Physics

Unit: - Electromagnetic Induction and Alternating Currents

Chapter– 6: Electromagnetic Induction

Class: 12th

EDUCATION SOURCE.IN

I. Electromagnetic induction: -

Electromagnetic induction is a phenomenon in physics where a changing magnetic field induces an electromotive force (EMF) or voltage in a conductor. This fundamental principle was first discovered by Michael Faraday in the early 19th century and is a key concept in the field of electromagnetism. Electromagnetic induction is the basis for the operation of many electrical devices, including generators and transformers.

The key components of electromagnetic induction are:

1. **Magnetic Field (B):** A magnetic field is essential for inducing an electromotive force. It can be produced by a permanent magnet or an electromagnet.

2. **Conductor or Coil (A):** A conductor, often in the form of a coil of wire, is placed in the magnetic field. The coil is usually made of a material with good electrical conductivity, such as copper.

3. **Relative Motion or Change in Magnetic Field (C):** For electromagnetic induction to occur, there must be a change in the magnetic field strength experienced by the conductor. This change can result from the motion of the conductor through a stationary magnetic field, a change in the strength of the magnetic field, or relative motion between the conductor and the magnetic field.

Faraday's law of electromagnetic induction describes the relationship between the change in magnetic flux (Φ) and the induced electromotive force (EMF or ε):

$$\boldsymbol{\varepsilon = -{}^{d\Phi}/_{dt}}$$

where:

- ε is the induced electromotive force,
- ${}^{d\Phi}/_{dt}$ is the rate of change of magnetic flux with respect to time.

This negative sign indicates the direction of the induced current or EMF, following Lenz's law, which states that the induced current will flow in a direction that opposes the change in magnetic flux that produced it.

Applications of electromagnetic induction include electric generators, transformers, inductors, and various types of sensors. Generators convert mechanical energy into electrical energy by rotating a coil in a magnetic field, while transformers transfer electrical energy between two coils through mutual induction. Inductors are components that store energy in a magnetic field, and sensors use electromagnetic induction for various purposes, such as detecting proximity or measuring current.

II. Faraday's laws: -

Faraday's laws of electromagnetic induction summarize the fundamental principles governing the relationship between a changing magnetic field and the induction of an electromotive force (EMF) in a conductor. Michael Faraday formulated these laws in the 1830s based on his experimental observations. There are two laws associated with electromagnetic induction:

1. Faraday's First Law:

- The change in magnetic flux (Φ) through a closed loop induces an electromotive force (EMF) in the conductor encircled by the loop.
- **Mathematically,** it can be expressed as:

$$\varepsilon = -{d\Phi}/{dt}$$

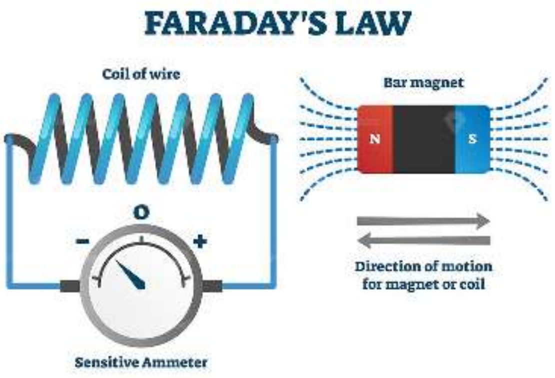
where:

- ε is the induced electromotive force (EMF),
- ${d\Phi}/{dt}$ is the rate of change of magnetic flux with respect to time.

The negative sign in the equation indicates the direction of the induced current or EMF, following Lenz's law. Lenz's law states that the induced current will flow in a direction that opposes the change in magnetic flux that produced it.

Experimental exaptation: -

In Faraday's experiment, when magnet is moved towards the coil, number of magnetic lines of force linked with the coil increases, i.e., magnetic flux increases. When the magnet is moved away, the magnetic flux linked with the coil decreases. In both the cases, galvanometer shows deflection indicating that emf is induced in the coil.



When there is no relative motion between the magnet and the coil, magnetic flux linked with the coil remains constant. That is why galvanometer shows no deflection. Thus, induced e.m.f. is produced when magnetic flux changes and induced e.m.f. continues so long as the change in magnetic flux continues.

2. Faraday's Second Law:

- The magnitude of the induced electromotive force (EMF) is directly proportional to the rate of change of magnetic flux.
- **Mathematically,** it can be expressed as:

$$\varepsilon = -N\, {d\Phi}/{dt}$$

where:

- ***N*** is the number of turns in the coil.

This law implies that increasing the rate of change of magnetic flux or increasing the number of turns in the coil will result in a larger induced EMF.

Experimental exaptation: -

In Faraday's experiment, when magnet is moved faster, the magnetic flux linked with the coil changes at a faster rate. Therefore, galvanometer deflection is more. However, when the magnet is moved slowly, rate of change of magnetic flux is smaller. Therefore, galvanometer deflection is smaller, i.e., induced e.m.f. is smaller. Hence magnitude of e.m.f induced varies directly as the rate of change of magnetic flux linked with the coil.

If ϕ_1 = The amount of magnetic flux link with coil at any time.
Φ_2 = The amount of magnetic flux link with coil at time T sec.

Rate of change of magnetic flux = $\frac{\varphi_1 - \varphi_2}{t}$

According to Faraday's second law of electro magnetic induction

$$e \,\alpha\, \frac{\varphi_2 - \varphi_1}{t}$$

$$e = k\,\frac{\varphi_2 - \varphi_1}{t}$$ (K = constant of proportionality)

If k = 1

$$e = \frac{\varphi_2 - \varphi_1}{t}$$

if dφ is small change then

$$e = -\frac{d\varphi}{dt}$$

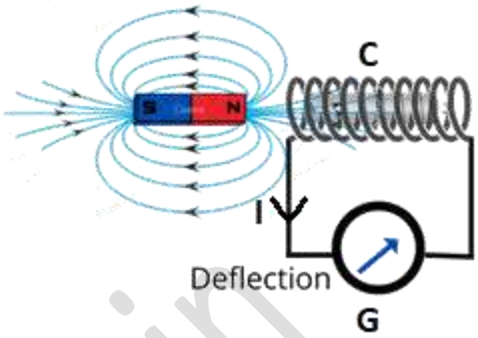
Negative sign indicates that e.m.f. always opposite to the change in magnetic flux.

Induced e.m.f. associated with the number of turns in the coil

$$e = -N\frac{d\varphi}{dt}$$

III. Experiments to show induced EMF and Current: -

(a) Current induced by a magnet: -



A coil or loop labelled as C, consisting of a few turns of insulated conducting material, is depicted. The coil is connected to a sensitive galvanometer, denoted as G. Faraday and Henry made the following observations regarding the induction of current by a magnet:

(i) When the north pole of a bar magnet is pushed towards the coil, a sudden deflection is observed in the galvanometer, indicating the induction of current in the coil.

(ii) The galvanometer deflection is temporary and occurs only while the bar magnet is in motion. No deflection is observed when the bar magnet is held stationary.

(iii) When the magnet is moved away from the coil, the galvanometer shows deflection in the opposite direction, signifying a reversal in the direction of the induced current.

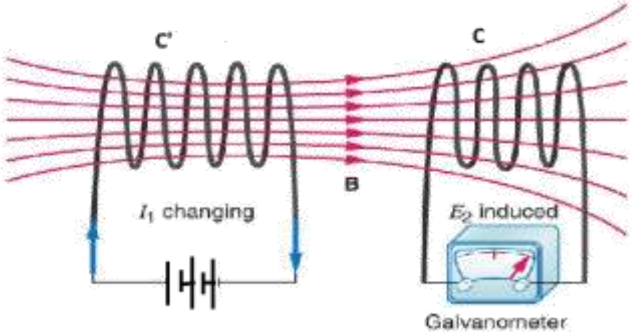
(iv) If the south pole of the bar magnet is moved towards or away from the coil, the galvanometer deflections are opposite to those observed with the north pole for similar movements. Additionally, it is noted that the galvanometer deflection (and consequently the induced current) is more pronounced when the magnet is pushed towards or pulled away from the coil at a faster rate.

(v) When the bar magnet is kept stationary, and the coil C is moved towards or away from the magnet, similar effects are observed. This observation emphasizes that the relative motion between the coil and the magnet is the factor responsible for the induction of electric current in the coil.

(b) Current induced in one coil due to current carry another coil: -

The described experiment involves two coils, one denoted as C and the other as C', both consisting of a few turns of insulated conducting material. Coil C is connected to a battery, generating a steady current that produces a uniform magnetic field along its axis.

(i) When coil C' is moved towards coil C, a deflection is observed in the galvanometer. This deflection signifies the induction of electric current in coil C as a result of the changing magnetic field caused by the relative motion between the two coils.



(ii) Conversely, when coil C' is moved away from coil C, the galvanometer shows a deflection in the opposite direction, indicating a reversal in the direction of the induced current in coil C.

(iii) The deflection in the galvanometer is temporary, lasting only as long as there is relative motion between the two coils.

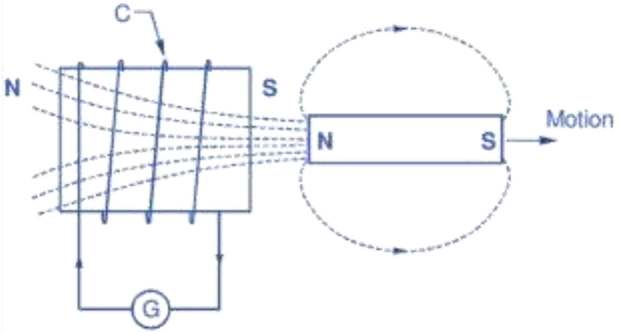
(iv) It is noted that the galvanometer deflection, and therefore the induced current, is more substantial when the coils are moved faster towards or away from each other. This observation aligns with the principles of electromagnetic induction, where a faster change in magnetic flux induces a larger electromotive force.

The observation that coils C, carrying current in the second experiment, behaves as a bar magnet in the first experiment further highlights the connection between moving coils and the induction of electric current, consistent with the principles of electromagnetic induction.

IV. LENZ's Law

According to Lenz's law, the polarity of the induced e.m.f. is such that it opposes the change in magnetic flux responsible for its production.

For example: - when north pole of a bar magnet is being pushed towards the coil, the number of magnetic flus linked with the coil increases. Current is induced in the coil in such a direction that it opposes the increase in flax. This is possible only when current induced in the coil is in anticlockwise direction with respect to an observer on the side of the bar magnet.



V. LENZ'S LAW AND ENERGY CONSEVATION: -

(a) Lenz's Law:

Lenz's Law states that the direction of the induced electromotive force (EMF) and the resulting induced current in a closed circuit will be such that it opposes the change in magnetic flux responsible for its production.

(b) Conservation of Energy:

The conservation of energy is a fundamental principle stating that the total energy of an isolated system remains constant over time. Energy can neither be created nor destroyed, only transferred or converted from one form to another.

(c) Connection between Lenz's Law and Conservation of Energy:

- When a change in magnetic flux occurs, such as when a coil is moved in or out of a magnetic field, Lenz's Law dictates that the induced current in the coil opposes this change. This opposition results in the conversion of mechanical energy (or other forms of energy causing the change in flux) into electrical energy.
- The work done against the induced EMF is manifested as electrical energy, and this process ensures that the total energy in the system is conserved.

Example:

- Consider a coil moving into a magnetic field. The work done against the induced EMF is converted into electrical energy. This electrical energy can then be used to power devices connected to the coil.

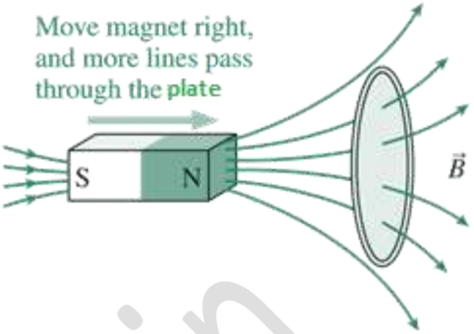
Q1: Why does a bird sitting on a high voltage line fly away when the current is switched on?

Ans: - When the current is switched on, induced currents flow through the bird's body. These induced currents in the wings experience a force of mutual repulsion, causing the wings to spread and the bird to take flight.

Q2: What method can be employed to detect the presence of a magnetic field on an unknown planet?

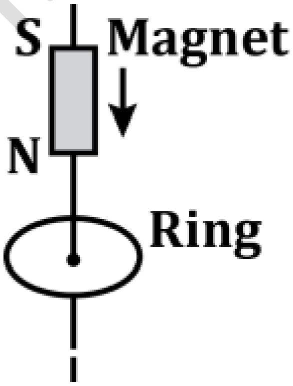
Ans: - To ascertain the existence of a magnetic field on an unknown planet, one can utilize a sensitive galvanometer connected to a coil. By rotating the coil, if the galvanometer exhibits a deflection, it indicates the presence of a magnetic field; otherwise, its absence can be inferred.

Q3: When a copper ring suspended in a vertical plane is subjected to the horizontal approach of one end of a magnet, how does the position of the ring change?



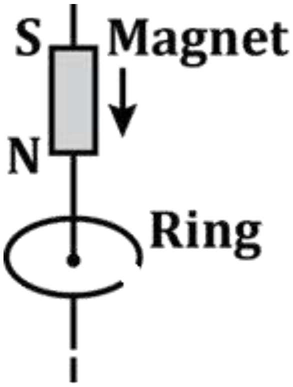
Ans: - The ring will move away from the magnet. This movement is attributed to Lenz's law, where an electromotive force (e.m.f.) is induced in the ring, resulting in a north pole facing the magnet. The repulsive force between the like poles (N pole of the ring and the approaching magnet) causes the ring to move away from the magnet.

Q4: When a bar magnet falls through a metal ring, is its acceleration equal to 'g'?



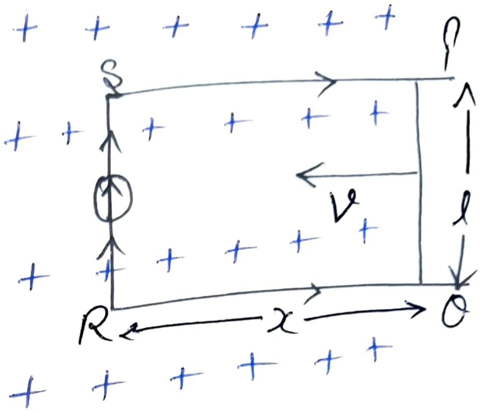
Ans: - No, the acceleration of the magnet will not be equal to 'g'. It will be less than 'g'. This is due to the changing magnetic flux linked with the ring as the magnet falls. The ring develops an induced current that opposes the downward motion of the magnet. Even after the magnet passes through the ring, the decreasing magnetic flux continues to induce a current, opposing the fall and resulting in an acceleration less than 'g'.

Q5: In the given scenario of a falling bar magnet through a metal ring, what happens if the ring is cut somewhere?



Ans: - If the metal ring is cut, an electromotive force (e.m.f.) will be induced, but no induced current can flow. Consequently, there will be no opposing force acting on the falling magnet. As a result, the acceleration of the falling magnet will remain equal to 'g' throughout the motion.

(VI) Producing Induced E.M.F. by changing the Area A :- Motional Electromotive Force :-



Let us Consider a Uniform magnetic field (B) ⊥ to the plane of paper and Coil PQRS in this magnetic field.

Let the Conductor PQ be moved toword the left with a Constant velocity (v). The area enclosed by the loop PQRS decreases. An emf. is induced in the loop as detected due to change in magnetic flux.

Here RO = x and PO = RS = l.

Now magnetic flux

$$\phi = B \times Area$$

$$\phi = B l x$$

Here (x) changes with time, amount of magnetic flux change and production of e.m.f.

$$e = -\frac{d\phi}{dt} = -\frac{d}{dt}(Blx)$$

$$e = Bl\left(-\frac{dx}{dt}\right) = Blv$$

$$\boxed{e = Blv}$$

where $\left[-\frac{dx}{dt} = v\right]$ is the velocity of the Conductor PQ toward the left.

(VII) Induced e.m.f. by Changing Relative orientation of Coil and magnetic field:-

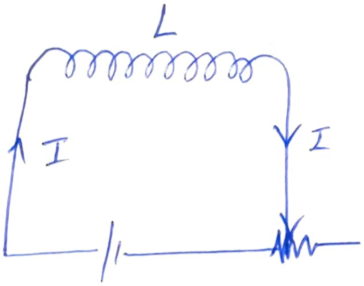
When a Coil is Ratated in a magnetic field, the angle (θ) between normal to the Coil and direction of magnetic field changes. ∴ magnetic flux linked with the Coil Changes and hence an e.m.f. is induced in the Coil.

e.g. This is basis of an AC. generator.

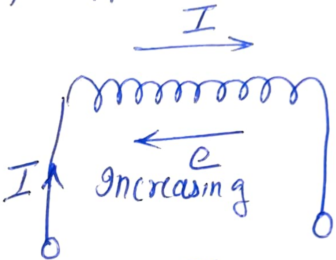
VIII SELF INDUCTION:-

Self Induction is the property of a Coil by virtue of which, the Coil opposes any change in the strength of current flowing through it by Inducing an e.m.f. in itself.

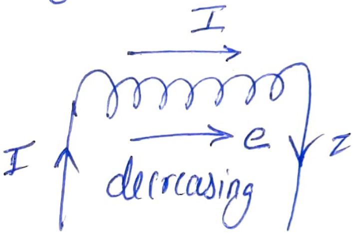
Self Induction is also called the Inertia of electricity.



if current in the Coil is changed a self Induction e.m.f appears in the Coil.



1) when current (I) is increasing, the self Induction e.m.f (e) appers opposite direction and it opposes the increases current.



2) when current (I) decreasing the self Induction e.m.f change the direction and opposes the decrease of current.

Coefficient of Self Induction :-

Suppose I = Strength of current
ϕ = amount of magnetic flux linked

$$\phi \propto I \Rightarrow \phi = LI$$

L = Constant (Coefficient of Self Induction) and It depends upon number of turns, area of cross-section and nature of material.

if $I = 1A$

ie $\phi = L$

$\therefore$ Coefficient of self Induction of a Coil is numerically equal to the amount of magnetic flux linked with the Coil when unit current flowing through the Coil.

We know that

$$e = -\frac{d\phi}{dt} = -\frac{d(LI)}{dt}$$

$$\boxed{e = -L\frac{dI}{dt}}$$

if $\frac{dI}{dt} = 1$ then $\boxed{e = -L}$

or $\boxed{L = -e}$

So Coefficient of self Induction of a Coil is equal to the e.m.f Induced in the Coil when Rate of change of current in Coil is unity.

S.I Units: $L = \frac{-e}{dI/dt} = \frac{\text{Volt}}{\text{Ampere/second}}$

$= \frac{\text{Volt Second}}{\text{Ampere}}$ or henry

dimensions :-

$$L = \frac{-e\,dt}{dI}$$

$$L = \frac{W}{q} \times \frac{dt}{dI} \qquad \boxed{e = \frac{W}{q}}$$

$$= \frac{[ML^2T^{-2}]}{[AT]} \times \frac{[T]}{[A]}$$

$$= [ML^2T^{-2}A^{-2}]$$

Example :- What e.m.f. will be induced in a 10H Inductor in which current changes from 10A to 7A in 9×10^{-2} s ?

Sol :- Here $e = ?$, $L = 10H$, $I_1 = 10A$, $I_2 = 7A$, $dt = 9 \times 10^{-2}$ s

$$e = -\frac{L\,dI}{dt}$$

$$= -\frac{10(I_2 - I_0)}{dt} = \frac{-10(7A - 10A)}{9 \times 10^{-2}}$$

$$= \frac{-10(-\cancel{3}) \times 10^2}{\cancel{9}\,3} = \frac{1000}{3} = 333.3 \text{ Volt}$$

$$\boxed{e = 333.3 \text{ Volt}}$$

Example 2:- A Coil of Inductance 0.5H is Connected to a 18V battery. Calculate the Rate of growth of current.

Solution:- $L = 0.5H$, $e = E = 18V$, $\frac{dI}{dt}$

$$e = \frac{dI}{dt}$$

$$\frac{dI}{dt} = \frac{e}{L}$$

$$= \frac{18}{0.5} = \frac{\cancel{180}^{36}}{\cancel{5}}$$

$$\boxed{\frac{dI}{dt} = 36\ As^{-1}}.$$

ix) Self Inductance of a long Solenoid:-

Let us Consider a long Solenoid
The magnetic field (B) at any Point inside it

$$B = \frac{\mu_0 N I}{l} \quad — ①$$

μ_0 = absolute Permeability.
l = length of solenoid
N = total number of turns.

∴ Magnetic flux through it (each turn)

$$B \times \text{area} = \left(\mu_0 \frac{N}{l} I\right) A$$

Total Magnetic flux

$$\phi = \left(\mu_0 \frac{N}{l} I\right) A \times N \quad \text{— (ii)}$$

if L is Coefficient of self Inductance of the Solenoid

$$\phi = L I \quad \text{— (iii)}$$

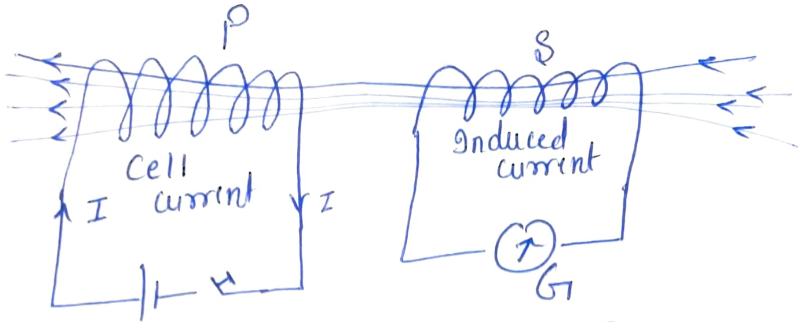
Comparing (ii) and (iii)

$$L \cancel{I} = \left(\mu_0 \frac{N}{l} \cancel{I}\right) A \times N$$

$$\boxed{L = \frac{\mu_0 N^2 A}{l}}$$

(X) Mutual Induction :-

Mutual Induction is the property of two Coils by virtue of which each opposes any change in the strength of current flowing through the other by developing an induced e.m.f.



Let us Consider two Coil P and S. where (P) is Connected with battery and (S) Connected with galvanometer.

on Pressing or releasing K, galvanometer shows sudden temporary deflection.

on pressing K, Current in (P) increases from zero to maximum value.

∴ magnetic flux linked with (P) is Increases as (S) is Close by, magnetic flux associated with (S) also Increase.

According Lenz's law the Induced current in (S) would oppose Increase in current in (P) by flowing in a opposite direction.

Coefficient of Mutual Inductance :-

Suppose

I = strength of current

ϕ = total amount of magnetic flux.

ie $\phi \propto I$

or $\boxed{\phi = MI}$

where M = Coefficient of mutual Induction.

if $I = 1$ ie $\boxed{\phi = M}$

Coefficient of Mutual Inductance of two Coils is numerically equal to the amount of magnetic flux linked with one Coil when unit current flows through the neighbouring Coil.

The emf Induced

$$e = -\frac{d\phi}{dt}$$

$$e = -\frac{d}{dt}(MI)$$

$$e = -M\frac{dI}{dt}$$

if $\frac{dI}{dt} = 1$ ie $\boxed{e = -M}$

Coefficient of mutual Inductance of two Coil is equal to the emf Induced in one coil when Rate of Change of current through the other Coil is unity.

The Coefficient of mutual Inductance of the coils depends :-

(I) Geometry of two Coils (Size, Shape, number of turns, Nature of material)

(II) distance between two Coils.

(III) Relative placement of two Coils.

Example :- A current of 10 A in primary of a circuit is reduced to zero at a uniform Rate in 10^{-3} s. if Coeff. of mutual Inductance is 3H, what is the induced e.m.f. in the Secondary?

Sol Here $I_1 = 10A$, $I_2 = 0$

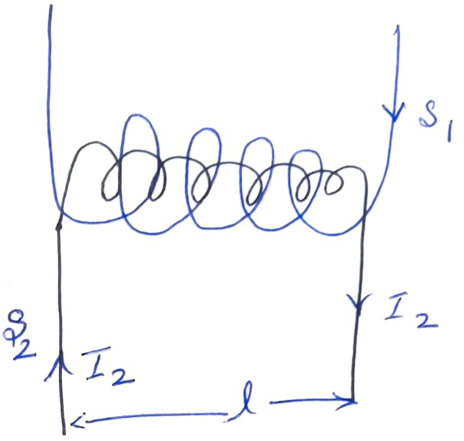
$dt = 10^{-3}s$, $M = 3$, $e = ?$

$$e = -M\frac{dI}{dt}$$

$$= -\frac{M(I_2 - I_1)}{dt}$$

$$e = \frac{-3(0-10)}{10^{-3}} = \underline{3\times10^{4}V}$$

(XI) Mutual Inductance of two Long Co-axial Solenoids ;



let us Consider two Co-axial long Solenoids of length (l)

n_1, r_1 be the number and Radius of Solenoid (S_1)

and n_2, r_2 be the number and Radius of Solenoid (S_2)

let I_2 = Current through (S_2)

ϕ_1 = Varying magnetic flux through S_1.

$$\therefore \phi_1 = M_{12}(I_2) \quad — ①$$

M_{12} = Coefficient of Mutual Induction.

Magnetic field due to (I_2) in (S_2)

$$B_2 = \mu_0 n_2 I_2$$

$\therefore$ Magnetic flux through (S_1)

$$\phi_1 = B_2 A_1 N_1$$

[$N_1 = n_1 l$ = total no. of turns]

$$\phi_1 = (\mu_0 n_2 I_2) A_1 (n_1 l)$$

$$\phi_1 = \mu_0 n_1 n_2 \pi r_1^2 l I_2 \quad \text{— (II)}$$

from (I) and (II)

$$M_{12} = \mu_0 n_1 n_2 \pi r_1^2 l \quad \text{— (III)}$$

if material of Relative magnetic permeability μ_r

$$M_{12} = \mu_r \mu_0 n_1 n_2 \pi r_1^2 l \quad \text{— (IV)}$$

Similarly

$$M_{21} = \mu_r \mu_0 n_1 n_2 \pi r_2^2 l \quad \text{— (V)}$$

No flux is ounde the Solenoid S_1

$$\text{ie } M_{21} = \mu_r \mu_0 n_1 n_2 \pi r_1^2 l \quad \text{— (VI)}$$

$$\text{ie } \boxed{M_{12} = M_{21} = \mu_r \mu_0 n_1 n_2 \pi r_1^2 l}$$

Here $n_1 = \frac{N_1}{l}$, & $n_2 = \frac{N_2}{l}$

$$M = \mu_0 \left(\frac{N_1}{l}\right)\left(\frac{N_1}{\cancel{l}}\right) \pi r_1^2 \times \cancel{l}$$

$$\boxed{M = \frac{\mu_0 N_1 N_2 A}{l}}$$

Problem:- Two circular Coils, on of small Radius (r_1) and the other of very large Radius (r_2) are placed Co-axially with Centres Conduciding. obtain the mutual Inductance of he arrangment?

Sol

Suppose a time varying current (I_2) is made to flow through the outer circular Coil.

∴ Magnetic field at the Centre of the Coil.

$$B_2 = \frac{\mu_0 I_2}{2r_2}$$

As the inner Coil placed Co-axially has very small Radius

the flux on it

$$\phi_1 = \pi r_1^2 B_2$$

$$= \pi r_1^2 \left(\frac{\mu_0 I_2}{2r_2}\right)$$

$$\phi_1 = \left(\frac{\mu_0 I r_1^2}{2r_2}\right) I_2$$

$$\boxed{\phi_1 = M_{12} I_2}$$

$$\therefore \left[M_{12} = \frac{\mu_0 I r_1^2}{2r_2}\right] = M_{21}.$$

Problem:- A 2m long Solenoid with diameter 4cm and 2000 turns has a Secondary of 1000 turns wound Closely near its mid Point, Calculate the mutual Inductance between the two Coils.

Sol:- Here

$$l = 2m, \quad r = \frac{4}{2} = 2\,cm = 2\times10^{-2}\,m$$

$$N_1 = 1000, \quad N_2 = 2000, \quad M = ?$$

$$M = \frac{\mu_0 N_1 N_2 A}{l} = \frac{4\pi\times10^{-7}\times1000\times\overset{1000}{\cancel{2000}}\times\pi\times4\times10^{-4}}{\cancel{2}}$$

$$M = 16\pi^2\,10^{-5}$$

$$M = 16\times\frac{22}{7}\times\frac{22}{7}\times10^{-5}$$

$$\boxed{M = 1.58\times10^{-3}\,H}$$