.

Shows that the magnetic field inside the solenoid is uniform strong and directed along the axis of the solenoid. The magnetic field outside a very long solenoid is very weak and can be neglected. the same may be taken as zero.

Let (P) be a point well with in the solenoid. Consider any rectangular loop ABCD passing through (P).

$\oint \vec{B} \cdot \vec{dl}$ = Line integral of magnetic field across the loop ABCD

$$\oint \vec{B} \cdot \vec{dl} = \int_A^B \vec{B} \cdot \vec{dl} + \int_B^C \vec{B} \cdot \vec{dl} + \int_C^D \vec{B} \cdot \vec{dl} + \int_D^A \vec{B} \cdot \vec{dl} \quad -(1)$$

$\vec{B}$ is perpendicular to path BC and AD. i.e angle b/w $\vec{B}$ and $\vec{dl}$ is 90° for these path

$$\therefore \int_B^C \vec{B} \cdot \vec{dl} = \int_D^A \vec{B} \cdot \vec{dl} = \int B\, dl \cos 90° = 0$$

Since path CD is out side the solenoid where $\vec{B}$ is taken as 0, so $\int_C^D \vec{B} \cdot \vec{dl} = 0$

For path AB, the direction of $d\vec{l}$ and $\vec{B}$ is same i.e $\theta = 0$

equation (1) become

$$\oint \vec{B}\cdot\vec{dl} = \int_A^B \vec{B}\cdot\vec{dl} = \int_A^B B\,dl \cos 0 = \int_A^B B\,dl$$

$$\oint \vec{B}\cdot\vec{dl} = B\int_A^B dl \qquad (\because B \text{ is uniform})$$

$$\oint \vec{B}\cdot\vec{dl} = Bl \text{ — (II)}$$

According to Ampere's circuital law

$\oint \vec{B}\cdot\vec{dl} = \mu_0 \times$ net current enclosed by loop ABCD

$$\oint \vec{B}\cdot\vec{dl} = \mu_0 n I l \text{ — (III)}$$

Comparing equation (II) and (III)

$$Bl = \mu_0 n I l$$

$$B = \mu_0 n I$$

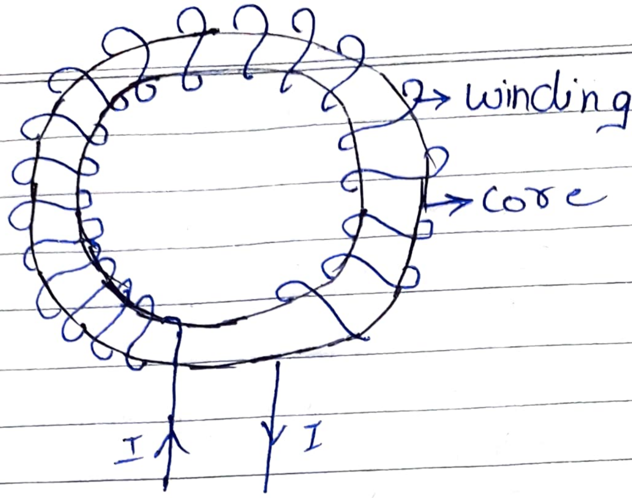
Thus magnetic field with in an infinitely long solenoid is given by

$$B = \mu_0 n I$$

$n = \frac{N}{l}$ where N = total no. of turns of solenoid

$$\boxed{B = \frac{\mu_0 N I}{l}}$$

(ii) Magnetic field due to a Toroid carrying Curren :-



Consider a toroid having (n) turns per unit length

Let (I) be the current flowing through the toroid. The magnetic lines of force mainly in the core of toroid and are in the form of concentric circles. Consider such a circle of mean of radius r.

$$\oint \vec{B}\cdot d\vec{l} = \oint B\,dl\cos\theta$$

By symmetry, magnetic field $\vec{B}$ in the coil is constant and is tangent to path $d\vec{l}$, therefore, angle θ between them is 0, hence

$$\oint \vec{B}\cdot d\vec{l} = \oint B\,dl\cos 0 = \oint B\,dl = B\oint dl$$

$$= B \times \text{Circumference of circle}$$

$$\oint \vec{B}\cdot d\vec{l} = B\times 2\pi r \quad \text{---(1)}$$

According to ampere's circuital law.

$\oint \vec{B}\cdot d\vec{l} = \mu_0 \times$ net current enclosed by the circle of radius r

$\oint \vec{B}\cdot d\vec{l} = \mu_0 \times$ total no. of turn $\times I$

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 (n \times 2\pi r) I \quad \text{—— (ii)}$$

Comparing equ. (i) and (ii)

$$B \times 2\pi r = \mu_0 (n \times 2\pi r) I$$

$$B = \mu_0 n I$$

Also N is the total no. of turns of a toroid then n

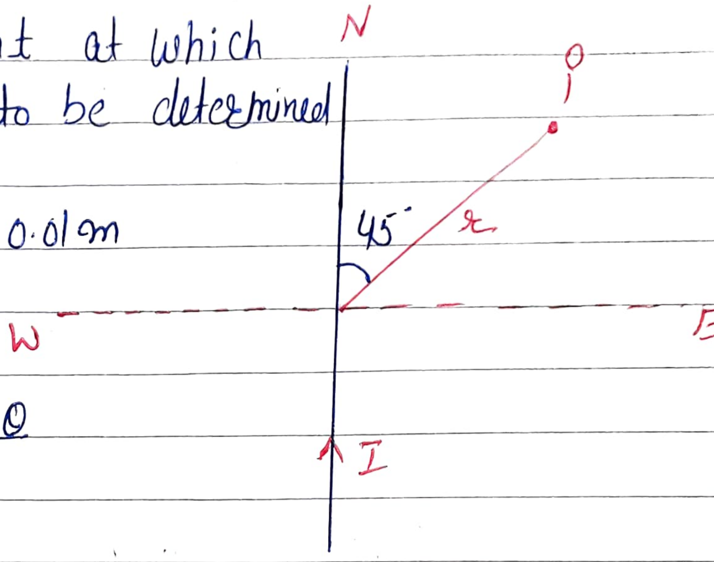
$$n = \frac{N}{2\pi r}$$

$$\boxed{B = \frac{\mu_0 N I}{2\pi r}}$$

Numerical:

Example: A Current of 5A is flowing from South to north in a straight wire. Find the magnetic field due to a 1cm piece of wire at a point 1m north east from the piece of wire.

Sol Here (P) is the Point at which magnetic field is to be determined



$I = 5A$, $dl = 1\,cm = 0.01\,m$

$r = 1m$, $\theta = 45$

$$dB = \frac{\mu_0}{4\pi} \frac{I\,dl \sin\theta}{r^2}$$

$$= \frac{10^{-7} \times 5 \times (0.01) \sin 45}{(1)^2}$$

$$dB = 10^{-7} \times 5\,(0.01)\, 1/\sqrt{2}$$

$$= \frac{5}{\sqrt{2}} \times 10^{-9} = 3.53 \times 10^{-9}\,T$$

$$\boxed{dB = 3.53 \times 10^{-9}\,T}$$

Q2 An element $\vec{dl} = dx\,\hat{i}$ (where $dx = 1cm$) is placed at the origin and carries a large current $i = 10A$. What is the magnetic field on the Y-axis at a distance of 0.5m.

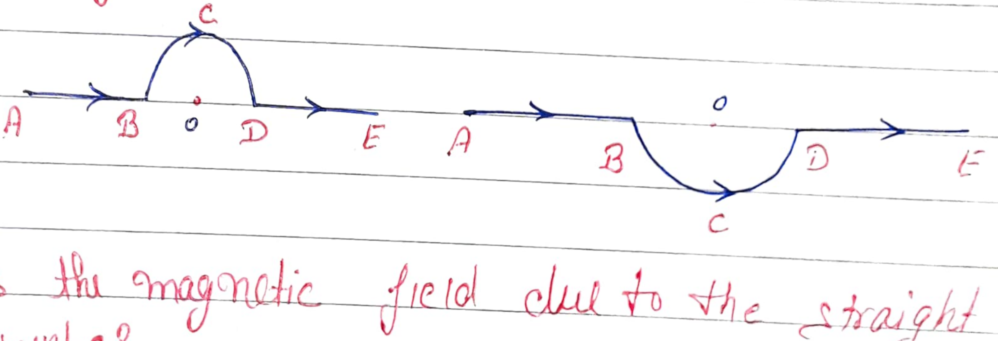
Sol: Here $dl = dx = 1cm = 10^{-2}m$, $I = 10A$, $r = 0.5m$

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\, d\vec{l} \times \vec{r}}{r^3} = \frac{\mu_0}{4\pi} \frac{I\, dl}{4\pi r^2} (\hat{i} \times \hat{j})$$

$$= \frac{\mu_0}{4\pi} \frac{I\, dx}{4\pi r^2} (\hat{k})$$

$$d\vec{B} = \frac{10^{-7} \times 10 \times 10^{-2}}{(0.5)^2} = 4 \times 10^{-8} \hat{k}\ T$$

Q:- A straight wire carrying a current of 12 A is bent into a semi-circular arc of Radius 2cm. Consider that magnetic field ($\vec{B}$) at the Centre of the arc.



a) What is the magnetic field due to the straight segements?

b) In what way the Contribution of $\vec{B}$ from the semi circle differs from that of a circular loop and in what way does it resemble.

c) Would your answer be different if the wire were bent into a semi circle arc of the same Radius but in the opposite way, as shown in fig

a) for a point O, $\vec{dl}$ and $\vec{r}$ for each element of the straight segment AB and DE are parallel.

$\therefore$ $\vec{dl} \times \vec{r} = 0$

hence magnetic field is zero

ie $\vec{B} = \sum \frac{\mu_0}{4\pi} \frac{I \vec{dl} \times \vec{r}}{r^3} = 0.$

b) The magnetic field Induction at (O) due to semicircular arc

$$B = \frac{\mu_0 I}{4\pi} \frac{\pi}{r}$$

$$= \frac{10^{-7} \times 12 \times 3.14}{2 \times 10^{-2}}$$

$$= 6 \times 3.14 \times 10^{-5}$$

$$B = 18.84 \times 10^{-5} = 1.88 \times 10^{-4} T$$

$$\boxed{B = 1.88 \times 10^{-4} T}$$

The direction of magnetic field due to Right ang hand Rule normal to the plane of paper and directed inwards.

(c) When the wire is in the form of opposite direction the magnetic field induction at point O will remain same in magnitude But its direction will be normal to the plane of paper directed outward.

Q A tightly wound 100 turn coil of Radius 10 cm carrying of 1A. What is the magnitude of the magnetic field at the centre of the coil.

Sol: Here $n = 100$, $I = 1A$, $r = 10\ cm = 0.1\ cm$.

$$B = \frac{\mu_0}{4\pi} \times \frac{2\pi n I}{r} = \frac{10^{-7} \times 2 \times (3.14) \times 100 \times 1}{0.1}$$

$$= 10^{-4} \times 2 \times 3.14$$

$$\boxed{B = 6.28 \times 10^{-4}\ T}$$

Q:- A Solenoid of length 0.5m has a Radius of 1cm and is made up of 500 turns. It carries a current of 5A. What is the magnitude of the magnetic field inside the solenoid.

Sol: Here, $N = 500$; $l = 0.5m$; $I = 5A$

$r = 1cm = 10^{-2} m$.

Now magnetic field induction at a point inside the solenoid due to long solenoid

$$B = \mu_0 \frac{NI}{l}$$

$$= \frac{\mu_0}{4\pi} \, 4\pi \frac{NI}{l}$$

$$= 10^{-7} \, 4 \times 3.14 \left(\frac{500 \times 5}{0.5} \right)$$

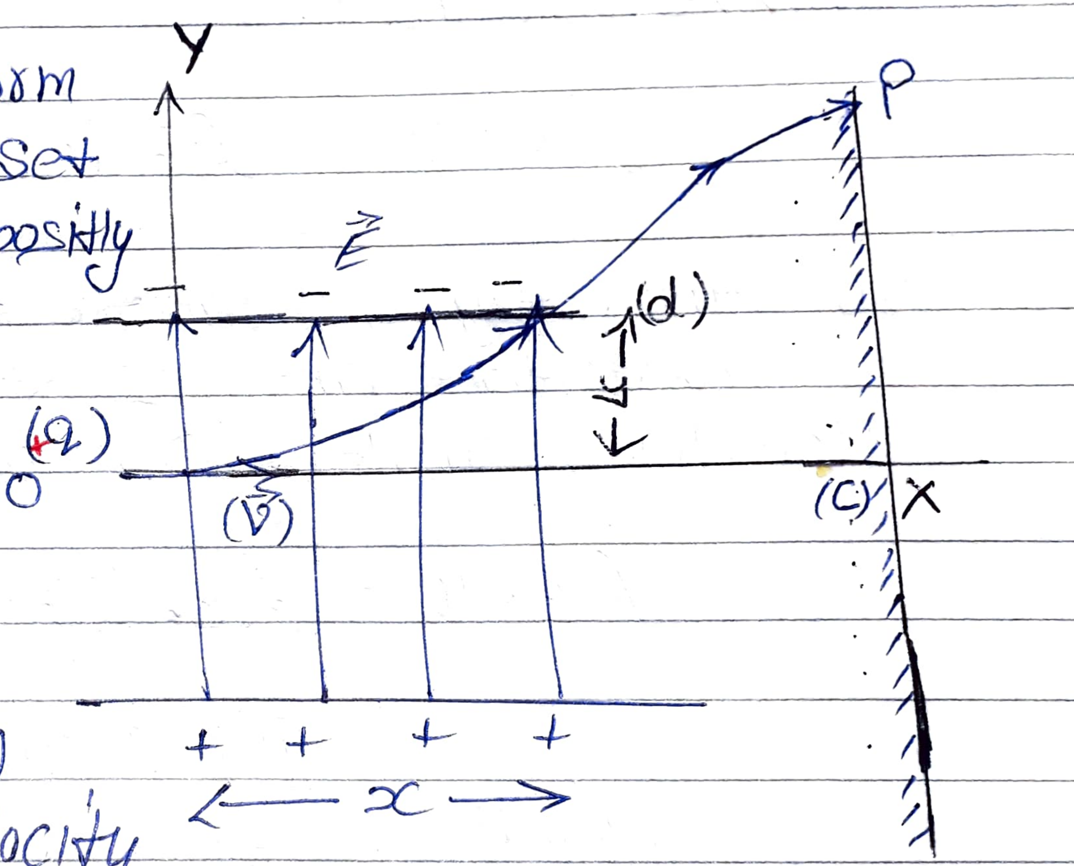
$$= \frac{\overset{4}{\cancel{20}} \times 500 \times 10 \times 3.14 \times 10^{-7}}{\cancel{5}}$$

$$= 20 \times 10^{-4} \times 3.14$$

$$\boxed{B = 6.28 \times 10^{-3} \text{ T}}$$

Q Show that A charge particle moving in a uniform electric field follow's a parabolic path?

Ans:- Consider a uniform electric field ($\vec{E}$) set up b/w two oppositly charged plate let a (+ve)ly charge particle having charge (+q) and mass (m) enter the region of electric field (E) at (O) with velocity (V) along (x-axis) direction.



Force acting on the charge (+q) due to electric field ($\vec{E}$) along in the direction of electric field.

$$\vec{F} = q\vec{E}$$

∴ acceleration produced in the Charged particle is give by

$$\vec{a} = \frac{\vec{F}}{m}$$

$$\vec{a} = \frac{q\vec{E}}{m} \quad \text{—— (i)}$$

Charge particle accelerate in the direction of electrical field ($\vec{E}$) as soon as the particle leaves region particle it self travell in the straght line due to the inertia of motion and hits the screen at (P). Let (t) is the time taken by the Charged particle to trance verce the region of electric field of length (x).

Then we know that

$$s = ut + \frac{1}{2}at^2 \quad \text{—— (ii)}$$

for horizontal motion $s = x$, $u = v$ and $a = 0$

$$x = vt$$

$$t = \frac{x}{v} \quad \text{—— (iii)}$$

for vertical motion $s = y$, $u = 0$, ~~a=t~~

$$y = \frac{1}{2}at^2 \quad \text{—— (iv)}$$

put the value of (a) in equation (IV)

$$y = \frac{1}{2}\left(\frac{q\vec{E}}{m}\right)t^2$$

But $t = \left(\frac{x}{v}\right)$ $\therefore y = \frac{1}{2}\left(\frac{q\vec{E}}{m}\right)\left(\frac{x}{v}\right)^2$

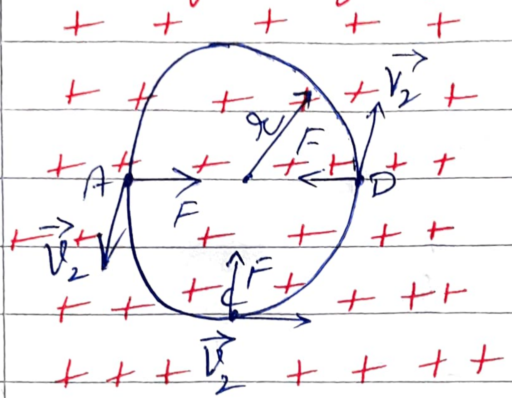
$$y = \frac{q\vec{E}x^2}{2mv^2} = kx^2$$

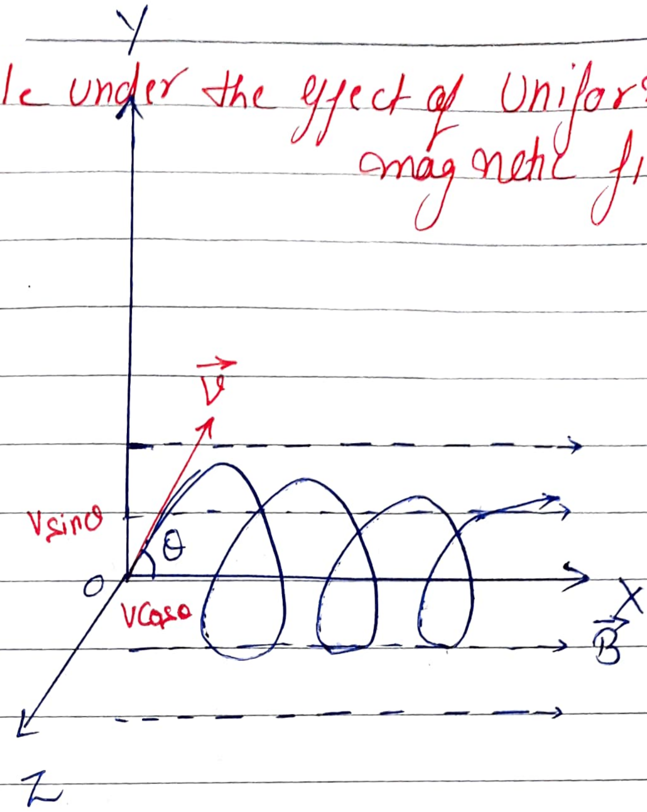
where $k = \frac{qE}{2mv^2}$ = Constant

$$y = kx^2$$

is the equation of parabola

⇒ Motion of charge particle under the effect of Uniform magnetic field.





Suppose a particle of mass (m) and charge (q) entering in a uniform magnetic field Induction $\vec{B}$ at O, with velocity ($\vec{V}$) making an angle θ with the direction of magnetic field.

So that we can Resolve the ($\vec{V}$) into two Rectangular Components ($V\cos\theta$) = V along magnetic field and ($V\sin\theta = V_2$) acts ⊥ to magnetic field.

for the velocity ($\vec{V_2}$), the force acting on the charge particle.

$$\vec{F} = q(\vec{V_2} \times \vec{B})$$

$$\vec{F} = q|\vec{V_2} \times \vec{B}| = qV_2 B \sin 90°$$

$$F = qV_2 B = q(V\sin\theta)B$$

$$\boxed{F = qBV\sin\theta} \quad \text{——} \quad (1)$$

The direction of this force ($\vec{F}$) is $\perp$ to the plane Containing $\vec{B}$ and $\vec{V_2}$ is directed as given Right hand Pam Rule.

or

Here magnetic field $\perp$ to the plane of paper directed inward and particle is moving in the plane of paper

The force (F) on the particle due to magnetic field provides the required Centripetal force $= (mV_2^2/r) = F$

$$\therefore \quad Bq\cancel{V_2} = mV_2^{\cancel{2}}/r$$

$$V_2 = \frac{Bqr}{m}$$

$$V\sin\theta = Bqr/m \quad \text{——} \quad (2)$$

The angular velocity of Ratation of the magnetic field will be

$$\omega = \frac{V\sin\theta}{r} = \frac{Bq\cancel{r}}{m\cancel{r}} = \frac{Bq}{m}$$

$$\boxed{\omega = \frac{Bq}{m}} \quad \text{——} \quad (3)$$

The frequency of Rotation

$$\boxed{\nu = \frac{\omega}{2\pi} = \frac{Bq}{2\pi m}} \quad \text{—— (4)}$$

The time period of revolution of particle

$$T = 1/\nu = \frac{2\pi m}{Bq}$$

$$\boxed{T = \frac{2\pi m}{Bq}} \quad \text{—— (5)}$$

from (4) and (5) we note that (ν) and (T) do not depend upon velocity ($\vec{v}$) of the particle.
The particle Revolving in circular path due to component velocities $\perp$ to the magnetic field. and the charge particle covers the linear distance in direction of the magnetic field with a constant $v\cos\theta$.

$\therefore$ under the combined effect of the two component velocities, the charge particle in magnetic field will cover linear path as well as circular path i.e. the path of charge particle will be "helical".

The linear distance cover by the charge particle in the magnetic field in the time equal to the one revolution of its circular path.

$$d = V_1 T = V\cos\theta \frac{2\pi m}{Bq}$$

$$\boxed{d = \frac{2\pi m V\cos\theta}{Bq}}$$

Important Points:-

(1) If a charge particle having charge (q) is at Rest in a magnetic field $(\vec{B})$ $(V=0)$

$$d = \frac{2\pi m(0)\cos\theta}{Bq}$$

$$\boxed{d = 0}$$

(2) If charge particle is moving $\parallel$rl to the magnetic field $\vec{B}$.

i.e $\theta = 0$ and 180.

In this case the charge particle will continue moving along the same path with the same velocity.

3) If charge particle is moving perpendicular to the direction of $\vec{B}$. It experiance a maximum force which acts $\perp$ to the direction of $\vec{B}$ as well as $\vec{v}$ and hence this force required centripetal force and the charge particle will describe a circular path in the magnetic field of Radius (r),

$$\boxed{\frac{mv^2}{r} = Bqv}$$

Sample problem

1) A beam of ions with velocity 2×10^5 m/s enters normally into a uniform magnetic field of 0.04 tesla. If the specific charge of ion is 5×10^7 c/kg, find the Radius of the circular path described.

Sol:- Here $v = 2\times10^5$ m/s

$B = 0.04$ T

$q/m = 5\times10^7$ C/kg (specific charge of ion)

Now $F = qvB$ (magnetic force)

$F = \frac{mv^2}{r}$ (Centripetal force)

$$qBv = \frac{mv^2}{r} \Rightarrow r = \frac{mv^2}{qBv}$$

$$r = \frac{v}{(q/m)\times B} = \frac{2\times10^5}{5\times10^7\times0.04} = \frac{\cancel{2}^1}{5\times\cancel{4}_2}\times10^{7-7}$$

$$\frac{1}{10} = 0.1\,m$$

2) A electron of mass 0.9×10^{-30} kg, under the action of magnetic field moves in a circular path of Radius 3×10^{6} m/s. If a proton of mass 1.8×10^{-27} kg were to move in a circle of the Radius in the magnetic field, find its speed.

Sol:- Here mass of electron $m_e = 0.9 \times 10^{-30}$ kg
mass of proton $m_p = 1.8 \times 10^{-27}$ kg
(r) Radius of circular path of electron =
$= 2$ cm
$= 0.02\,m$

Speed of electron $= 3 \times 10^{6}$ m/s

We know that

$$qVB = mv^2/r$$

$$V = \frac{qrB}{m}$$

$$V_p = \frac{qrB}{m_p} \quad \text{and} \quad V_e = \frac{qrm}{m_e}$$

$$\frac{V_p}{V_e} = \frac{\cancel{qrB}}{m_p} \times \frac{m_e}{\cancel{qrB}}$$

$$\frac{V_p}{V_e} = \frac{m_e}{m_p}.$$

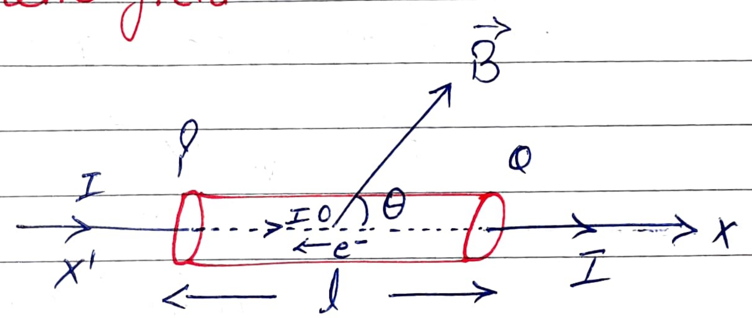
$$V_p = \frac{m_e}{m_p} \times V_e$$

$$= \frac{3 \times 10^6 \times 9 \times 10^{-31}}{1.8 \times 10^{-27}}$$

$$\boxed{V_p = 1.5 \times 10^3 \, m/s}$$

Q Force ON a current carrying conductor placed in a magnetic field?

Sol:-



Consider a straight cylindrical conductor (PQ) of length l, Area of cross-section (A).

I = carrying current placed in magnetic field B. ie make angle θ with the $X'X$-axis.

Since current is due to the flow of electron e^-

ie let V_d = drift velocity of electron.

$-e$ = charge on each electron.

The magnetic Lorentz force on an electron is given by

$$\vec{f} = -e(\vec{V_d} \times \vec{B}) \quad \text{--- (I)}$$

If n = number charge density of free electron.

N = total number of free electron.

ie $N = n(Al)$

$\therefore$ Total force on the Conductor

$$\vec{F} = N\vec{f}$$

$$= nAl(-e)(\vec{V_d} \times \vec{B})$$

$$\vec{F} = -enAl(\vec{V_d} \times \vec{B}) \quad \text{--- (II)}$$

We know that current through a Conductor is related to the drift velocity

ie $I = nAeV_d$

multiplying l both side

$$Il = nAeV_d l \quad \text{--- (III)}$$

Il = current element

using equation (III) in equation (II)

$$\vec{F} = I\vec{l} \times \vec{B}$$

$$\boxed{F = IlB\sin\theta}$$

where θ is the angle between $\vec{l}$ and $\vec{B}$.

Spicial Cases :

Case I If $\theta = 0$ and $\theta = 180^\circ$
ie $\sin\theta = 0$

$$\boxed{F = \text{minimum}}$$

It mean a linear Conductor Carrying a current 'I' if place parallel to the direction of magnetic field experience no force.

Case II : If $\theta = 90^\circ$, $\sin\theta = 1$

ie $F = IlB$ (Maximum).

It mean linear conductor Carrying current (I) if placed $\perp$ to the direction of magnetic field experience maximum force.

Problem :- The Horizontal Component of earth's magnetic field at a Certain place is 3×10^{-5} T and the direction of the field is from the geographic south to the geographic north. A very long straight Conductor carrying a steady Current of 1A. What is the force per unit length on it when it placed on a horizontal table and the direction of the current is (a) east to west (b) south to north?

Sol :- Here $B = 3\times10^{-5}$ T, $I = 1A$; $l = 1m$

a) When current is flowing from east to west ie $\theta = 90$ ($B \perp l$)

$$F = IlB \sin\theta = IlB \sin 90$$

$$F = IlB = 1\times1\times3\times10^{-5}$$

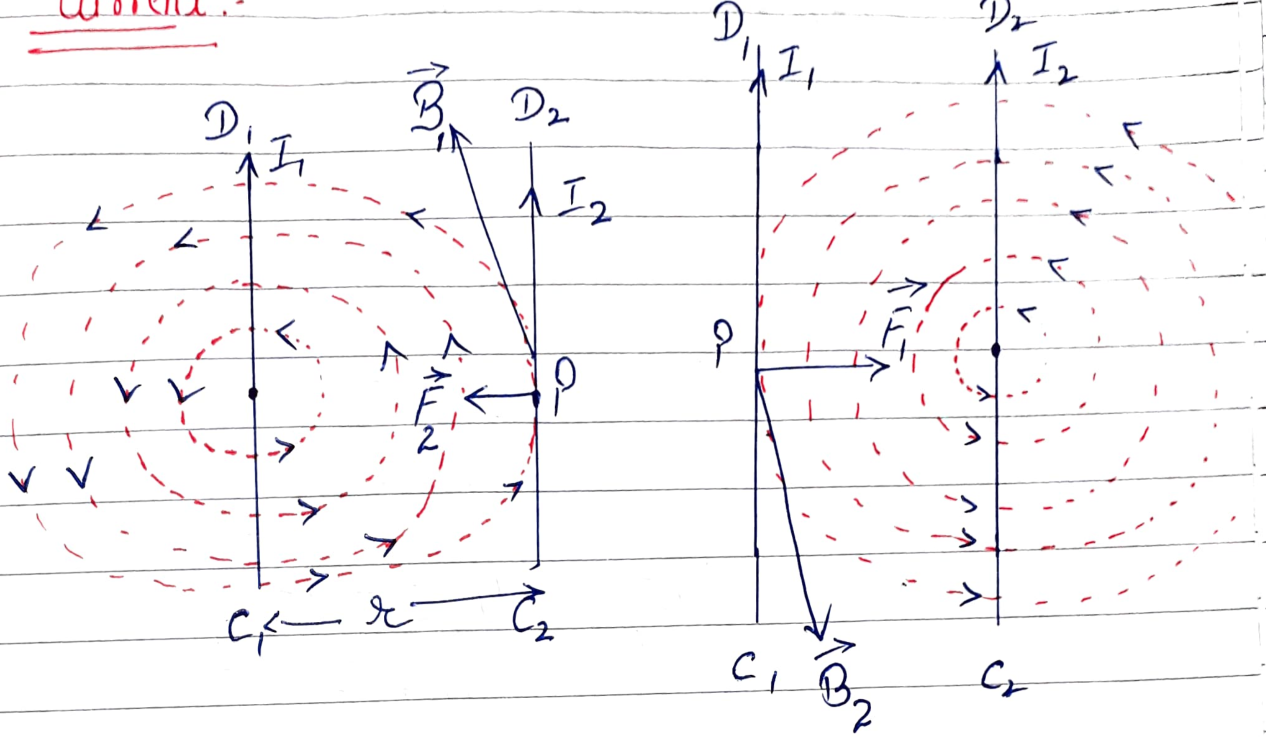
$$F = 3\times10^{-5} N/m.$$

(b) When Current is flowing from south to north $\theta = 0$

$$F = lIB\sin\theta = 1\times1\times3\times10^{-5}\sin(0) = 0$$

Hence no force is acting per unit length on the Conductor.

Force Between two Parallel Conductors Carrying Current:-



Consider two Conductors C_1D_1 and C_2D_2 Carrying a Current In same direction (I_1) and (I_2) held parallel to each other at a distance (r) apart from each other.

Now they produced magnetic field $\vec{B}$ due to this magnetic field each conductor experiance a force (F).

Here Magnetic field Induction at a Point (P) on (C_2D_2) due to (C_1D_1)

$$\left[B_1 = \frac{\mu_0}{4\pi} \frac{2I_1}{r} \right] \quad \text{——— (1)}$$

Acc. to Right hand Rule, the direction of magnetic field ($\vec{B}_1$) ⊥ to the plane of the paper.

as the current carrying conductor C_2D_2 lies in the magnetic field $\vec{B_1}$
$\therefore$ the length of C_2D_2 will experience a force given by

$$F_2 = B_1 I_2 \times 1 = B_1 I_2$$

$$\boxed{F_2 = \frac{\mu_0 2 I_1 I_2}{4\pi r}}$$

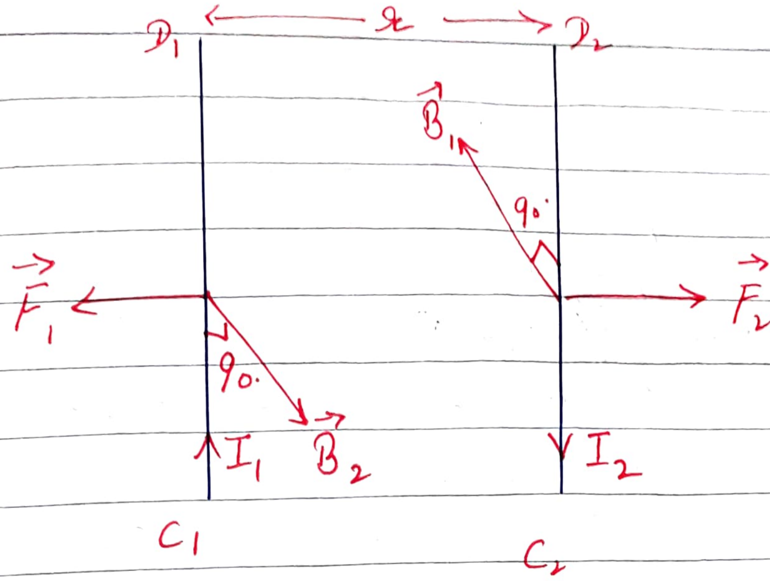
||rly $$\boxed{F_1 = \frac{\mu_0 2 I_1 I_2}{4\pi r}}$$

According to fleming's left hand Rule.

The direction of current, magnetic field and force.

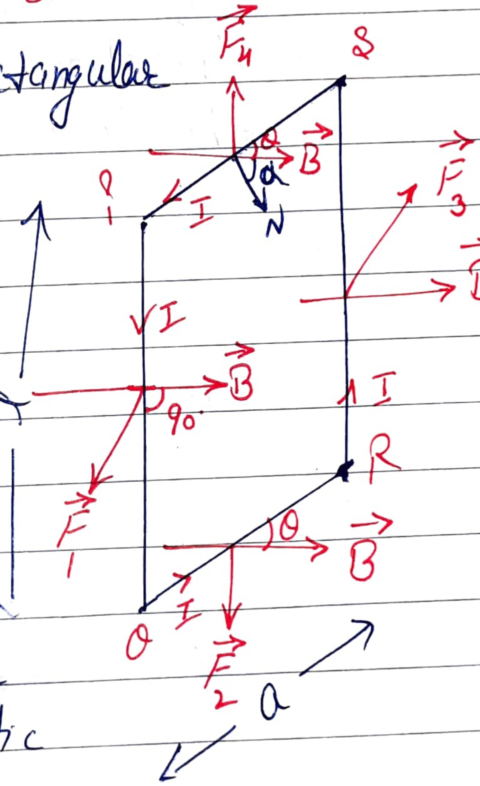
⇒ $\therefore$ The direction of force directed inward.
It means two linear parallel conductors carrying currents in the same direction attract to each other

⇒ And the direction of current is opposite in both conductors repell each other.



Q Discuss the experiment Torque on a Current Carrying Coil in a Magnetic field.

Sol:- Let us consider a rectangular Coil PQRS place in magnetic field $\vec{B}$.
Let $PQ = RS = l$
and $PS = QR = b$
Let I = Current flowing through it.
Here θ be the angle which plane of the Coil makes with the direction of magnetic field.



Let F_1, F_2, F_3 and F_4 be the force acting on the four arms of the Coil.

The force on arm SP

$F_4 = I(SP) B \sin(180 - \theta)$
$F_4 = IbB \sin\theta$

θ is the angle between $(\vec{SP})$ and $\vec{B}$
force in the plane of coil directed upward
The force on the arm OR

$\vec{F_2} = I(\vec{OR} \times \vec{B})$
$= I(OR) B \sin\theta$
$F_2 = IbB \sin\theta$

The direction of this force is in the plane of the coil directed downward.

Since $\vec{F_2}$ and $\vec{F_4}$ are equal in magnitude but opposite in direction along the straight line ie they can cancel each other.

Now force on PQ

ie $\vec{F_1} = I(\vec{PQ} \times \vec{B})$
$= IlB \sin 90^\circ = IlB$
$F_1 = IlB$ $[\because PQ \perp B]$

Acc. to fleming's left hand Rule the direction of magnetic field directed outward. $\perp$ to the plane of the coil. (or paper).

Simillarly for the phase RS

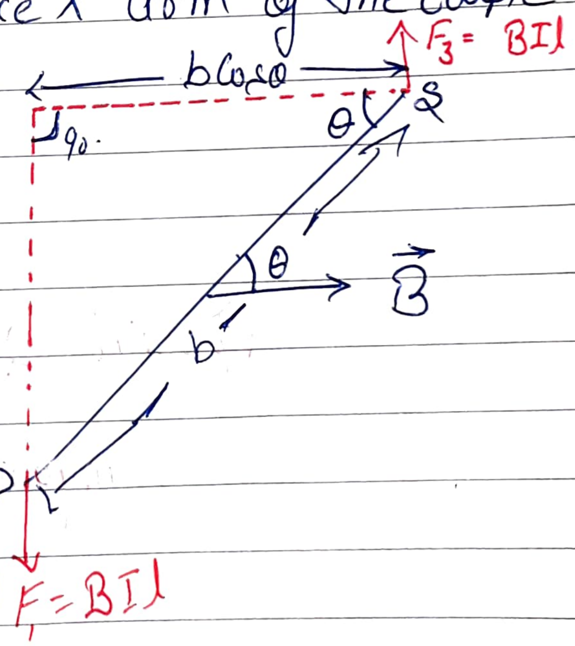
$$\vec{F_3} = I(\vec{RS} \times \vec{B})$$
$$= I\, PQ\, B \sin e$$
$$= IlB \sin 90$$
$$\boxed{|\vec{F_3}| = lIB.} \qquad (\vec{RS} \perp \vec{B})$$

acc. to fleming left hand Rule, the direction of magnetic field is $\perp$ to the Coil and also plane of the paper directed Inward.

The force acting on the arm PQ and RS are equal and $\parallel$el and acting in opposite direction having different lines of action form a Couple.

The torque on the Coil.

τ = either force × arm of the couple



arm of the couple $\Rightarrow \cos\theta = \dfrac{PS}{b}$

$$PS = b\cos\theta$$

$$\tau = lIB\, b\cos\theta$$
$$\tau = IBA \cos\theta$$
$$[\because l \times b = A]$$

If Coil has (n) number of turns

$$\tau = IABn\cos\theta$$

$$\boxed{\tau = nIAB\cos\theta} \longleftarrow (1)$$

Note If we draw a Normal (N) on the plane on the Coil make an angle (α) with the direction of magnetic field.

ie $\theta + \alpha = 90$

$\theta = 90 - \alpha$

using in equation (1)

$$\tau = nIAB\cos(90-\alpha)$$
$$\tau = nIAB\sin\alpha.$$
$$\tau = nI(\vec{A}\times\vec{B})$$

[Here $nIA = M$]

M = magnetic dipole moment

$$\tau = MB\sin\alpha$$
$$\boxed{\vec{\tau} = |\vec{M}\times\vec{B}|}$$

Special Case I :- If the Coil is set with its plane ||r| to the direction of magnetic field B.

$\theta = 0^\circ$ and $\cos\theta = 1$

$\therefore$ Torque $\tau = nIBA$ (maximum)

2. If the Coil is set with its plane ($\perp$) to the direction of magnetic field B.

$\theta = 90°$ and $\cos 90 = 0$

$\therefore$ Torque $\tau = nIAB(0) = 0$ (minimum)

Problem:- A circular Coil of 20 turns and Radius 10 cm carries a current of 5 A. It is placed in a uniform magnetic field of 0.10 T. Find the torque acting on the coil when the magnetic field is applied

a) normal to the plane of the coil.
b) In the plane of Coil.
also find out the total force acting on the Coil.

Sol:- Here $n = 20$

$r = 10\text{ cm} = 10 \times 10^{-2}\text{ m}$

$B = 0.10\text{ T}$

$I = 5\text{ A}$

Torque acting on the Coil

$\tau = nIAB\sin\alpha$

Area of Coil $A = \pi r^2 = \frac{22}{7} \times (0.1)^2$

$= \frac{22}{7} \times 0.01$

$A = 0.0314\ m^2$

a) Here $\alpha = 0$

$\tau = nIAB\sin\alpha$

$= 20(5)(0.0314)(0.1)\sin(0) = 0$

$(\tau = 0)$

b) Here $\alpha = 90°$

$\tau = nIAB\sin\alpha$

$= 20(5)(0.0314)(0.1)\sin 90$

$= 20(5)(0.0314)\ 0.1$

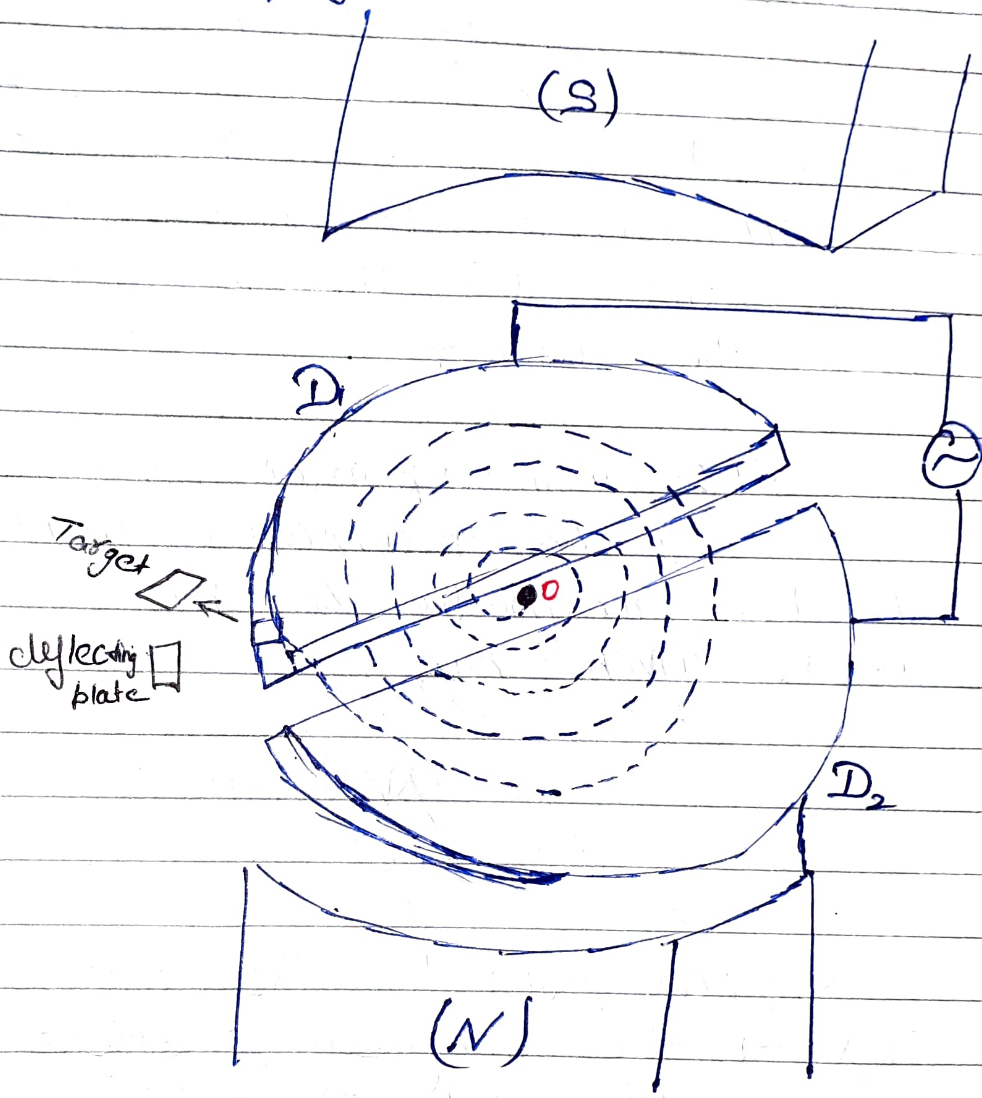
$\tau = 0.314\ (N\text{-}m)$.

Coil carrying current acts as a magnetic dipole. The force acting on magnetic dipole in a uniform magnetic field is zero.

Q:- What is Cyclotron. Discuss its principle, working, Figure and Theory ?

Sol:- Cyclotron is a device used to accelerate positively charged particles (like protons, α-particles, deutron, ions etc.) to acquire enough energy to carry out nuclear disintegrations.

Principle :- Cyclotron is an application of cross field ie. electric field (E) and magnetic field 'B' are at 90° to each other. When a positively charged particle is made to move time and again in a high frequency electric field and using strong magnetic field, it gets accelerated and acquires sufficiently large amount of energy.



<u>Working</u>:- If a positively charged particle is emitted from O, when dee D_2 is negatively charged and dee D_1 is positively charged, it will accelerate towards D_2. As soon as it enters D_2, it is shielded from the electric field by the metallic chamber. Inside D_2, it moves at right angle to the magnetic field and hence describes a semi-circle inside it. After completing the semi-circle, it enters the gap b/w the dees at the time, when polarities of dees have been reversed. Now the

proton is further acc. toward D_1. Then it enters D_1 and again describes the semi-circle due to the magnetic field which is perpendicular to the motion of the proton. This process continues till the proton reaches the periphery of the dee system. At this stage, the proton is deflected by the deflecting plate, which then comes out through the window (W) and hits the target.

Theory :- When a proton moves at right angle to the magnetic field ($\vec{B}$) inside the dee, magnetic Lorentz force acting on it is given by,

$$F = qVB \sin 90^\circ$$
$$F = qVB$$

This force provides the centripetal force $\frac{mv^2}{r}$ to the charged particle to move in a circular path of radius (r).

$$qVB = \frac{mv^2}{r}$$

$$r = \frac{mV}{qB} \quad \text{—(1)}$$

Time taken by the particle to complete the semi-circle inside the dee.

$$t = \frac{\text{Distance}}{\text{Speed}} = \frac{\pi r}{V} = \frac{\pi}{\cancel{V}} \frac{m\cancel{V}}{qB}$$

$$t = \frac{\pi m}{qB}$$

This shows that the time taken by the positively charged particle to Complete any semi-circle is same.

Time period: Let (T) be the time period of the high frequency electric field, then the polarities of dees will charge after time $T/2$

The particle will be acc. if time taken by it to describe semi-circle is equal to $T/2$

$$T/2 = t = \frac{\pi m}{qB}$$

$$T = \frac{2\pi m}{qB}$$

Cyclotron frequency:- $\nu = \frac{1}{T} = \frac{1}{\frac{2\pi m}{qB}} = \frac{qB}{2\pi m}$

Energy gained:- Energy gained by the positively charged particle is given by

$$E = \frac{1}{2} m v^2$$

we know $v = \frac{qBr}{m}$

$$E = \frac{1}{2} m \times \left(\frac{qBr}{m}\right)^2 = \frac{1}{2} \frac{q^2B^2r^2}{m}$$

Maximum energy gained by the positively charged particle

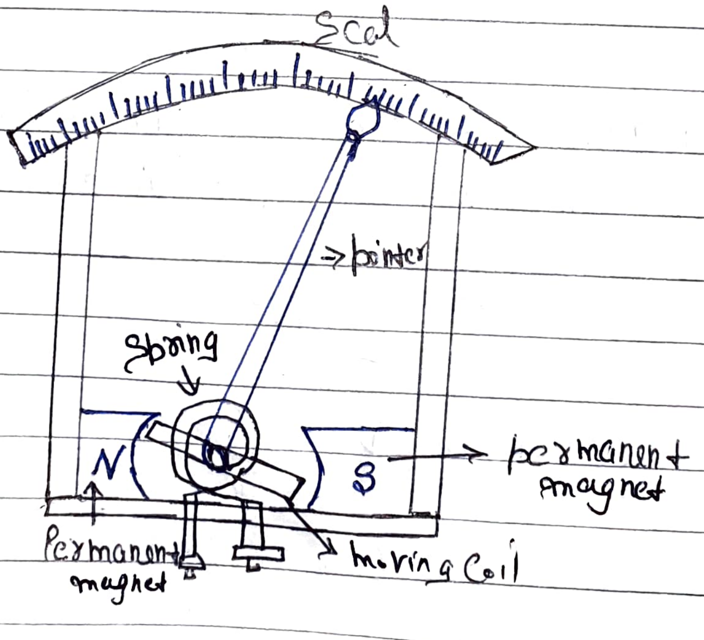
$$E_{max} = \left(\frac{q^2 B^2}{2m} \right) r_{max}^2$$

Thus, the positively charged particle will acquire maximum energy when it is at the periphery of the dees (where (r) is maximum)

Q:- Drive and explain Moving Coil Galvanometer Discous its principal, figure, Theory?

Sol:- Moving Coil Galvanometer is a device used to detect / measure small electric current flowing in the electric circuit.

Principle:- Moving coil galvanometer is based on the fact that when a current carrying loop or coil is placed in the uniform magnetic field, it experiences a torque.



Theory:- Let B = Intensity of magnetic field
I = Current flowing through the coil.
l = Length of coil
(b) = Breadth of the coil
$(l \times b) = A$ = Area of the coil.
N = Number of turns in the coil.

When current flows through the coil, it experiences a torque, which is given by

$$\tau_N = NIAB \sin\theta.$$

If angle is 90° the $\sin\theta = \sin 90$
$= 1$

$$\tau_N = NIAB$$

This torque is known as the deflecting torque.

As the coil gets deflected, the suspension wire is twised and a restoring torque is developed in it. If (k) is the restoring torque per unit twist of the suspension wire, then the restoring torque for the deflection α is given by

$$\tau_N' = k\alpha$$

For equilibrium of the coil, deflecting torque = Restoring torque

$$NIAB = k\alpha$$

$$I = \frac{k\alpha}{NAB}$$

$$I = G\alpha$$

$G = \dfrac{K}{NAB}$ and is called galvanometer Constant

$$I \propto \alpha.$$

Q:- Define sensitivity of a galvanometer? And discuss its type?

Ans:- A galvanometer is said to be sensitive if a small current flowing through the coil of galvanometer produces a large deflection in it.

(i) **Current sensitivity :-**

The current sensitivity of a galvanometer is defined as the deflection produced in the galvanometer per unit current flowing through it.

i.e Current sensitivity $= \dfrac{\alpha}{I} = \dfrac{\alpha(NAB)}{K\alpha}$

$= \dfrac{NAB}{K}$ $\left[\because I = \dfrac{K\alpha}{NAB}\right.$

Current sensitivity of galvanometer can be increased either by:-

(a) increasing the magnetic field B by using a strong permanent horse-shoe shaped magnet.

(b) increasing the number of turns N.

(c) increasing the area of the coil A
(d) decreasing the value of restoring force constant (K) by using a flat strip of phosphor-bronze instead of a circular wire of phosphor-bronze because the value of K is small in case of a flat strip than a round wire of phosphor bronze.

(ii) **Voltage Sensitivity:** Voltage sensitivity of a galvanometer is defined as the deflection produced in the galvanometer per unit voltage applied to it.

$$\text{Voltage Sensitivity} = \frac{\alpha}{V} = \frac{\alpha}{IR} = \frac{NBA}{KR}$$

Voltage sensitivity can be increased by
(a) increasing No. of turns (N)
(b) increasing magnetic field (B)
(c) increasing Area (A)
(d) decreasing (K) and (e) decreasing (R).

Q:- What is Ammeter and How can Conversion of Galvanometer into A Ammeter with the help of shunt?

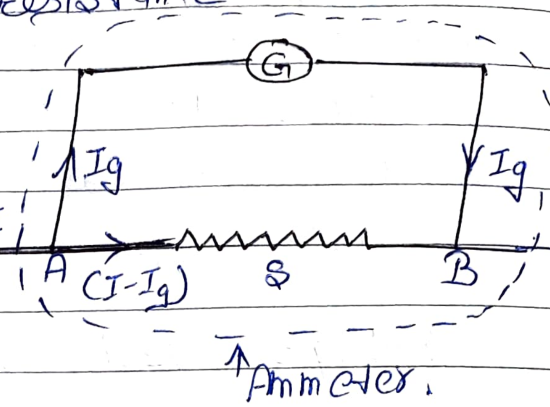
Ans:- An ammeter is an instrument used to measure electric current in an electric circuit.

Conversion of Galvanometer into a Ammeter:-

A galvanometer can be converted into an ammeter by connecting a low resistance called shunt

||rl to the galvanometer.

Let (G) and (S) be the resistance of a galvanometer and shunt respectively. Let (I) be the total current to be measured by an ammeter in the circuit.



Let (Ig) be the current flowing through the galvanometer corresponding to which galvanometer given the full scale deflection.

The remaining current (I - Ig) is to flow through the shunt.

Since (G) and (S) are ||rl, the potential diff. across them is same

ie $I_g G = (I - I_g) S$

$$S = \left(\frac{I_g}{I - I_g}\right) G$$

This is the required value of shunt resistance to convert a galvanometer into an ammeter of range (0-I) ampere.

Effective resistance of ammeter:-

The total effective resistance R_{eff} of an ammeter is given by

$$\frac{1}{R_{eff}} = \frac{1}{G} + \frac{1}{S} = \frac{G+S}{GS}$$

$$R_{eff} = \frac{GS}{G+S}$$

Since $G >> S$, so $(G+S) \simeq G$

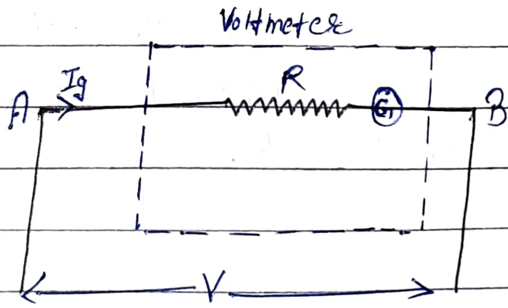
$$R_{eff} = \frac{GS}{G} = S$$

Thus, an ammeter is a low resistance device
Resistance of an ideal ammeter is zero.

Q:- Define Voltmeter and How can Conversion of Galvanometer into Voltmeter?

Ans: A voltmeter is an instrument used to measure the potential difference across the two ends of a circuit element.

Conversion of Galvanometer into Voltmeter :-



A galvanometer can be converted into a voltmeter by connecting a large resistance in series to the galvanometer.

Let (G) and (R) be the resistance of a galvanometer and a Conductor Connected in series with it respectively.

Let (V) Volt be the potential diff. to be measured by the Voltmeter.

Let (I_g) be the Current flowing in the circuit Corresponding to which the Volt meter gives the full Scale deflection.

Now potential diff. b/w points A and B

$$V = I_g R + I_g G$$

$$V = I_g (R+G)$$

$$R+G = \frac{V}{I_g}$$

$$\boxed{R = \frac{V}{I_g} - G}$$

This is the required value of resistance which must be Connected in Series to the galvanometer to Convert it into a Voltmeter of range (0-V) Volt.

What is the Radius of the path of an electron (mass 9×10^{-31} kg and charge 1.6×10^{-19} c) moving at a speed of 3×10^{7} m/s in a magnetic field of 6×10^{-4} T perpendicular to it? What is its frequency? Calculate its energy in keV?

Sol:- Here $m = 9\times10^{-31}$ kg, $q = 1.6\times10^{-19}$ c

$v = 3\times10^{7}$ m/s, $B = 6\times10^{-4}$ T

$$r = \frac{mv}{qB} = \frac{9\times10^{-31}(3\times10^{7})}{1.6\times10^{-19}\times6\times10^{-4}}$$

$$= \frac{9\times\cancel{3}^{1}}{1.6\times\cancel{6}_{2}}\times10^{-31+7+19+4}$$

$$= \frac{9}{3.2}\,10^{-1} = \frac{9}{32} = 0.28\text{ m}$$

$$\boxed{r = 0.28\text{ m}}$$

$$\nu = \frac{v}{2\pi r} = \frac{\cancel{v}\,qB}{2\pi m\cancel{v}} = \frac{qB}{2\pi m} = \frac{\cancel{6}^{3}\times10^{-4}\times1.6\times10^{-19}}{\cancel{2}(3.14)(\cancel{9}_{3}\times10^{-31})}$$

$$= \frac{1.6}{3.14\times3}\,10^{-4-19+31}$$

$$= 0.169\times10^{8}$$

$$\boxed{\nu = 1.7\times10^{7}\text{ Hz}}$$

$$E_K = \frac{1}{2} m v^2 = \frac{1}{2} \times (9 \times 10^{-31}) \times (3 \times 10^7)^2$$

$$= \frac{1}{2} \cancel{81}^{40.5} \times 10^{-31+14}$$

$$= 40.5 \times 10^{-17} \text{ J}$$

$$E_K = 40.5 \times 10^{-17} \text{ J}$$

In killo electorn volt

$$E_K = \frac{40.5 \times 10^{-17}}{1.6 \times 10^{-19} \times 1000} \text{ kev}$$

$$= \frac{40.5 \times 10^{-1}}{1.6} \text{ kev}$$

$$= \frac{4.05}{1.6} = 2.53 \text{ kev}$$

$$\boxed{E_K = 2.53 \text{ kev}}$$

A cyclotron oscillator frequency is 10 MHz what should be the operating magnetic field for accelerating protons? If the Radius of its dees is 60 cm, what is the kinetic energy of the proton beam produced by the accelerator in Mev?

Here $e = 1.6 \times 10^{-19} C$, $m_p = 1.67 \times 10^{-27} kg$, $1 MeV = 1.6 \times 10^{-13} J$

Sol:- $$B = \frac{2\pi \nu m}{q} = \frac{2 \times \frac{22}{7} \times (10 \times 10^6)(1.6 \times 10^{-27})}{1.6 \times 10^{-19}}$$

$$B = \frac{44 \times \cancel{1.6}}{\cancel{7} \cancel{1.6}} \times 10^{7-27+19}$$

$$= 6.28 \times 10^{-1}$$

$$\boxed{B = 0.628\ T}$$

$$E_K = \frac{B^2 q^2 r_0^2}{2m} = \frac{(0.628)^2 * (1.6 \times 10^{-19})^2 (0.60)^2}{2(1.67 \times 10^{-27})}$$

$$= \frac{0.394 \times 2.56 \times 0.36 \times 10^{-38+27}}{2 \times 1.67}$$

$$= 0.108 \times 10^{-11}$$

$$E_K = \frac{0.108 \times 10^{-11}}{1.6 \times 10^{-19} \times 10^6} = 0.0675 \times 10^{-11-6+19}$$

$$= 0.0675 \times 10^2$$

$$\boxed{E_K = 6.75\ MeV}$$

A straight wire of mass 200g and length 1.5m carries a current of 2A. It is suspended in mid air by a uniform magnetic field ($\vec{B}$) Horizontally. What is the magnitude of $\vec{B}$?

Sol: Here $m = 200g = 0.200kg$

$I = 2A$, $l = 1.5m$

for mid air suspension

$$F = BIl$$

$$mg = BIl$$

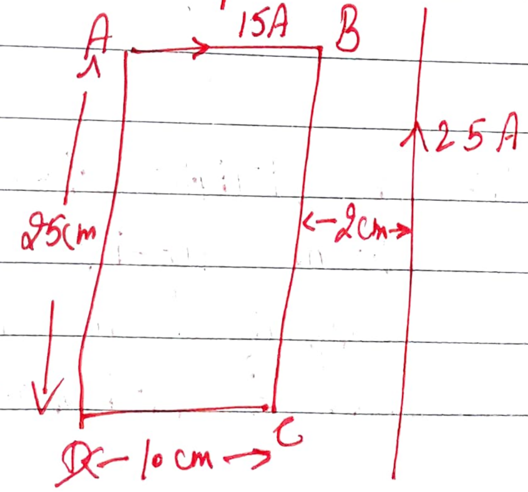
$$0.200(9.8) = B\, 2 \times 1.5$$

$$B = \frac{(0.200)(9.8)}{2 \times 1.5}$$

$$= \frac{0.100 \times \cancel{9.8}^{6.53}}{\cancel{1.5}} \times 10$$

$$\boxed{B = 0.653\,T}$$

Q# Show that a rectangular current-carrying loop placed 2 cm away from a long straight current carrying conductor. What is the direction and magnitude of the net force acting on the loop?



Sol:

Here $I_1 = 15A$ (ABCD)

$I_2 = 25\,A$ (XY)

$r_1 = 2\times10^{-2}\,m$ (distance b/w loop and wire)(BC)

$r_2 = (2+10)\times10^{-2}\,m$

$= 12\times10^{-2}\,m$ [distance b/w loop and wire) (AD)

force on BC, $F_1 = \frac{\mu_0}{4\pi}\frac{2I_1I_2}{r_1} \times$ length BC

$= 10^{-7}\times\frac{\cancel{2}(15)(25)}{\cancel{2}\times\cancel{10^{-2}}}\times(25\times\cancel{10^{-2}})$

$= 9375\times10^{-7}$

$(F_1 = 9.375\times10^{-4}\,N)$ (Repulsive)

force on DA, $F_2 = \frac{\mu_0}{4\pi}\frac{2I_1I_2}{r_2}$ length of DA

$F_2 = 10^{-7}\frac{\cancel{2}(\cancel{15}^{5})(25)}{\cancel{12}_{\cancel{6}\,3}\times\cancel{10^{-2}}}(25\times\cancel{10^{-2}})$

$= 15625\times10^{-7}$

$= 1.5625\times10^{-4}\,N$ (attrative)

Net force $F = F_1 - F_2 = (9.375 - 1.5625)\times10^{-4}$

$= 7.8175\times10^{-4}\,N$

$\boxed{F = 7.8175\times10^{-4}}\,N$ (Repulsive).

A circular coil of 100 turns, Radius (10 cm) Carries a current of 5A. It is suspended vertically in a uniform horizontal magnetic field of 0.5 T, the ~~dir~~ field lines making an angle of 60° with the plane of coil. Calculate the magnitude of the torque that must be applied on it to prevent it from turning.

Sol:- Here $n = 100$, $r = 10\,cm = 0.010\,m$

$I = 5A$, $B = 0.5\,T$

$\theta = 60°$

Torque $(\tau) = nIBA \sin\alpha$

$\alpha = 90 - 60 = 30$

$A = \pi r^2 = \frac{22}{7} \times (0.1)^2 = 3.14 \times (0.1)^2 = 0.0314\,m^2$

$\tau = 100(5)(0.5)(0.0314) \sin 30$

$= \frac{78.5 \times \frac{1}{2}}{10}$

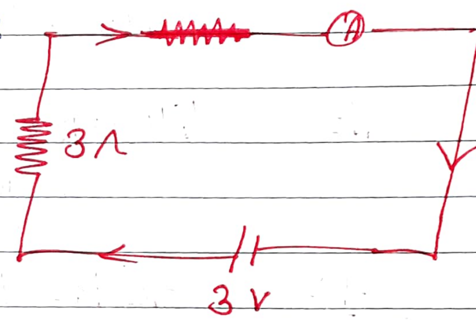
$= \frac{7.85}{2} = 3.925\,N\text{-}m$

$$\boxed{\tau = 3.927\,N\text{-}m}$$

11 In the circuit shown below the current is to be measured. What is the value of the current if the ammeter shown

(I) is a galvanometer with a resistance $R_G = 60\,\Omega$
(II) is a galvanometer with described in (i) but converted to an ammeter by a shunt resistance $r_s = 0.02\,\Omega$
(III) is an ideal ammeter with zero resistance.

Sol (I) Let (G) be the resistance of ammeter
ie current in circuit



$$I = \frac{E}{R+G} = \frac{3}{3+60}$$

$$= \frac{3}{63} = 0.048A$$

(II) When galvanometer is shunted with Resistance

$$R_P = \frac{GS}{G+S} = \frac{60 \times 0.02}{60 + 0.02} = 0.02\,\Omega$$

$$I = \frac{E}{R+R_P} = \frac{3}{3+0.02} = 0.99A$$

(III) for the ideal ammeter :- $I = \frac{3}{3} = 1\,A.$

The energy of Charged particle moving in a uniform magnetic field does not Change Explain ?

Sol: When a Charge particle is moving in a uniform parpendicular magnetic field, it experiance a force in a direction perpendicular to its direction of motion Due to which the speed of charged particle remain unchanged and hence Kinetic energy remains same.

difference between electric field and magnetic field.

Sol:-

	Electric field	Magnetic field
(1)	electric field is exert force on the charge particle whethere at rest and or motion.	Magnetic field exert force only those charge particle which are in motion.
(2)	Monopole or dipole exist	only dipole exist
3)	force $\vec{F} = \vec{E}q$	$\vec{F} = qvB \sin\theta$

Why is ammeter connected in series and Voltmeter is parallel in a circuit?

Sol:- An ammeter is a low resistance galvanometer. It is used to measure the current in amperes. To measure the current of a circuit, the ammeter is connected in series to the circuit so that the current to be measured must pass through it. Since, the resistance of ammeter is low, so it's inclusion in series in the circuit does not change the Resistance and hence the main current in the circuit.

A Voltmeter is a high Resistance galvanometer. It is used to measure potential difference between two points of the circuit in Volt. To measure the potential difference b/w the two points of a circuit, the Voltmeter is connected in parallel to the circuit. The Voltmeter Resistance being high, it draws minimum current from the main circuit and the potential difference to be measured is not affected materially.

Formulas :-

1. **Magnetic field: -**

$$\vec{F} = q(\vec{V} \times \vec{B})$$

$$F = q(VB)\sin\theta.$$

2. **Biot - Savart law: -**

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I(d\vec{l} \times \vec{r})}{r^3}$$

$$dB = \frac{\mu_0}{4\pi} \frac{I\,dl\sin\theta}{r^2}$$

$$B = \int \frac{\mu_0}{4\pi} \frac{I\,dl\sin\theta}{r^2}$$

3. Magnetic field due to long straight infinite wire :-

$$B = \frac{\mu_0 I}{4\pi a}\left[\sin\theta_2 - \sin\theta_1\right]$$

4. Magnetic field due to Circular Coil on axial line :-

$$B = \frac{\mu_0}{4\pi} \frac{2\pi n I a^2}{(a^2+x^2)^{3/2}}$$

5. **Ampere's law: -**

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I.$$

6. Magnetic field to Solenoid :-

$$B = \mu_0 n I \quad \text{or} \quad \mu_0 N I$$

7. Magnetic field due to Toroid:

$$B = \frac{\mu_0 N I}{2 \pi r}$$

8. Charge particle in uniform Electric field :-

$$y = \frac{qE x^2}{2 m v^2} \quad \left(K = \frac{qE}{2mv^2}\right)$$

9. Motion of charge particle under Magnetic field :-

Angular velocity :- $\left[\omega = \frac{Bq}{m}\right]$

Frequency :- $\frac{\omega}{2\pi} = \frac{Bq}{2\pi m}$.

Time period :-

$$T = \frac{2\pi m}{Bq}$$

Distance travelled by particle :-

$$d = \frac{2\pi m v}{Bq} \cos\theta$$

10. **Force on a current-carrying conductor in a uniform magnetic field: -**

$$F = I l B \sin\theta.$$

$$\vec{F} = I \vec{l} \times \vec{B}$$

11. **Force between two parallel current - carrying conductors:-**

$$F_1 = \frac{\mu_0 2 I_1 I_2}{4\pi r}$$

12. **Torque experienced by a current loop in uniform magnetic field: -**

$$\tau = MB\sin\alpha$$

$$\tau = |\vec{M} \times \vec{B}|$$

13. **Moving coil galvanometer: -**

$$I = \frac{K\alpha}{NAB}$$

14. **Current sensitivity and conversion to ammeter and voltmeter: -**

Current sensitivity :-

$$I = \frac{K\alpha}{NAB}$$

Voltage sensitivity

$$\frac{NBA}{KR}$$

Conversion into ammeter :-

(Shunt Resistance) $S = \left(\frac{I_g}{I - I_g}\right) G$

Conversion into Voltmeter :-

$$\boxed{R = \frac{V}{I_g} - G}$$