Physics

Alternating Current

Chapter:- 7

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Class: - 12th

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Unit :- IV

Chapter :- 7th

Alternating Current :-

Alternating current :- It is a current in which magnitude of current changes continuously with time and its direction is reversed periodically.

$$\left[\begin{array}{l} I = I_0 \sin\omega t \\ \text{or} \\ I = I_0 \cos\omega t \end{array} \right]$$

Here $\left[\omega = \frac{2\pi}{T} = 2\pi \nu \right]$

Alternating E.m.f. :-

$$\left[E = E_0 \sin\omega t \text{ or } E = E_0 \cos\omega t \right]$$

I ⇒ MEAN VALUE or Average Value of Alternating Current :-

The mean or average value of a.c. over any half cycle is defined as that value of steady current which would send the same amount of charge through a circuit in the time of half cycle $(T/2)$ through the same circuit as is sent by the a.c. through the same circuit in the same time.

alternating current

$$I = I_0 \sin\omega t \quad \text{— (I)}$$

we know that

$$I = \frac{dq}{dt}$$

$$dq = I\,dt$$

Intigrating both side $(0 - T/2)$

$$q = \int^{T/2} I\,dt$$

$$q = \int_0^{T/2} I_0 \sin\omega t\,dt$$

$$= I_0 \left[-\frac{\cos\omega t}{\omega} \right]_0^{T/2}$$

$$= \frac{-I_0}{\omega} \left[\cos\omega \tfrac{T}{2} - \cos 0 \right]$$

$$q = \frac{-I_0}{\omega} \left[\cos \frac{2\pi}{\cancel{T}} \times \frac{\cancel{T}}{2} - \cos 0 \right]$$

$$q = -\frac{I_0}{\omega} \left[\cos\pi - \cos 0 \right]$$

$$q = -\frac{I_0}{\omega} \left[-1 - 1 \right] = \frac{2I_0}{\omega}$$

$$q = \frac{2I_0}{\omega} \quad \text{— (II)}$$

If Im represents the mean or average value of a.c.

$$q = I_m \times \frac{T}{2} \quad \text{— (III)}$$

equating (II) and (III)

$$I_m \times T/2 = \frac{\alpha I_0}{\omega}$$

$$I_m = \frac{4}{T} \frac{I_0}{\frac{2\pi}{T}}$$

$$I_m = \frac{2 I_0}{\pi} = 0.637 I_0$$

$$\boxed{I_m = 0.637 I_0}$$

Hence mean or average value of a.c. over positive half cycle is 0.637 times the peak value of a.c. i.e 63.7% of peak value.

II ⟶ Mean value of Average value of Alternating E.M.F. :

It the average value of Alternating e.m.f. over a half cycle is that value of Constant e.m.f. which would send the same amount of charge through a circuit in the time of half cycle $(T/2)$, as is sent by alternating e.m.f through the same circuit in the same time.

Here $E = E_0 \sin\omega t$ (alternating e.m.f) if I is the value of current

ie $I = \frac{E}{R} = \frac{E_0 \sin wt}{R}$

Here $dq = I\,dt$

$dq = \frac{E_0 \sin wt}{R} dt$

Total charge in half cycle $(0 - T/2)$

$$q = \int_0^{T/2} \frac{E_0 \sin wt}{R} dt$$

$$q = \frac{E_0}{R} \int_0^{T/2} \sin wt \, dt$$

$$q = \frac{E_0}{R} \left[-\frac{\cos wt}{w} \right]_0^{T/2}$$

$$q = -\frac{E_0}{Rw} \left[\cos w \tfrac{T}{2} - \cos w(0) \right]$$

$$q = -\frac{E_0}{Rw} \left[\cos \frac{2\pi}{T} \times \frac{T}{2} - 1 \right]$$

$$q = -\frac{E_0}{Rw} [-1 - 1]$$

$$\boxed{q = +\frac{2E_0}{Rw}} \quad \text{——} (1)$$

if E_m is mean or average value of alternat
e.m.f.

$$q = \frac{E_m}{R} \times \frac{T}{2} \quad \text{--- (II)}$$

equating (I) and (II)

$$\frac{E_m}{\cancel{R}} \times \frac{T}{2} = \frac{2E_0}{\cancel{R}\omega}$$

$$E_m = \frac{2}{T} \times \frac{\cancel{2}E_0}{\cancel{2}\pi/T} \qquad \left[\because \omega = \frac{2\pi}{T}\right]$$

$$\boxed{E_m = \frac{2E_0}{\pi}}$$

$$\boxed{E_m = 0.637\,E_0}$$

Here mean or average value of alternating e.m.f. over positive half cycle is 0.637 time the peak value of alternating emf.

Similarly, over the negative half cycle $\left(\frac{T}{2} \to T\right)$

$$E_m = \frac{-2E_0}{\pi} = -0.637\,E_0$$

Hence average value of alternating e.m.f. over the full cycle = zero.

III Root Mean Square Value of Alternating Current :-

It is defined as that value of steady current, which would generate the same amount of heat in a given resistance in a given time, as is done by the a.c., when passed through the same resistance for the same time.

It is also called effective value or virtual value of a.c. It is denoted by (I_{rms}) or I_{eff} or I_v.

Alternating current

$$I = I_0 \sin\omega t \quad \text{—— (I)}$$

Amount of heat produced through R.

$$H = I^2 R\, dt \quad \text{—— (II)}$$

Total amount of heat produced.

$$H = \int_0^T I^2 R\, dt$$

$$H = I_0^2 R \int_0^T \sin^2\omega t\, dt$$

$$= I_0^2 R \int_0^T \left(\frac{1 - \cos 2\omega t}{2}\right) dt$$

$$H = \frac{I_0^2 R}{2}\left[\int_0^T 1\,dt - \int_0^T \cos 2\omega t\,dt\right]$$

$$= \frac{I_0^2 R}{2}\left[t - \frac{\sin 2\omega t}{2\omega}\right]_0^T$$

$$= \frac{I_0^2 R}{2}\left[T - \frac{\sin 2\omega T}{2\omega} - 0\right]$$

$$= \frac{I_0^2 R}{2}\left[T - \frac{\sin 2\,\frac{2\pi}{T}(T)}{2\omega}\right]$$

$$= \frac{I_0^2 R}{2}\left[T - \frac{\sin 4\pi}{2\omega}\right] \qquad [\sin 4\pi = 0]$$

$$H = \frac{I_0^2 R}{2}[T]$$

$$\boxed{H = \frac{I_0^2 R T}{2}} \text{ ——— (III)}$$

if r.m.s. value of a.c. is represented by (I_v) Then amount of heat

$$H = I_v^2 R T \text{ —— (IV)}$$

Comparing (III) and (IV)

$$I_v^2 \cancel{R}\cancel{T} = \frac{I_0^2 \cancel{R}\cancel{T}}{2}$$

$$I_v = \frac{I_0}{\sqrt{2}} = 0.707\, I_0$$

Hence the r.m.s. value ~~of~~ of a.c. is 0.707 times the peak value of a.c. ie 70.7% of the peak value of a.c.

Problem:- The Instantaneous Current from an a.c. source is $I = 6 \sin 314t$. What is the r.m.s value of the current?

Sol:- given $I = 6 \sin 314t$

Comparing $I = I_0 \sin \omega t$

Here $I_0 = 6$

$$I_v = \frac{I_0}{\sqrt{2}} = 6(0.707)$$

$$\boxed{I_v = 4.242\, A}$$

IV Root Mean Square value of Alternating E.M.F.

It is defined as that value of steady voltage, which would generate the same amount of heat in a given resistance in a given time, as is done by the alternating e.m.f. when applied to the same resistance for the same time.

The r.m.s value is also called effective value of virtual value of alternating e.m.f. (It is denoted by E_{rms} or E_{eff} or E_v.

The alternating e.m.f.

$$E = E_0 \sin\omega t \quad \text{—} (1)$$

The amount of heat produced when alternating emf is applied.

$$dH = \frac{E^2}{R} dt$$

$$= \frac{(E_0 \sin\omega t)^2}{R} dt$$

$$dH = \frac{E_0^2 \sin^2\omega t}{R} dt$$

Total amount of heat produced

$$H = \int_0^T \frac{E_0^2}{R} \sin^2\omega t \, dt$$

$$= \frac{E_0^2}{R} \int_0^T \left(\frac{1 - \cos 2\omega t}{2} \right) dt$$

$$= \frac{E_0^2}{2R} \left[\int_0^T dt - \int_0^T \cos 2\omega t \, dt \right]$$

$$= \frac{E_0^2}{2R} \left[(t)_0^T - \left[\frac{\sin 2\omega t}{2\omega} \right]_0^T \right]$$

$$= \frac{E_0^2}{2R} \left[(T - 0) - \left(\frac{\sin 2\omega T}{2\omega} - 0 \right) \right]$$

$$= \frac{E_0^2}{2R} \left[T - \sin 2 \left(\frac{2\pi}{\cancel{T}} \right) \cancel{T} \right)$$

$$H = \frac{E_0^2 T}{2R} \quad \text{--- (II)}$$

If (E_r) is the rms value of alternating emf.

$$H = \frac{E_v^2 T}{R} \quad \text{--- (III)}$$

equating (II) and (III)

$$\frac{E_v^2}{\cancel{R}} \cancel{T} = \frac{E_0^2 \cancel{T}}{2\cancel{R}}$$

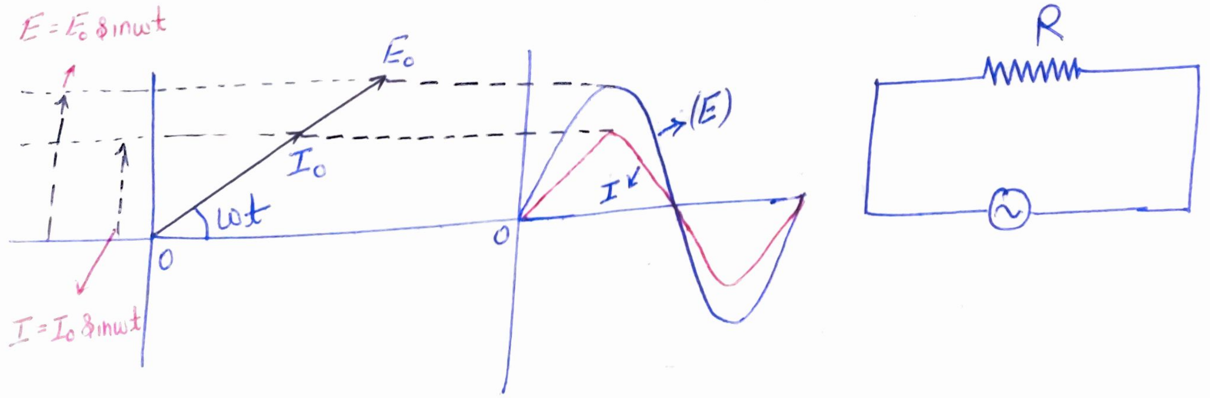
$$E_v = \frac{E_0}{\sqrt{2}}$$

$$E_v = \frac{E_0}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{E_0 \times 1.414}{2}$$

$$\boxed{E_v = 0.707 E_0}$$

Hence r.m.s. value of alternating e.m.f. is 0.707 times the peak value of alternating e.m.f.

V :- A.C. Circuit Containing Resistance only:



Let us consider a Resistance (R) with a alternating e.m.f.

$$E = E_0 \sin\omega t \quad \text{——} \ (I)$$

Let (I) be the current in the circuit at instant (t).

The Potential difference

$$E = IR \quad \text{——} \ (II)$$

equating (I) and (II)

$$IR = E_0 \sin\omega t$$

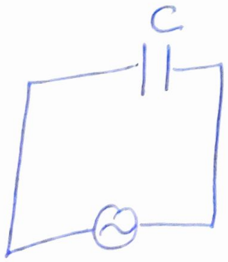
$$I = \frac{E_0}{R} \sin\omega t \qquad \left[I_0 = \frac{E_0}{R} \right]$$

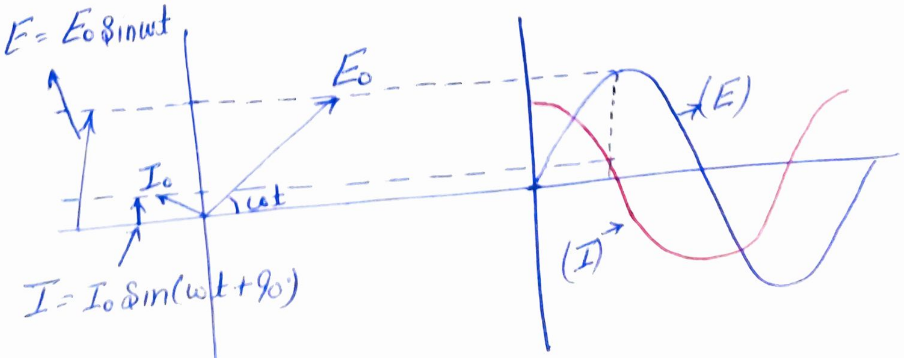
(maximum value to current)

$$\boxed{I = I_0 \sin\omega t}$$

∴ In an a.c. Circuit Containing (R) only, the Voltage and current are in the same phase.

VI :- A.C. CIRCUIT Containing Capacitance only :-





Let the source of alternating e.m.f be connected to a capacitor only of capacitance (C).

$$E = E_0 \sin\omega t \quad \text{—— (1)}$$

Potential difference across the plates of capacitor

$$V = q/C$$

The potential difference (V) must be equal to the e.m.f. applied.

$$V = \frac{q}{C} = E = E_0 \sin\omega t$$

$$q = C E_0 \sin\omega t$$

if I is instantaneous value of current in the circuit.

$$I = \frac{dq}{dt} = \frac{d}{dt}(C E_0 \sin\omega t)$$

$$I = C E_0 \frac{d}{dt}(\sin\omega t)$$

$$= C E_0 (\cos\omega t)(\omega)$$

$$I = C\omega (E_0 \cos\omega t)$$

$$I = \frac{E_0}{\frac{1}{C\omega}} \cos(\omega t)$$

$$I = \frac{E_0}{X_C} \sin(\omega t + \pi/2) \qquad \left[X_C = \frac{1}{C\omega} \right]$$

$\downarrow$
Capacitative Resistance.

$$I = I_0 \sin(\omega t + \pi/2) \quad \left[I_0 = \frac{E_0}{X_C} \right]$$

We find that in an ac. circuit containing (C) only, alternating current (I) leads the alternating e.m.f. by a phase angle of 90°.

Problem

(1) Calculate the capacitative reactance of a 5μF capacitor for a frequency of 10^6 Hz.

Sol.: $C = 5\mu F = 5 \times 10^{-6} F$

$\nu = 10^6$ Hz

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi \nu C} = \frac{1}{2 \times \pi \times 10^6 \times 5 \times 10^{-6}}$$

$$X_C = \frac{1}{2\pi \times 5} = \frac{1}{10\pi}$$

$$X_C = \frac{1}{10} \times \frac{7}{22} = 0.032\,\Omega$$

(2) One microfarad capacitor is joined to 200V, 50Hz alternator. Calculate the rms current through capacitor.

Here $C = 1\mu F = 10^{-6} F$

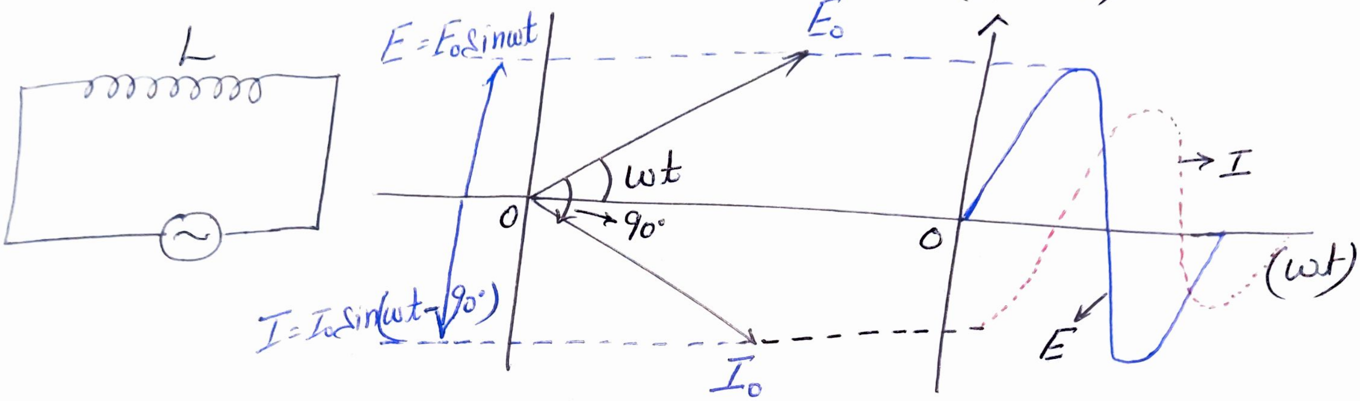
$E_v = 200V$, $\nu = 50Hz$, $I_v = ?$

$$I_v = \frac{E_v}{X_C} = \frac{200}{\frac{1}{2\pi\nu C}} = 200 \times 2 \times \frac{22}{7} \times 50 \times 10^{-6}$$

$$= \frac{88}{7} \times 5 \times 10^{-3} = \frac{440}{7} \times 10^{-3}$$

$$\boxed{I_v = 6.28 \times 10^{-2} A}$$

VII AC circuit Containing ~~Resistance~~ Inductance only :-



Let a source of alternating e.m.f. be connected to a circuit containing a pure inductance (L) only.

alternating e.m.f.

$$E = E_0 \sin\omega t \quad — (I)$$

E.m.f. in the Inductor

$$E = -L\frac{dI}{dt} \quad — (II)$$

The applied voltage and flow of current must be equal and opposite.

$$E = -\left(-L\frac{dI}{dt}\right) = E_0 \sin\omega t$$

$$dI = \frac{E_0}{L} \sin\omega t \, dt$$

Integrating both side

$$I = \frac{E_0}{L}\int \sin\omega t \, dt$$

$$I = \frac{E_0}{L}\left[-\frac{\cos\omega t}{\omega}\right]$$

$$I = -\frac{E_0}{\omega L}\cos\omega t$$

$$I = \frac{-E_0}{\omega L} \sin\left(\frac{\pi}{2} - \omega t\right)$$

$$I = \frac{-E_0}{\omega L}\left(-\sin(\omega t - \pi/2)\right)$$

$$I = \frac{E_0}{\omega L}\left[\sin(\omega t - \pi/2)\right] \quad \text{— (III)}$$

The current will be maximum $(\sin(\omega t - \pi/2) = 1)$

ie $I = I_0 = \frac{E_0}{\omega L}$ — (IV)

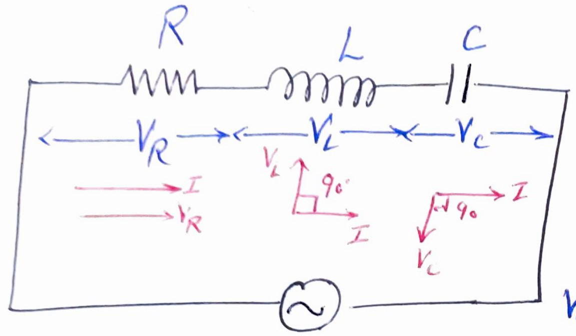
$[X_L = \omega L]$

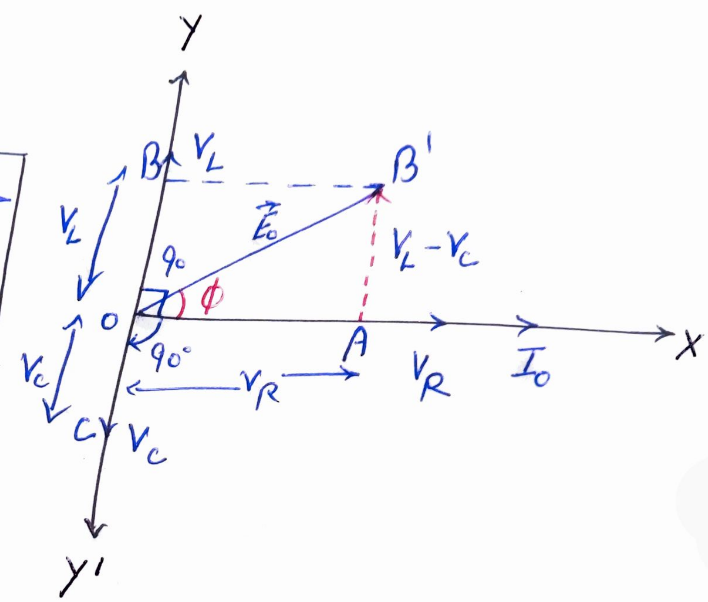
from (III) and (IV)

$$I = I_0 \sin(\omega t - \pi/2)$$

In an a.c. circuit containing (L) only, alternating current I lags behind the alternating voltage (E) by a phase angle of 90°.

VIII :- A.C. Circuit Containing Resistance, Inductance and Capacitance in Series (RLC Circuit) :-





Let a pure Resistance(R), Inductance(L) and Capacitance (C) be connected in series to a source of alternating e.m.f.

Current through the circuit is

$$I = I_0 \sin \omega t \quad -(1)$$

1) The maximum Voltage across Resistance (R)

$$\vec{V_R} = \vec{I_0} R$$

The current and Voltage $\vec{V_R}$ in the same phase.

i.e $\vec{OA}$ along OX.

2) The maximum Voltage across (L) is

$$\vec{V_L} = \vec{I_0} X_L .$$

As voltage across the Inductor leads the current by $90°$ ie Represented by $\vec{OB}$ along OY.

(III) The maximum Voltage across Capacitor (C)

$$\vec{V_c} = \vec{I_o} X_c$$

As voltage across the capacitor lags behind the alternating current by $90°$, it is Represented by $\vec{OC}$

Here phase difference between L and C is $180°$

The net reactive voltage is $(\vec{V_L} - \vec{V_C})$, assuming that $(\vec{V_L} > \vec{V_C})$.

The resultant Represented By (AB')

Now from $\Delta OAB'$

$$OA = V_R \quad , \quad AB' = V_L - V_C$$

$$OB' = \sqrt{(OA)^2 + (AB')^2}$$

$$= \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$= \sqrt{(I_o R)^2 + (I_o X_L - I_o X_C)^2}$$

$$E_o = \sqrt{I_o^2 (R^2) + I_o^2 (X_L - X_C)^2}$$

$$E_o = I_o \sqrt{R^2 + (X_L - X_C)^2}$$

$$Z = \frac{E_o}{I_o} = \sqrt{R^2 + (X_L - X_C)^2}$$

$Z = \frac{E_0}{I_0}$ = Impedance.

It is clear that circuit contain L, C and R in a
the voltage leads the current by a phase angle ϕ.

$$\tan\phi = \frac{AB'}{OA}$$

$$= \frac{V_L - V_C}{V_R}$$

$$= \frac{I_0 X_L - I_0 X_C}{I_0 R}$$

$$\tan\phi = \frac{\cancel{I_0}(X_L - X_C)}{\cancel{I_0} R}$$

$$\boxed{\tan\phi = \frac{(X_L - X_C)}{R}}$$

$\therefore$ Alternating emf in the LRC circuit

$$E = E_0 \sin(\omega t + \phi)$$

Case:-

(1) when $X_L = X_C$

$$\tan\phi = 0$$

ie $\phi = 0$.

Hence Voltage and current in same phase.
The AC Circuit is non-Inductive

(ii) when $X_L > X_C$

$\tan\phi$ is positive

$\therefore \phi$ is (+ve)

hence voltage leads the current by phas angle ϕ.

The a.c. Circuit is. Inductance dominated.

(iii) when $X_L < X_C$

$\tan\phi$ is negative

$\therefore \phi$ is (-ve)

hence voltage lags behind the current by phase angle ϕ.

The a.c. Circuit is Capacitance dominated.

Problem

A Resistor of 12Ω, a Capacitor of reactance 14Ω and a Pure Inductor of Inductance 0.1H are joined in series and placed across 200V, 50Hz a.c. supply. Calculate

(i) current in the circuit

(ii) phase angle between current and voltage. (Take $\pi = 3$)

given $R = 12\Omega$, $X_C = 14\Omega$, $L = 0.14H$, $E_V = 220V$,

$\nu = 50\ Hz$, $I_V = ?$, $\phi = ?$

$X_L = L \times 2\pi\nu = 0.1 \times 2 \times 3 \times 50$

$$\boxed{X_L = 30\Omega}$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$= \sqrt{144 + (30 - 14)^2}$$

$$= \sqrt{144 + (16)^2}$$

$$= \sqrt{144 + 256} = \sqrt{400}$$

$$\boxed{Z = 20\Omega}$$

$$I_V = \frac{E_V}{Z} = \frac{\cancel{200}^{10}}{\cancel{20}} = 10A$$

$$\boxed{I_V = 10A}$$

$$\tan\phi = \frac{X_L - X_C}{R} = \frac{30 - 14}{12}$$

$$= \frac{16}{12} = 1.3$$

$$\phi = \tan^{-1}(1.3)$$

$\boxed{\phi = 53^\circ}$ Voltage lead the current.

Electric Resonance :

(a) Series Resonance Circuit :-

A circuit in which Inductance L, Capacitance C, and Resistance (R) are connected in series and the circuit admits maximum current corresponding to a given frequency of a.c., is called series resonance circuit.

Impedance $(Z) = \sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}$

At very low frequency

$X_L = \omega L$ (negligible)

But $X_C = \frac{1}{\omega C}$ [very high]

As frequency of alternating e.m.f. applied to the circuit is increased, X_L goes on increasing and X_C goes on decreasing.

at particular value

$(\omega = \omega_r)$

$$X_L = X_C$$

$$\omega_r L = \frac{1}{\omega_r C} \Rightarrow \omega_r^2 = \frac{1}{LC}$$

$$\omega_r = \frac{1}{\sqrt{LC}}$$

$$2\pi \nu_r = \frac{1}{\sqrt{LC}} \quad \text{or} \quad \boxed{\nu_r = \frac{1}{2\pi\sqrt{LC}}}$$

At this particular frequency ν_r

$$[X_L = X_C]$$

$$\therefore \quad Z = \sqrt{R^2 + (X_L - X_C)}$$

$$Z = \sqrt{R^2} \Rightarrow \boxed{Z = R} \rightarrow \underline{\text{minimum}}$$

Here at this frequency Impedance (Z) is minimum and current is maximum.

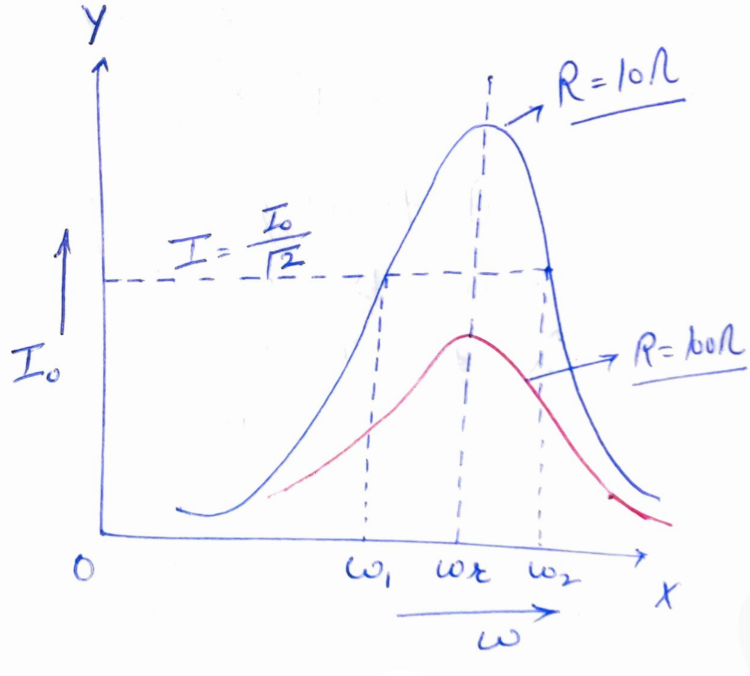
$$\text{ie } I_0 = \frac{E_0}{Z} = \frac{E_0}{R}.$$

This frequency is known as resonance frequency.

A series Resonance circuit admits maximum current

$\therefore$ It is also called

<u>Acceptor Circuit</u>



⇒ Q - Factor of <u>Resonance Circuit</u> (or) <u>Sharpness of Resonance</u> :-

It is defined as the Ratio of the voltage developed across the Inductance or Capacitance at resonance to the applied voltage across the R.

It is denoted (Q)

ie $Q = \dfrac{\text{Voltage across L or C}}{\text{applied voltage}}$

(across L) $Q = \dfrac{(\omega_r L)\cancel{I}}{R\cancel{I}}$ $\left[Q = \dfrac{V_L}{V_r}\right]$

$Q = \dfrac{\omega_r L}{R}$ $\left[\begin{array}{l} V_L = X_L I \\ V_r = IR \end{array}\right]$

$Q = \dfrac{L}{R\sqrt{LC}} = \dfrac{1}{R}\sqrt{\dfrac{L}{C}}$ $\left[\omega_r = \dfrac{1}{\sqrt{LC}}\right]$

(across C) $Q = \dfrac{\left(\dfrac{1}{\omega_r C}\right)\cancel{I}}{R\cancel{I}} = \dfrac{1}{(\omega_r C)R}$

$Q = \dfrac{\sqrt{LC}}{RC}$

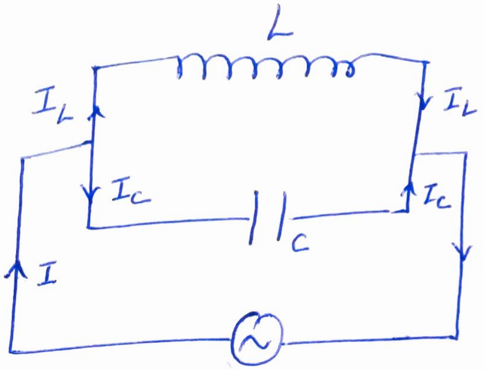
$$\boxed{Q = \frac{1}{R}\sqrt{\frac{L}{C}}}$$

Thus $\boxed{Q = \dfrac{1}{R}\sqrt{\dfrac{L}{C}}}$

Thus Q-factor may also be taken as voltage multiplication.

⇒ **Parallel Resonance Circuit :-**

A parallel Resonance Circuit Consist of a Coil of Inductance and Capacitor. joined in parallel to a source of e.m.f.



The alternating e.m.f.

$$E = E_0 \sin\omega t \quad \text{—— (I)}$$

we know that current lags behind in Inductor.

$$I_L = \frac{E_0}{X_L} \sin(\omega t - \pi/2) \quad \text{— (II)}$$

and leads in capacitors.

$$I_C = \frac{E_0}{I_C} \sin(\omega t + \pi/2) \quad \text{— (III)}$$

Total current in the circuit

$$I = I_L + I_C$$

$$I = \frac{E_0}{X_L} \sin(\omega t - \pi/2) + \frac{E_0}{X_C} \sin(\omega t + \pi/2)$$

$$I = \frac{E_0}{X_L} (-\cos\omega t) + \frac{E_0}{X_C} \cos\omega t$$

$$I = E_0 \cos\omega t \left[-\frac{1}{X_L} + \frac{1}{X_C} \right]$$

$$I = E_0 \cos\omega t \left[-\frac{1}{\omega L} + \omega C \right]$$

if $\omega C = \frac{1}{\omega L}$ $[I = 0]$

$$\therefore \quad I = E_0 \cos\omega t (0)$$

$$\boxed{I = 0}$$

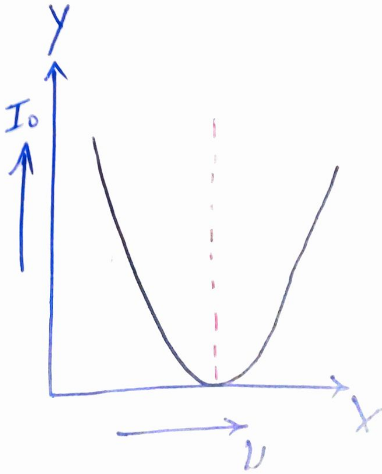
Or $\omega C = \frac{1}{\omega L}$

$$\omega^2 = \frac{1}{LC}$$

$$\omega = \frac{1}{\sqrt{LC}} \Rightarrow 2\pi\nu = \frac{1}{\sqrt{LC}}$$

$$\boxed{\nu = \frac{1}{2\pi\sqrt{LC}}} \qquad [\because \omega = 2\pi\nu]$$

frequency of applied alternating e.m.f become equal to natural frequency of oscillation of the circuit.



⇒ Power In AC Circuits :-

a) **Average Power Associated with Resistance :-**

Power is defined as the rate of doing work.
The Alternating current and E.m.f. across the circuit

$$E = E_0 \sin\omega t$$

$$I = I_0 \sin\omega t$$

Instantaneous Power

$$P = EI = (E_0 \sin\omega t)(I_0 \sin\omega t)$$

$$P = E_0 I_0 \sin^2\omega t$$

$$dw = E_0 I_0 \sin^2\omega t \, dt \qquad \left[P = \frac{dw}{dt} \right]$$

Integrating both side to Calculate total work done

$$\int dw = \int_0^T E_0 I_0 \sin^2\omega t \, dt$$

$$W = E_0 I_0 \int_0^T \sin^2\omega t \, dt$$

$$= E_0 I_0 \int_0^T \left(\frac{1 - \cos 2\omega t}{2} \right) dt$$

$$W = \frac{E_0 I_0}{2} \left[\int_0^T dt - \int_0^T \cos 2\omega t \, dt \right]$$

$$W = \frac{E_0 I_0}{2}\left[[t]_0^T - \left[\frac{\sin 2\omega t}{2\omega}\right]_0^T\right]$$

$$W = \frac{E_0 I_0}{2}\left[(T-0) - \frac{1}{2\omega}\left[\sin 2\left(\frac{2\pi}{T}\right)T - \sin 0\right]\right]$$

$$W = \frac{E_0 I_0}{2}\left[T - 0\right] \qquad \left[\because \sin 4\pi = 0\right]$$

$$W = \frac{E_0 I_0 T}{2}$$

$\therefore$ Average Power supplied to R

$$P = \frac{W}{T} = \frac{E_0 I_0 \cancel{T}}{2\cancel{T}}$$

$$P = \frac{E_0}{\sqrt{2}} \frac{I_0}{\sqrt{2}} = E_v \cdot I_v$$

$$\boxed{P = E_v \cdot I_v}.$$

hence average Power over a Complete Cycle of a.c. through the resistor is the product of virtual voltage and virtual Current.

b) Average Power Associated with An Inductor :-

In a circuit having Inductor (L), Current in it grows from zero to maximum (I_0) and an Induced e.m.f. develops in the Inductor, which opposes the growth of current.

Now alternating emf.

$$E = E_0 \sin \omega t$$

we know current lags behind (E) by a phase $\pi/2$

$$I = I_0 \sin(\omega t - \pi/2)$$

$$I = I_0(-\sin(\pi/2 - \omega t))$$

$$I = -I_0 \cos \omega t$$

Work done in one cycle.

$$W = \int_0^T E I \, dt$$

$$= \int_0^T E_0 \sin \omega t \, (-I_0 \cos \omega t) \, dt$$

$$= \int_0^T (-E_0 I_0) \sin \omega t \cos \omega t \, dt$$

$$= -\frac{E_0 I_0}{2} \int_0^T 2 \sin \omega t \cos \omega t \, dt$$

$$= -\frac{E_0 I_0}{2} \int_0^T \sin 2\omega t \, dt$$

$$= -\frac{E_0 I_0}{2} \left[-\frac{\cos 2\omega t}{2} \right]_0^T$$

$$W = +\frac{E_0 I_0}{2\times 2}\left[\cos 2\frac{2\pi}{T}\times T - \cos 0\right]$$

$$W = \frac{E_0 I_0}{4}\left[\cos 4\pi - \cos 0\right]$$

$$= \frac{E_0 I_0}{4}\left[1-1\right]$$

$$\boxed{W = 0}$$

Thus net Power $\left(P = \frac{W}{T} = 0\right)$ supplied by the source in a complete cycle is zero.

c) <u>Average Power Associated with a Capacitor</u>:-

A.C. Source connected to a capacitor of capacitance C, the charge grows from zero to maximum steady value Q_0.

Alternating Voltage

$$E = E_0 \sin \omega t$$

Current leads the e.m.f. by phase angle $\pi/2$

$$I = I_0 \sin(\omega t + \pi/2)$$

$$I = I_0 \cos \omega t$$

Work done over a complete cycle

$$W = \int_0^T E I \, dt$$

$$W = \int_0^T (E_0 \sin\omega t)(I_0 \cos\omega t)\, dt$$

$$= \int_0^T E_0 I_0 \sin\omega t \cos\omega t \, dt$$

$$= \frac{E_0 I_0}{2} \int_0^T 2 \sin\omega t \cos\omega t \, dt$$

$$= \frac{E_0 I_0}{2} \int_0^T \sin 2\omega t \, dt$$

$$= \frac{E_0 I_0}{2} \left[\frac{\cos 2\omega t}{2\omega} \right]_0^T$$

$$= \frac{E_0 I_0}{4\omega} \left[\cos 2\left(\frac{2\pi}{T}\right) T - \cos 0 \right]$$

$$W = \frac{E_0 I_0}{4\pi} [1 - 1]$$

$$\boxed{W = 0}$$

$\therefore$ Average Power Supplied to an ideal Capacitor by the Source over a Complete Cycle of a.c. is also zero.

d) Average Power In RLC Circuit :-

let the alternating e.m.f.

$$E = E_0 \sin wt \quad \text{—(I)}$$

alternating current

$$I = I_0 \sin(wt - \phi) \quad \text{—(II)}$$

Power at instant $= \frac{dw}{dt} = EI$

$$= (E_0 \sin wt)(I_0 \sin(wt - \phi))$$

$$= E_0 I_0 \sin wt (\sin wt \cos\phi - \cos wt \sin\phi)$$

$$= E_0 I_0 [\sin^2 wt \cos\phi - \sin wt \cos wt \sin\phi]$$

$$= E_0 I_0 [\sin^2 wt \cos\phi - \frac{2 \sin wt \cos wt}{2} \sin\phi]$$

$$= E_0 I_0 [\sin^2 wt \cos\phi - \frac{\sin 2wt \sin\phi}{2}]$$

$$dw = (E_0 I_0 \sin^2 wt \cos\phi - \frac{E_0 I_0}{2} \sin 2wt \sin\phi) dt$$

Intigrating both side to calculate total work done

$$\int dw = \int_0^T E_0 I_0 \sin^2 wt \cos\phi \, dt - \frac{E_0 I_0}{2} \int_0^T \sin 2wt \sin\phi \, dt$$

$$W = E_0 I_0 \cos\phi \underbrace{\int_0^T \sin^2\omega t \, dt}_{I} - \frac{E_0 I_0}{2} \sin\phi \underbrace{\int_0^T \sin 2\omega t \, dt}_{II} \quad \text{(III)}$$

$$I = \int_0^T \sin^2\omega t \, dt = \int_0^T \frac{1-\cos 2\omega t}{2} \, dt$$

$$= \frac{1}{2}\int_0^T (1-\cos 2\omega t) \, dt$$

$$= \frac{1}{2}\left[t - \frac{\sin 2\omega t}{2\omega} \right]_0^T$$

$$= \frac{1}{2}\left[\left(T - \frac{\sin 2\left(\frac{2\pi}{T}\right)T}{2\omega}\right) - 0\right]$$

$$= \frac{1}{2}\left[T - 0\right]$$

$$\int_0^T \sin^2\omega t \, dt = T/2$$

$$II = \int_0^T \sin 2\omega t \, dt = \left[\frac{-\cos 2\omega t}{2\omega}\right]_0^T = \frac{-1}{2\omega}\left[\cos 2\frac{2\pi}{T}(T) - 1\right]$$

$$= \frac{1}{2\omega}\left[1-1\right] = 0$$

Using in equation (III)

$$W = E_0 I_0 \cos\phi \left(\frac{T}{2}\right) - 0$$

$$W = E_0 I_0 \cos\phi \left(T/2\right)$$

Power $P = \frac{W}{T} = \frac{E_0 I_0 \cos\phi(\cancel{T})}{2\cancel{T}}$

$$P = \frac{E_0}{\sqrt{2}} \times \frac{I_0}{\sqrt{2}} \cos\phi$$

$$\boxed{P = E_v \times I_v \cos\phi}$$

hence average Power over a Complete cycle in a inductive Circuit is the product of virtual e.m.f, Virtual current and Cosine of the phase angle.

Problem:- A Coil of 1H and 100Ω Resistance has a peak Voltage of $5\sqrt{2}$ Volt, 50 Hz Connected across it. Calculate the current through the Coil and Power absorbed.

Sol:- $L = 1H$, $R = 100\Omega$, $E_0 = 5\sqrt{2}$ Volt, $\nu = 50$ Hz, $I_v = ?$, $P = ?$

$$X_L = \omega L = 2\pi \nu L$$

$$= 2\pi \times 50 \times 1$$

$$= 100\pi$$

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{(100)^2 + (100\pi)^2}$$

$$= 100 \times 3.3$$

$$= 3.3 \times 10^2 \Omega.$$

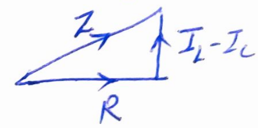
$$I_V = \frac{E_V}{Z} = \frac{5\cancel{\sqrt{2}}}{\cancel{\sqrt{2}}\times 3.3\times 10^{2}}$$

$$I_V = \frac{5\times 10^{-2}}{3.3} = 1.52\times 10^{-2}A$$

$$\boxed{I_V = 1.52\times 10^{-2}A}$$

<u>Power</u> $P = E_V I_V \cos\phi$ $\qquad\left[\cos\phi = \frac{R}{Z}\right]$

$$= E_V I_V\left(\frac{R}{Z}\right)$$



$$= \frac{5\cancel{\sqrt{2}}}{\cancel{\sqrt{2}}}\times 1.52\times 10^{-2}\left(\frac{100}{3.3\times 10^{2}}\right)$$

$$= \frac{5\times 1.52\times 100}{3}\times 10^{-4}$$

$$= 253.3\times 10^{-4}$$

$$= 2.53\times 10^{-2} \text{ watt}$$

⇒ <u>Power factor</u> :- It is defined as the Ratio of true Power to the apparent Power.

$$\text{Power factor} = \frac{\text{true Power}}{\text{apparent Power}} = \cos\phi$$

$$= \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}}$$

$$\cos\phi = \frac{R}{Z} = \frac{\text{Resistance}}{\text{Impedance}}$$

In a non-Inductive circuit

$$(X_L = X_C)$$

$$\text{Power factor} = \cos\phi = \frac{R}{\sqrt{R^2}} = \frac{R}{R} = 1$$

$$\text{ie } \boxed{\phi = 0}$$

This is the maximum value of Power factor.

In Pure Inductor or an ideal Capacitor, $\phi = 90°$

$$\text{Power factor} = \cos 90 = 0$$

∴ Average Power Consumed in a pure inductor or ideal Capacitor $P = E_v I_v \cos 90$

$$\boxed{P = \text{Zero}}$$

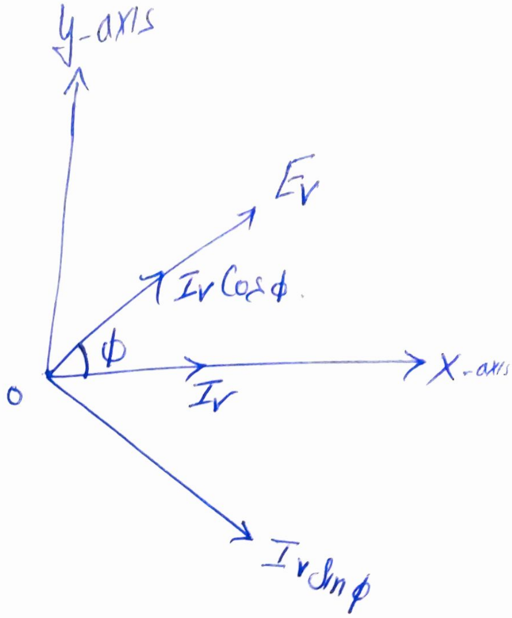
Current through Pure (L) or pure (C), which Consumes no Power for its maintenance in the circuit is Called Idle Current or wattless Current.

⇒ <u>WAttless</u> <u>Current</u>:-

The current which Consumes no Power for its maintenance in the circuit is Called wattless Current.

Average Power over a Complete cycle

$$P = E_V I_V \cos\phi$$



Suppose the phase angle b/w E_V and I_V is ϕ.

So I_V can be resolve into two Rectangular Components

$I_V \cos\phi$ along E_V

And $I_V \sin\phi$ ⊥ to E_V.

As the phase angle between $I_V \cos\phi$ and E_V is zero.

ie $P = E_V (I_V \cos\phi) \cos(0)$

$$\boxed{P_{av} = E_V I_V \cos\phi}$$

and phase angle b/w $I_V \sin\phi$ and E_V is 90.

ie $P = E_V (I_V \cos\phi) (\cos 90)$

$$\boxed{P_{av} = 0}$$

∴ The average Power Consumed in the a.c. circuit is wholly due to Component $I_V \cos\phi$ of the virtual Current.

No Contribution of $I_V \sin\phi$ ie it is called Idle Component or wattless Component.

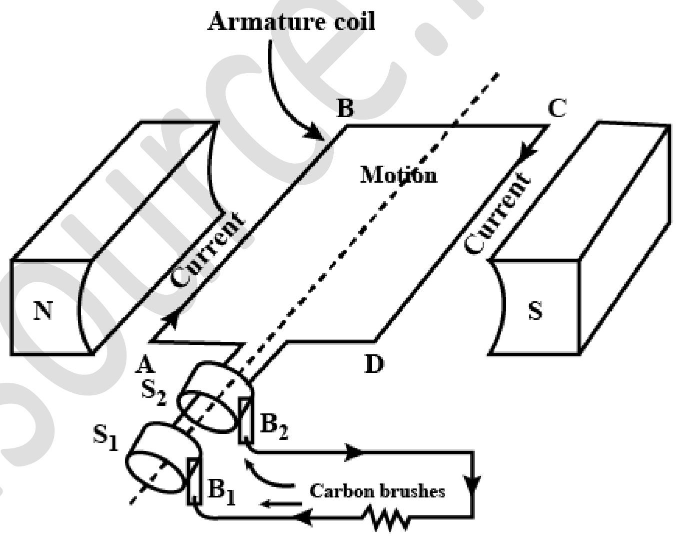
A.C. Generator or A.C. Dynamo: -

An a.c. generator/dynamo is a machine that produces alternating current energy from mechanical energy.

Principles: -

An a.c. generator/dynamo is based on the phenomenon of electromagnetic induction, i.e., whenever the amount of magnetic flux linked with a coil change, an e.m.f. is induced in the coil. It lasts so long as the change in magnetic flux through the coil continues.

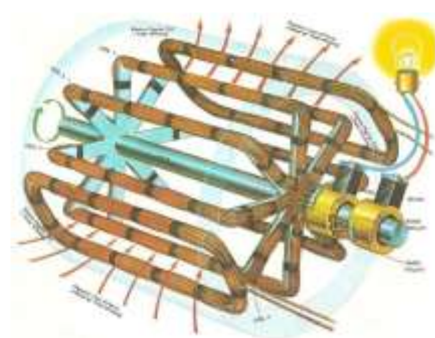
Construction: -



The essential parts of an a.c. dynamo is

1. **Armature.** ABCD is a rectangular armature coil. It consists of a large number of turns of insulated copper wire wound over a laminated soft iron core, /. The coil can be rotated about the central axis.

2. **Field Magnets.** N and S are the pole pieces of a strong electromagnet in which the armature coil is rotated. Axis of rotation is perpendicular to the magnetic field lines.



3. **Slip Rings.** S_1 and S_2 are two hollow metallic rings, to which two ends of the armature coil are connected. These rings rotate with the rotation of the coil.

4. **Brushes.** B_1 and B_2 are two flexible metal plates or carbon rods. They are fixed and are kept in light contact with S_1 and S_2 respectively. The purpose of brushes is to pass on current from the armature coil to the external load resistance R.

Theory and Working: -

As the armature coil is rotated in the magnetic field, angle & between the field and normal to the coil changes continuously. Therefore, magnetic flux linked with the coil changes. An e.m.f. is induced in the coil.

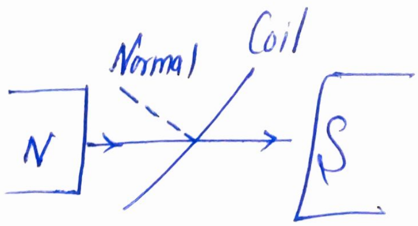
To Calculate the magnitude of e.m.f. Induced:-

N = Number of turns in the Coil.

A = Area enclosed by each turn

$\vec{B}$ = Magnetic field

θ = angle which normal to the Coil



$\therefore$ Magnetic flux linked with the coil

$$\phi = N(\vec{B}\cdot\vec{A}) = NBA\cos\theta \qquad \left[\omega = \frac{\theta}{t},\ \theta = \omega t\right]$$

$$\phi = NBA\cos\omega t \quad \text{--- (I)}$$

ω = angular velocity

At the instant t, if (e) is the e.m.f. induced in the Coil.

$$e = -\frac{d\phi}{dt} = -\frac{d}{dt}(NBA\cos\omega t)$$

$$e = -NBA\frac{d}{dt}(\cos\omega t)$$

$$e = -NBA(-\sin\omega t)\,\omega$$

$$e = NBA\omega\sin\omega t \quad \text{--- (II)}$$

Case I: Maximum Induced e.m.f.

$$\sin\omega t = 1$$

$$\omega t = \pi/2$$

$$e_{max} = e_0 = NAB\omega \quad \text{—— (III)}$$

Using in equation (II)

$$\boxed{e = e_0 \sin\omega t} \quad \text{—— (IV)}$$

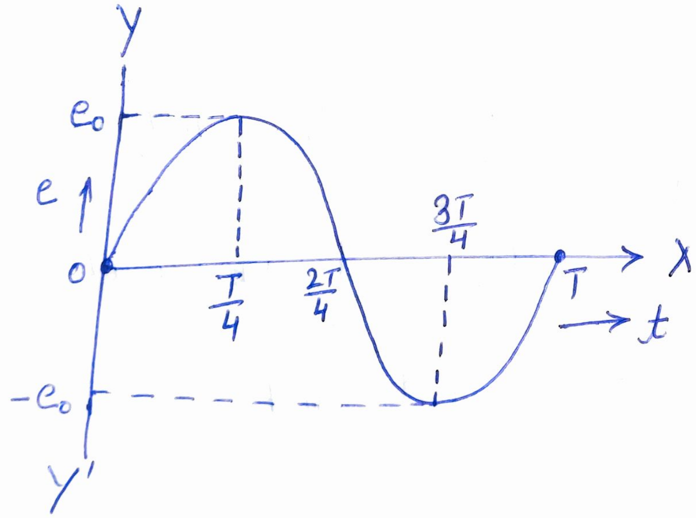
The Current supplied by the a.c. generator

$$i = \frac{e}{R} = \frac{e_0 \sin\omega t}{R}$$

$$i = \frac{e_0}{R} \sin\omega t = i_0 \sin\omega t$$

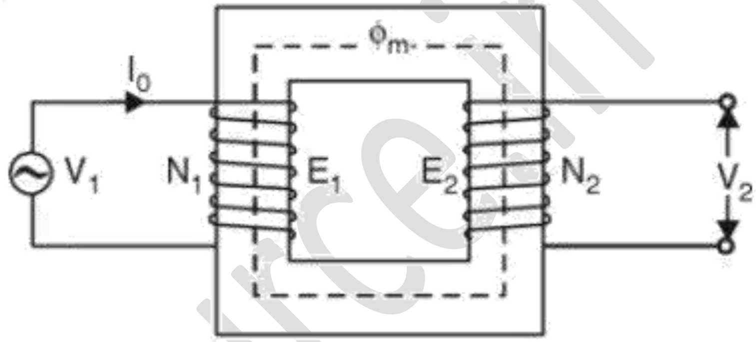
$$\boxed{i = i_0 \sin\omega t} \qquad \left[i_0 = \frac{e_0}{R} \right]$$

i_0 → Maximum current.



<u>Transformer: -</u> A transformer is an electrical device which is used for changing the a.c. voltages. A transformer which increases the ac voltages is called a step-up transformer. A transformer which decreases the a.c. voltages is called a step-down transformer.

Principle: - A transformer is based on the principle of mutual induction, i.e., whenever the amount of magnetic flux linked with a coil change, an e.m.f. is induced in the neighboring coil.



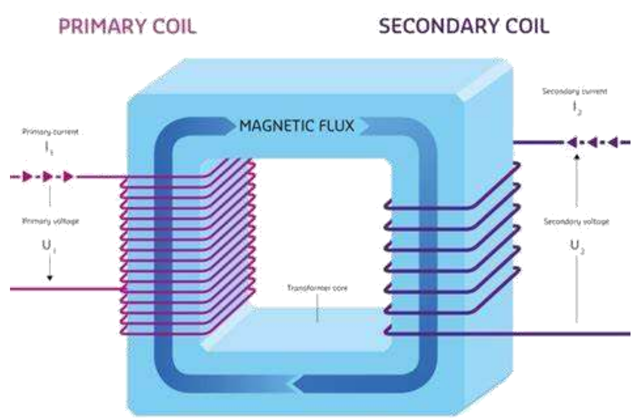
<u>Construction: -</u> A transformer consists of a rectangular soft iron core made of laminated sheets, well insulated from one another. There are two coils primary coil and secondary coil. The source of alternating e.m.f.(V_1) is connected to the primary coil and a load resistance R is connected to the secondary coil.

N_1 and N_2 be the number of turns in the primary and secondary coil.
E_1 and E_2 be the voltage across primary and secondary coil.

<u>Theory and working: -</u>

Let the alternate e.m.f. supplied by the a.c. source connected to primary coil

$$E = E_o \sin\omega t \quad \ldots\ldots\ldots\ldots\ldots\ldots(1)$$



The alternating primary current induces an alternating magnetic flux on in the iron core. Because the core extends through the secondary winding, the induced flux also extends through the turns of secondary coil.

According to Faraday's law of induction, the induced e.m.f. per turn is same for both the primary and secondary. Also, the voltage Eacross the primary is equal to the e.m.f. induced in the secondary.

$$E_{turn} = \frac{d\phi_B}{dt} = \frac{E_p}{n_p} = \frac{E_s}{n_s}$$

Here n_p and n_s be the number of turns in Primary and Secondary Coil.

$$\boxed{E_s = E_p \frac{n_s}{n_p}}$$

for step up: $n_s > n_p$ ie $E_s > E_p$

for step down:- $n_p > n_s$ ie $E_p > E_s$.

$$\frac{n_s}{n_p} = k = \text{transformation Ratio.}$$

As we assume no energy is lost along.

$$I_p E_p = I_s E_s$$

$$\therefore I_s = I_p \frac{E_p}{E_s}$$

for step up: $E_s > E_p$ ie $I_s < I_p$

for step down:- $E_s < E_p$ ie $I_s > I_p$.

Efficiency:- $\boxed{\eta = \frac{\text{Output power}}{\text{Input power}} = \frac{E_s I_s}{E_p I_p}}$