Physics

Chapter: - 15th

Waves

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Physics

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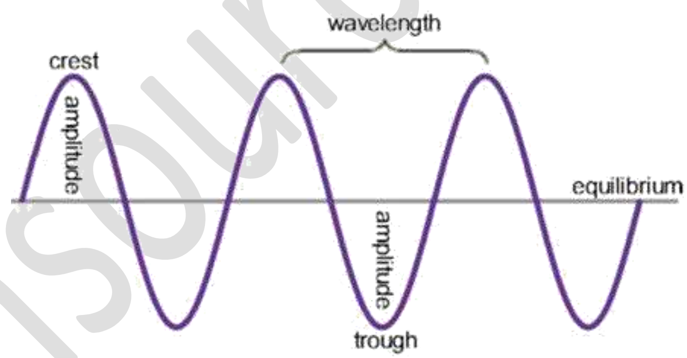
Waves

I. **Wave Motion: -** Wave motion refers to the transfer of energy from one point to another without the transfer of material, in the form of oscillations or disturbances that propagate through a medium or in some cases even in a vacuum (like electromagnetic waves).

Type of wave motion: - Depending on the relationship between the direction of oscillation of individual particles and wave propagation, the waves are classified as: transverse and longitudinal waves.

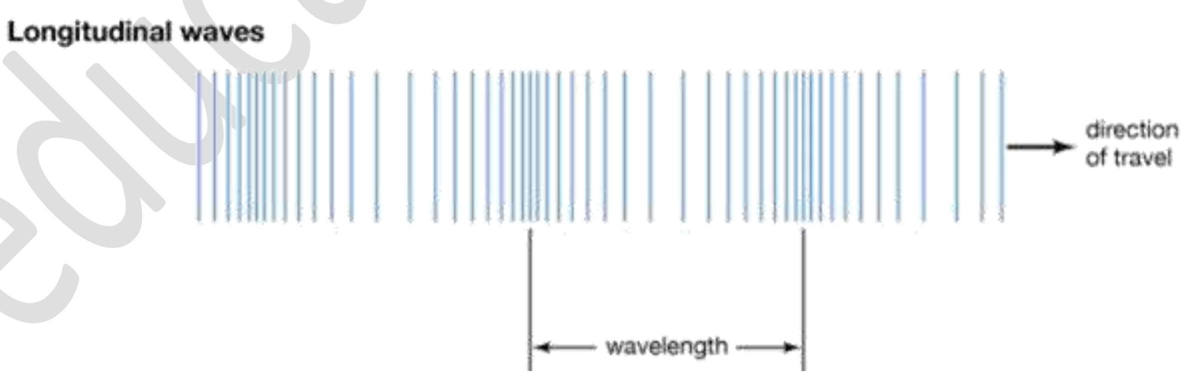
a) **Transverse wave motion**: - It is that wave motion in which the individual particles of the medium execute simple harmonic motion about their mean positions in a direction perpendicular to the direction of propagation of the wave.

e.g., Waves in string, ripples on the surface of water, etc. This form of wave motion travels in the form of **crests** and **troughs.**



b) **Longitudinal Wave: -** The waves in which the individual particles of the medium execute simple harmonic motion about their mean positions along the direction of propagation of the wave are called longitudinal waves.

e.g., Waves in spring, sound waves, etc.



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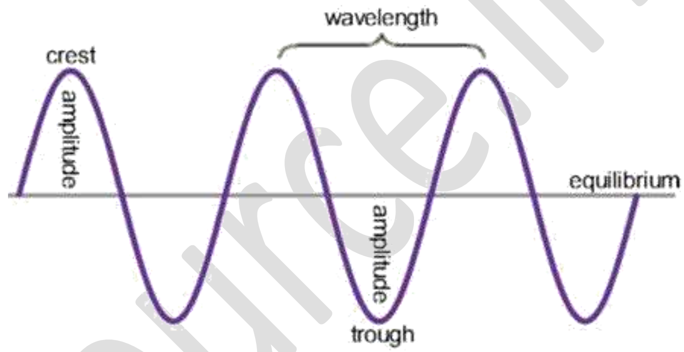
II. Some Important Terms Connected with Wave Motion: -

a) **Amplitude: -** The amplitude of a wave is the magnitude of maximum displacement of the particles from their equilibrium position, as the wave passes through them.

b) **Wavelength: -** Wavelength of a wave is the length of one wave or refers to the distance between two consecutive peaks (crests or troughs) of a wave.

The wavelength can be found by dividing the speed of the wave by its frequency:



$$\lambda = \frac{v}{f}$$

Where:
λ is the wavelength.
v is the speed of the wave.
f is the frequency of the wave.

c) **Angular Wave number: -** Angular wave number of a wave is also called propagation constant of the wave.
It is 2π times the number of waves that can be accommodated per unit length.

$$K = \frac{2\pi}{\lambda}$$

SI units of K: - Radian/m or simply m^{-1}.

d) **Frequency: -** Frequency of vibration of a particle is defined as the number of vibrations completed by particle in one second. As one vibration is equivalent to one wavelength.

Or

Frequency of a wave as the number of complete wavelengths traversed by the wave in one second.

Angular frequency: - Angular frequency of the wave is 2π times the frequency of the wave. It is represented by ω and is measured in $rad s^{-1}$.

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e) **Time period: -** We know that time period of vibration of a particle is defined as the time taken by the particle to complete one vibration about its mean position. As one vibration is equivalent to one wavelength, therefore,

Time period of a wave is equal to time taken by the wave to travel a distance equal to one wavelength.
It is represented by T.

Relation between ν and T: -

$$\text{Time} = \frac{1}{Frequency}$$

$$T = \frac{1}{\nu}$$

III. Character sticks of Wave Motion: -

a) Wave motion is a sort of disturbance which travels through a medium.
b) A material medium is essential for the propagation of mechanical waves.
c) When a wave motion passes through a medium, particles of the medium only vibrate simple harmonically about their mean position. They do not leave their position and move with the disturbance.
d) There is a continuous phase difference amongst successive particles of the medium.
e) The velocity of the particles during their vibration is different at different positions.
For example: - All the particles cross their mean position with maximum velocity and at extreme positions, their velocity is zero.
f) The velocity of wave motion through a particular medium is constant. It depends only on the nature of the medium.
g) Energy is propagated along with the wave motion without any net transport of the medium.

IV. Speed of a Transverse wave in a solid: -

$$V = \nu\lambda = \frac{\lambda}{T}$$

The above equation is general relation for all progressive waves, shows that the wave pattern travels a distance equal to the wavelength of the wave.

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V. Displacement Relation for a progressive wave: -

Let us Consider the wave travelling in positive x-direction. The displacement y(x, t) denotes the transverse displacement of the element at position x at time t and is given by Displacement,

$$y(x, t) = a \sin (kx - \omega t + \phi) \qquad ...(i)$$

If the wave travelling in the negative direction of X-axis can be represented by

$$y(x,t) = a \sin (kx + \omega t + \phi) \qquad ...(ii)$$

Where,

y(x, t)= displacement of vibrating element or particle as a function of position x and time t.

and the quantity **$(kx - \omega t + \phi)$** is called the phase of the wave.

VI. The principle of superposition of waves: -

When any number of waves meet simultaneously at a point in a medium, the net displacement at a given point and given time is the algebraic sum of the displacements due to each wave at the given point at the same time.

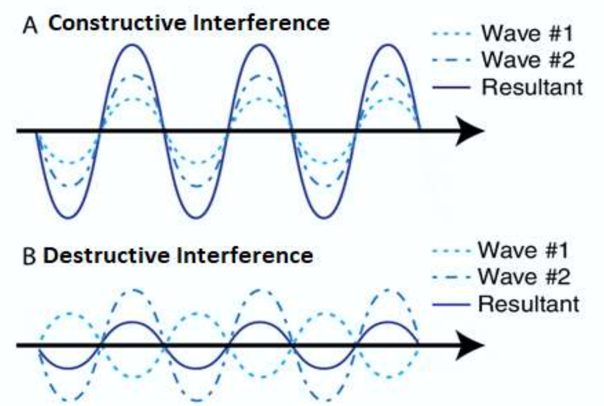
Displacement of two different waves is $y_1(x,t)$ and $y_2(x,t)$ superimpose to each other.

i.e., the final wave is **$y(x,t) = y_1(x,t) + y_2(x,t)$.**

Two types of Interference of wavs: -

(a) Constructive Interference.

(b) Destructive Interference



a) **Constructive Interference: -** when the two waves overlap to each other in phase means crust or trough of one wave fall on the crust or trough of another wave.
   Phase Difference, **$\phi = 2n\pi$**
   Path difference, **$\Delta x = n\lambda$**

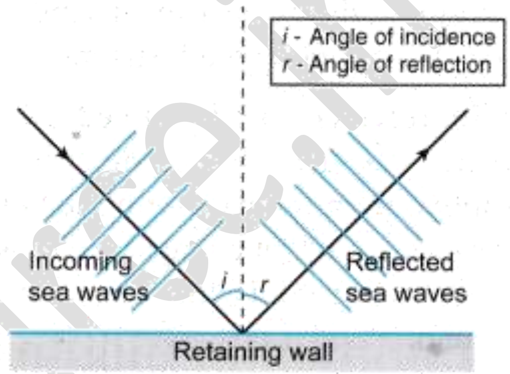
b) **Destructive Interference: -** when two waves overlap to each other in out of phase like crust or trough of one wave fall on trough or crust of another wave.
   Phase Difference, **$\phi = (2n + 1)\pi$**
   Path difference, **$\Delta x = (2n + 1)\lambda/2$**
   Where n = 0,1,2,3,…… and λ is wave length.

VII. Reflection of Waves: Reflection of waves refers to the phenomenon where wavefronts change direction upon striking a boundary that does not absorb the energy of the wave. The angle of incidence, which is the angle at which the wave strikes the boundary, is equal to the angle of reflection.

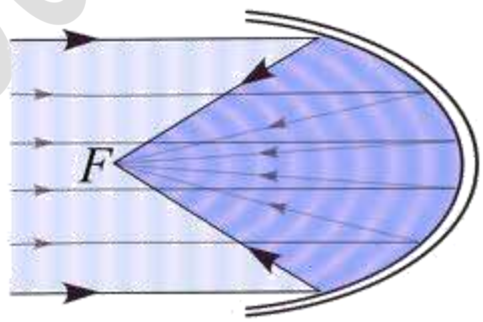
Reflection of Waves from Different Surfaces:

1. **Plane Surface**:
   - Waves reflecting off a flat or plane surface will do so in a uniform manner.
   - For example, light reflecting off a flat mirror will produce a clear and undistorted reflection.

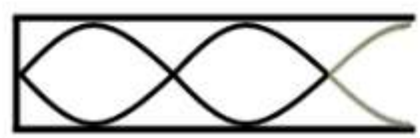
   ∠I = ∠r



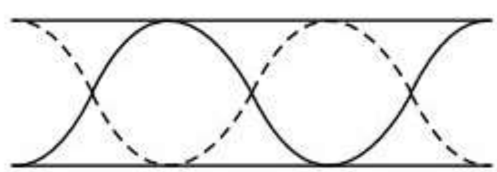
2. **Concave Spherical Surface**:
   - A concave surface curve inward.
   - Waves reflecting off a concave surface converge at a focal point.
   - For instance, light reflecting off a concave mirror will converge at a point called the focal point.



3. **Closed-end**:
   - Pertains mostly to tubes or pipes and is commonly associated with sound waves.
   - When a wave reflects off a closed end, there is usually a phase reversal. For example, in a closed-end pipe, a compression wave will reflect as a rarefaction and vice versa.



4. **Open-end**:
   - Like the closed end, this also pertains mainly to tubes or pipes.
   - Waves reflecting off an open end will do so without a phase change. In the context of sound waves in an open pipe, a compression wave will reflect as a compression.



Applications:

1. **Plane Surface**:
    - **Mirrors**: Used in various daily activities, and also in devices like periscopes.
    - **Solar panels**: Some designs use flat surfaces to reflect sunlight onto a specific area.
2. **Concave Spherical Surface**:
    - **Telescopes**: Concave mirrors can focus light from distant stars and galaxies onto a specific point, aiding in observation.
    - **Headlights**: Car headlights use concave mirrors to focus light into a beam.
3. **Closed-end**:
    - **Musical Instruments**: Instruments like clarinets function based on the reflection of sound waves in a tube with one closed end.
    - **Noise Control**: Understanding wave reflection can help in designing spaces or devices that minimize noise.
4. **Open-end**:
    - **Organ Pipes**: Many organ pipes are open at both ends, and the sound they produce is based on the reflection and interference of sound waves in the pipe.
    - **Wind Instruments**: Flutes and some other wind instruments rely on open-ended tubes to produce sound.

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(v) Speed of a Transverse wave in a solid :-

The speed of Transverse wave (v) on a string depends upon

(i) linear mass density (mass per unit length)

(ii) tension (T) in the string.

ie $V \propto m^a T^b$

$V = k m^a T^b$ —— ①

Using dimension Analysis

(velocity) $V = [M^0 L T^{-1}]$

(mass per unit length) $m = [M L^{-1} T^0]$

(Tension) $T = [M L T^{-2}]$

Using in equation ①.

$$[M^0 L T^{-1}] = k [M L^{-1} T^0]^a [M L T^{-2}]^b$$

$$[M^0 L T^{-1}] = k [M^{a+b} L^{-a+b} T^{-2b}]$$

Comparing both side.

$$a + b = 0 \quad \text{—— (a)}$$

$$-a + b = 1 \quad \text{—— (b)}$$

$$+2b = +1$$

$$\boxed{b = \tfrac{1}{2}} \quad \text{—— (c)}$$

using (b) in (a)

$$a + \tfrac{1}{2} = 0$$

$$\boxed{a = -\tfrac{1}{2}}$$

Using the value of a and b in (1)

$$v = k\, m^{-1/2}\, T^{1/2}$$

$$v = k \sqrt{T/m}$$

Consider $(k = 1)$

$$\boxed{v = \sqrt{\frac{T}{m}}}$$

Now speed of transverse wave in a solid

$$\boxed{v = \sqrt{\frac{n}{\rho}}}$$

where n = modulus of rigidity and ρ is density of material of the solid.

Speed of Longitudinal waves :-

we know that in Longitudinal wave there is forward and backward motion in the propagation of the wave. we also said the longitudinal waves are pressure waves. during the motion there is change in volume $\left(\frac{\Delta V}{V}\right)$ when pressure is change (ΔP).

ie Bulk Modulus

$$B = \frac{\Delta P}{\Delta V / V} \quad \text{——— (I)}$$

Now velocity of longitudinal waves depends upon

$$v \propto B^a \rho^b$$

$$v = k \cdot B^a \rho^b \quad \text{——— (II)}$$

Using dimension Analysis

(velocity) $v = [M^0 L T^{-1}]$

(Bulk Modulus) $B = [M L^{-1} T^{-2}]$

(density) $\rho = [M^1 L^{-3} T^0]$

using in equation (II)

$$[M^0 L T^{-1}] = k [M L^{-1} T^{-2}]^a [M^1 L^{-3} T^0]^b$$

$$[M^0 L T^{-1}] = k [M^a L^{-a} T^{-2a}] [M^b L^{-3b} T^0]$$

$$[M^0 L T^{-1}] = k[M^{a+b} L^{-a-3b} T^{-2a}]$$

Comparing all values

$a+b=0$ —— (1)

$-a-3b=1$ —— (2)

$-2a=-1$

$\boxed{a=\frac{1}{2}}$

using in (1)

$\frac{1}{2}+b=0$

$\boxed{b=-\frac{1}{2}}$

using the value of a and b in (11)

$$v = k\sqrt{\frac{B}{\rho}}$$

using $k=1$

$$\boxed{v=\sqrt{B/\rho}}$$

this is the speed in fluid.

Note :- speed in solid

$$v=\sqrt{\frac{Y}{\rho}}$$ (∵ Y = young's Modulus)

Newton's Formula for velocity of sound in Gases:-

The equation of velocity of sound is

$$v = \sqrt{B/\rho} \quad \text{—— (i)}$$

B = Bulk modulus of the gas and ρ = density of gas.

Newton assumed that during the motion of the sound through gas Compression there is lost of heat and ~~rare~~fraction there is gain of energy. He consider entire process isothermal.

$$\text{i.e. } v = \sqrt{B_i/\rho} \quad \text{—— (ii)}$$

Calculation of B_i

We know that for isothermal

$$PV = \text{Constant}$$

differentiating

$$PdV + VdP = 0$$

$$PdV = -VdP$$

$$P = -\frac{VdP}{dV} \Rightarrow -\frac{dP}{dV/V} = B_i$$

$$\boxed{P = B_i}$$

from equation (ii)

$$\boxed{v = \sqrt{P/\rho}}.$$

⇒ **Error In Newton's Formula**:-

speed of light acc. to Newton's formula

$P = h\rho' g$ and $h = 0.76m$

$\rho' = 13.6 \times 10^{-3}\ kg\,m^{-3}$

ρ (density of air) $= 1.293\ kg/m^3$ $\quad g = 9.8\ m/s^2$

$$v = \sqrt{P/\rho}$$

$$v = \sqrt{\frac{0.76 \times 13.6 \times 10^{-3} \times 9.8}{1.293}}$$

$$v = 280\ m/s$$

experimental speed of light = 332 m/s

ie $332 - 280 = 52\ m/s$

$$\text{Percentage Error} = \frac{52}{332} \times 100 = 15.7 \approx 16\%$$

∴ there is 16% error in velocity of sound.

LAPLACE's Correction in velocity of sound:-

In this Correction Laplace said that the Newton's assumption was wrong. The propagation of sound is not isothermal it is a adiabatic.

So there is no exchange of heat during Compression and rarefraction.

ie $V = \sqrt{\frac{B_a}{\rho}}$ —— (I)

Calculation of "B_a"

We know that

$$PV^{\gamma} = \text{Constant}$$

where $\gamma = C_p/C_v$

$$P(\gamma V^{\gamma-1} dV) + V^{\gamma} dP = 0$$

$$\gamma P V^{\gamma-1} dV = -V^{\gamma}(dP)$$

$$\gamma P = -\frac{V^{\gamma}}{V^{\gamma-1}}\left(\frac{dP}{dV}\right)$$

$$= -V^{\cancel{\gamma}-\cancel{\gamma}+1} \frac{dP}{dV}$$

$$\gamma P = \frac{-V\,dP}{dV} = \frac{-P}{dV/V} = B_a \text{ —— (II)}$$

from (i) and (iv)

$\therefore B_a = \gamma P$

ie $V = \sqrt{\frac{\gamma P}{\rho}}$

Here $\sqrt{P/\rho} = 280 m/s$

$V = \sqrt{\gamma} \times 280 = \sqrt{1.41} \times 280$

$\boxed{V = 332.5 \ m/s}$

⇒ **Factors Affecting Velocity of Sound :-**

a) **Effect of density :-** The velocity of sound in a gaseous medium is given by

$$V = \sqrt{\frac{\gamma P}{\rho}}$$

The velocity of sound in a gas is inversely proportional to the square root of density of the gas.

example :- density of oxygen 16 times the density of hydrogen.

ie $\rho_o = 16 \rho_H$

$$\frac{V_H}{V_o} = \sqrt{\frac{\rho_o}{\rho_H}} = \sqrt{\frac{16 \cancel{\rho_H}}{\cancel{\rho_H}}} = 4$$

$$\boxed{V_H = 4V_o}$$

the velocity of sound in Hydrogen is four time the velocity of sound in oxygen.

b) **Effect of Pressure :-** we know that

$$V = \sqrt{\frac{\gamma P}{\rho}}$$

Put: $\rho = \frac{M}{V}$ $\therefore V = \sqrt{\frac{\gamma P V}{M}}$

when $T =$ Constant, $PV =$ Constant

$\therefore V =$ Constant

Hence velocity of sound is independent of the change in pressure of the gas, provided temperature remain constant.

c) **Effect of Temperature :-**

we know that

$$V = \sqrt{\frac{\gamma P}{\rho}}$$

According to standard gas equation.

$$PV = RT$$

$$P = \frac{RT}{V}$$

$$v = \sqrt{\frac{\gamma RT}{\rho \times V}} = \sqrt{\frac{\gamma RT}{M}} \quad [\because \rho \times V = M]$$

Here $v \propto \sqrt{T}$

or $$\frac{V_t}{V_0} = \sqrt{\frac{T}{T_0}} \quad -① $$

Hence, velocity of sound in a gas is directly proportional to the square root of its absolute temperature.

__Temperature Coefficient of velocity of sound in air (α):-__

The temperature Coefficient of velocity of sound in air is defined as the change in the velocity of sound in air, when temperature change by 1°c.

Here V_0 = ~~Value~~ Initial velocity at 0°c

V_t = velocity of sound at t°c

ie $$\alpha = \frac{V_t - V_0}{t}$$

S.I. Unit of α is $ms^{-1}c^{-1}$.

equation ① become

$$\frac{V_t}{V_0} = \sqrt{\frac{T}{T_0}} = \sqrt{\frac{273+t}{273+0}} = \left(1 + \frac{t}{273}\right)^{1/2}$$

Expanding Binomially

$$\frac{V_t}{V_0} = \left(1 + \frac{1}{2}\frac{t}{273}\right)$$

$$\frac{V_t}{V_0} - 1 = \frac{t}{546} \Rightarrow \frac{V_t - V_0}{V_0} = \frac{t}{546}$$

$$\frac{V_t - V_0}{t} = \alpha = \frac{V_0}{546} \Rightarrow \alpha = \frac{332}{546} \quad [\because V_0 = 332 m/s]$$

$$\alpha = 0.608 \, m/sc$$

Hence, velocity of sound in air increases approximately by 0.61 m/s for every 1°c rise in temperature.

d) <u>Effect of Humidity</u> :- The presence of water vapour in air changes its density.

ρ_m = density of moist air,

ρ_d = density of dry air,

V_m = velocity of sound in moist air,

V_d = velocity of sound in dry air.

Let us assume that effect of γ is negligible.

$$V_m = \sqrt{\frac{\gamma P}{\rho_m}} \quad \text{and} \quad V_d = \sqrt{\frac{\gamma P}{\rho_d}}$$

dividing

$$\boxed{\frac{V_m}{V_d} = \sqrt{\frac{\rho_d}{\rho_m}}}$$

Here

$\rho_m < \rho_d$ [because water vapours reduces the density of air]

ie $\boxed{V_m > V_d}$

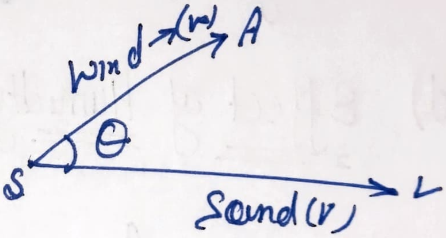
Hence, velocity of sound in moist air is greater than the velocity of sound in dry air.

e) Effect of Wind velocity:- The velocity of sound in air is affected by the velocity of wind.

The velocity of sound in a medium is the algebraic sum of velocity of sound and Component of wind velocity.

V = Velocity of sound

w = Velocity of wind



∴ Resultant velocity

$\boxed{SL = V + w\cos\theta}$

⇒ Progressive wave:- A wave which travels Continuously in the same direction ~~from~~ without any Change in its amplitude is called progressive wave.

To show the principle let us consider two waves of same frequency, $\left(\omega = \frac{2\pi}{T}\right)$

angular wave length $k = 2\pi/\lambda$

amplitude $= a$

and consider that moving in positive x-direction.

$$y_1(x,t) = a\sin(\omega t - kx) \quad \text{—(I)}$$

$$y_2(x,t) = a\sin(\omega t - kx + \phi) \quad \text{—(II)}$$

According to superposition

$$y = y_1 + y_2$$

$$= a\sin(\omega t - kx) + a\sin(\omega t - kx + \phi)$$

$$= a[\sin(\omega t - kx) + \sin(\omega t - kx + \phi)]$$

using $\sin\alpha + \sin\beta = 2\cos\left(\frac{\alpha-\beta}{2}\right)\sin\left(\frac{\alpha+\beta}{2}\right)$

$$y(x,t) = a\left[2\cos\left(\frac{\cancel{\omega t} - \cancel{kx} - \cancel{\omega t} + \cancel{kx} - \phi}{2}\right)\right.$$

$$\left.\sin\left(\frac{\omega t - kx + \omega t - kx + \phi}{2}\right)\right]$$

$$y(x,t) = 2a\cos\frac{\phi}{2}\sin\left(\frac{2\omega t - 2kx + \phi}{2}\right)$$

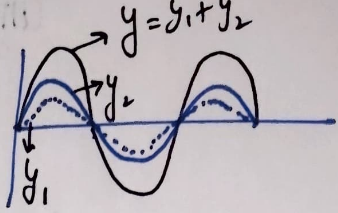
$$y(x,t) = 2a\cos\phi/2\,\sin\left(\omega t - kx + \phi/2\right) \quad \text{—(III)}$$

Here $R = 2a\cos\frac{\phi}{2}$

if $\phi = 0$ ie the two waves are in phase

ie Constructive Interference

ie $\boxed{y = 2a \sin(\omega t - kx)}$

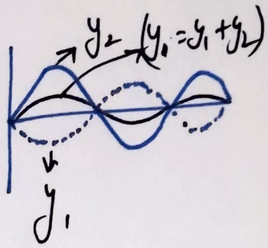


if $\phi = 180$ ie the two wave are out of phase

ie distructive Interference

ie $y = -2a \sin(\omega t - kx + 90)$

$\boxed{y = -2a \cdot \cos(\omega t - kx)}$



⇒ Standing waves In strings AND Normal Modes of Vibration :

when two sets of progressive wave trains of the same type having same amplitude and same time period/wavelength/frequency travelling with same speed along a straight line in opposite direction superimpose, a new set of waves are formed. These are called Stationary or standing waves.

Let us consider a string of length (L). ~~fixed from~~ both ends of the string are fixed.

Pulse moves from left to right

$$y_1(x,t) = a \sin(\omega t - kx)$$

and from right to left

$$y_2(x,t) = a \sin(\omega t + kx + \pi)$$

$$= -a \sin(\omega t + kx)$$

According to superposition

$$y = y_1 + y_2$$

$$y = y_1(x,t) + y_2(x,t)$$

$$= a \sin(\omega t - kx) - a \sin(\omega t + kx)$$

$$= -a\left[\sin(\omega t + kx) - \sin(\omega t - kx)\right]$$

using $\sin\alpha - \sin\beta = 2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$

$$y = -a\left[2\cos\left(\frac{\omega t + kx + \omega t - kx}{2}\right)\sin\left(\frac{\omega t + kx - \omega t + kx}{2}\right)\right]$$

$$y = -2a\cos\left(\frac{2\omega t}{2}\right)\sin\left(\frac{2kx}{2}\right)$$

$$y = -2a\cos\omega t \sin kx$$

$$\boxed{y = -(2a\sin kx)\cos\omega t}$$

($2a\sin kx$ = Amplitude)

At one end of the string; where $x=0$

$$y = -2a \sin(0) \cos\omega t$$

$$y = 0$$

$\therefore$ This end is a node.

At the other end of the string $x=L$

$$y = -2a \cos\omega t \sin kL$$

as this end is fixed

ie $y=0$ (node)

$$-2a \cos\omega t \sin kL = 0$$

$$\sin kL = 0 = \sin n\pi \quad (\text{where } n = 0, 1, 2, 3 \ldots)$$

$$kL = n\pi \qquad \left(k = \frac{2\pi}{\lambda}\right)$$

$$\frac{2\pi}{\lambda} L = n\pi$$

$$\lambda = \frac{2\cancel{\pi} L}{n\cancel{\pi}}$$

$$\boxed{\lambda = \frac{2L}{n}} \quad (n = 1, 2, 3 \ldots)$$

The value of n corresponds to 1st, 2nd, 3rd normal modes of vibration of the string.

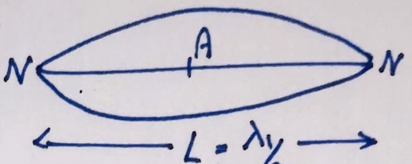
(I) First Normal mode of Vibration :-

ie $n = 1$

$\lambda_1 = \frac{2L}{1} = 2L \Rightarrow \boxed{L = \lambda/2}$

The string vibrates as a whole in one segment.

frequency $\nu_1 = \frac{v}{\lambda_1} = \frac{v}{2L}$



Here $v = \sqrt{T/m}$ [T = Tension, m = mass per unit string]

$$\boxed{\nu_1 = \frac{1}{2L}\sqrt{\frac{T}{m}}}$$

This is known as normal mode of vibration is called fundamental mode.

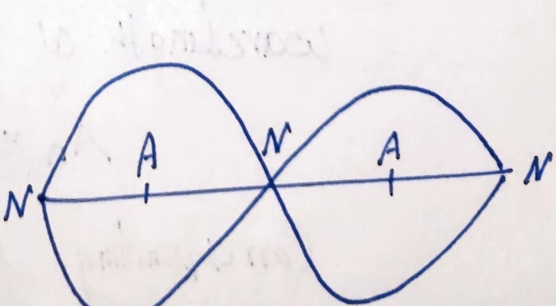
This is also known as first harmonic.

(II) Second Normal mode of Vibration :-

ie $n = 2$

$\lambda_2 = \frac{2L}{2} = L$

$\boxed{\lambda_2 = L}$



frequency $\nu_2 = \frac{v}{\lambda_2} = \frac{v}{L}$

$\nu_2 = \frac{2v}{2L} \Rightarrow \boxed{\nu_2 = 2\nu_1}$ $\left[\because \nu_1 = \frac{v}{2L}\right]$

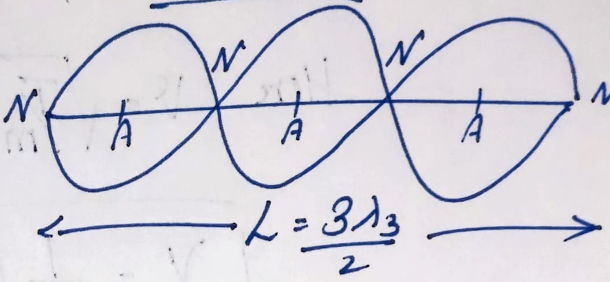
ie frequency of vibration of string becomes twice the fundamental frequency or second Harmonic or first overtone.

(III) Third Normal mode of Vibration :-

ie $n = 3$

$$\lambda_3 = \frac{2L}{3} \text{ or } \boxed{L = \frac{3\lambda_3}{2}}$$

String vibrate in three segments of equal length.



frequency

$$\nu_3 = \frac{v}{\lambda_3} = \frac{v}{2L/3} = 3\left(\frac{v}{2L}\right) = 3\nu_1$$

$$\boxed{\nu_3 = 3\nu_1}$$

This is know as third harmonic or second overtone.

In general :-

wavelength of n^{th} mode of vibration

$$\lambda_n = \frac{2L}{n}$$

Corresponding frequency

$$\nu_n = \frac{v}{\lambda_n} = \frac{v}{2L/n} = n\left(\frac{v}{2L}\right)$$

$$\boxed{\nu_n = n\nu_1}$$

This frequency is n times fundamental frequency. or nth harmonic or $(n-1)^{th}$ overtone.

Laws of Vibrations of Stretched Strings:-

The fundamental frequency of vibration of a streched string

$$\nu = \frac{1}{2L}\sqrt{\frac{T}{m}}$$

We deduce the following laws:-

1) **Law of length**:- Here T and m = Constant

Then $\nu \propto \frac{1}{L}$

This is law of length.

i.e frequency of streched string inversely proportional to the length of the string (L)

2) **Law of tension**:- Here L and m are Constant

Then $\nu \propto \sqrt{T}$.

This is law of Tension.

The fundamental frequency of vibration of a stretched string is directly proportional to the square root of tension (T).

③ Law of mass :- if L and T are Constant

then $\nu \propto \frac{1}{\sqrt{m}}$

This is the law of mass

i.e. the fundamental frequency of vibration of a streched string inversely proportional to the square root of mass per unit length.

There are two additional laws.

(i) Law of diameter :- $\boxed{\nu \propto \frac{1}{D}}$

(ii) Law of density :- $\boxed{\nu \propto \frac{1}{\sqrt{\rho}}}$.

⇒ Standing Waves in Closed organ PIPES :-

organ Pipes are the musical Instruments which are used for producing musical sound by blowing air into the pipe.

There are two types of organ pipes:

① Closed orgnal pipes (one end is Close)
(ii) open orgnal pipes (both ends are open).

① **Close organ Pipe :-**

we know that sound wave : longitudinal wave

$$S(x,t) = -2a \sin kx \cos\omega t \quad \text{— ①}$$

$\downarrow$

displacement

At the Close end of the pipe, $x = 0$

ie $\sin kx = \sin(0) = 0$

$S = 0$ [ie a node is formed]

At the open end of the pipe of length (L)

ie ($S = L$)

an antinode to be formed ie $S = max$.

when $\sin kL = max = \pm 1$

$\sin kL = \sin(2n-1)\frac{\pi}{2}$

$kL = (2n-1)\frac{\pi}{2}$ $\quad [n = 1, 2, 3 \ldots]$

$\frac{2\pi L}{\lambda} = (2n-1)\frac{\pi}{2}$ $\quad \left[k = \frac{2\pi}{\lambda}\right]$

$\lambda = \frac{2\pi L}{(2n-1)\frac{\pi}{2}}$

$$\boxed{\lambda = \frac{4L}{(2n-1)}} \quad \text{— ⑪}$$

(1) First Normal mode of vibration

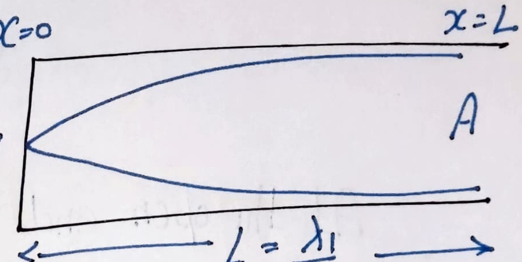
Let λ_1 be the wavelength of standing waves.

i.e. $n = 1$

$$\lambda_1 = \frac{4L}{(2-1)} = \frac{4L}{1}$$

$$\lambda_1 = 4L \Rightarrow L = \frac{\lambda_1}{4}$$

There is one node and one antinode.



The frequency of vibration

$$\nu_1 = \frac{v}{\lambda_1} = \frac{v}{4L}$$

$$\boxed{\nu_1 = \frac{v}{4L}}$$

This is the lowest frequency of vibration and is called the fundamental frequency. or fundamental mode or first Harmonic.

(II) Second normal mode of vibration :-

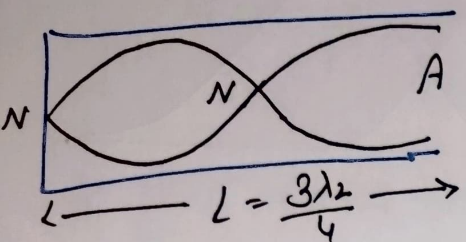
Let λ_2 be the wavelength of standing when $n = 2$

$$\lambda_2 = \frac{4L}{4-1} = \frac{4L}{3} \Rightarrow \lambda_2 = \frac{4L}{3} \text{ or } \lambda_2 = \frac{\lambda_1}{3}$$

$$L = \frac{3\lambda_2}{4}$$

There are two nodes and two antinodes

The frequency of vibration



$$\nu_2 = \frac{v}{\lambda_2} = \frac{v}{4L/3} = \frac{3v}{4L} = 3\nu_1$$

$$\boxed{\nu_2 = 3\nu_1}$$

The frequency of vibration in 2nd normal mode is Thrice the fundamental frequency. or Called third Harmonic or first overtone.

Third Normal mode of vibration

Let λ_3 be the wave length of standing wave when $(n=3)$

$$\lambda_3 = \frac{4L}{6-1} = \frac{4L}{5}$$

$$L = \frac{5\lambda_3}{4}$$

There are three nodes and three antinodes

frequency

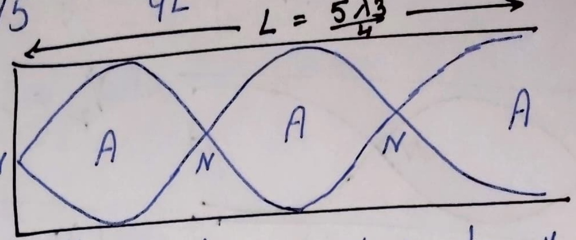
$$\nu_3 = \frac{v}{\lambda_3} = \frac{v}{4L/5} = \frac{5v}{4L}$$

$$\nu_3 = 5\left(\frac{v}{4L}\right) = 5\nu_1 \Rightarrow \boxed{\nu_3 = 5\nu_1}$$

ie the frequency of vibration

$$\nu_3 = \frac{v}{\lambda_3} = \frac{v}{4L/5} = 5\frac{v}{4L} = 5\nu_1$$

$$\boxed{\nu_3 = 5\nu_1}$$



i.e the frequency of vibration in the 3rd normal mode of is five time the fundamental frequency.

Now n^{th} Normal mode of vibration of closed orgnal pipe

$$\nu_n = \frac{(2n-1)v}{4L} = (2n-1)\nu_1$$

$$\boxed{\nu_n = (2n-1)\nu_1}$$

Standing waves in open organ Pipes:-

An open organ pipe is open at both ends.

∴ Antinode is formed at each end.

So in this case $S = max$

at $x = 0$ also at $x = L$

$$\lambda = \frac{2L}{n} \quad \text{or} \quad L = \frac{n\lambda}{2}$$

where $n = 1, 2, 3 ---$

(I) First Normal Mode of vibration :-

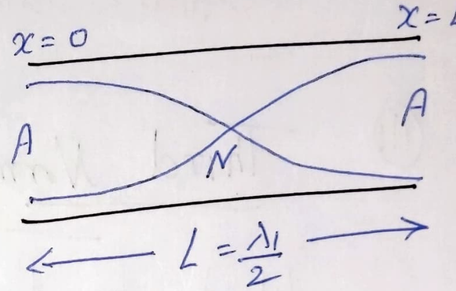
Let λ_1 be the wave length of standing wave when $n = 1$

$$\lambda_1 = \frac{2L}{1} \text{ or } \boxed{L = \lambda_1/2}$$

This mode of Vibration
There are two antinodes and one node

The frequency of vibration is

$$\nu_1 = \frac{v}{\lambda_1} = \frac{v}{2L}$$



$$\boxed{\nu_1 = \frac{v}{2L}}$$

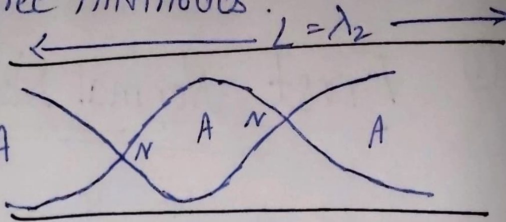
This is lowest frequency of vibration and is called the fundamental frequency.
The note of this frequency is called fundamental note or first harmonic.

(II) Second Normal mode of Vibration :-

Let λ_2 be the wave length of stationary waves when $n = 2$.

$$\lambda_2 = \frac{2L}{2} \Rightarrow \boxed{\lambda_2 = L}$$

There are two nodes and three Antinodes.



<u>frequency</u>

$$\nu_2 = \frac{v}{\lambda_2} = \frac{v}{L}$$

$$\nu_2 = \frac{2v}{2L} = 2\nu_1 \qquad \left[\nu_1 = \frac{v}{2L}\right]$$

$$\boxed{\nu_2 = 2\nu_1}$$

ie frequency of vibration in second normal mode is twice the fundamental frequency.

(iii) <u>Third Normal mode of Vibration</u>

Let λ_3 be the wavelength of standing waves when $(n = 3)$

$$\lambda_3 = \frac{2L}{3} \text{ or } L = \frac{3\lambda_3}{2}$$

This mode of vibration has Three nodes and four antinodes.

$$\nu_3 = \frac{v}{\lambda_3} = \frac{v}{\frac{2L}{3}} = \frac{3v}{2L}$$

$$\boxed{\nu_3 = 3\nu_1}$$

ie frequency of vibration in third normal mode of open organ pipe is thrice the fundamental frequency. or third Harmonic

In general

$$\boxed{\nu_n = n\nu_1}$$

or we can said that $(n-1)^{th}$ overtone.

⇒ **BEATS :-**

The phenomenon of alternate variation in the intensity of sound with time at a particular position, when two sound waves of nearly same frequency and amplitude superimpose on each other is called beats.

Analytical Method :-

Let us consider two wave trains of equal amplitude (a) and slightly ~~of~~ different frequency ν_1 and ν_2 travelling in a medium in the same direction

Let

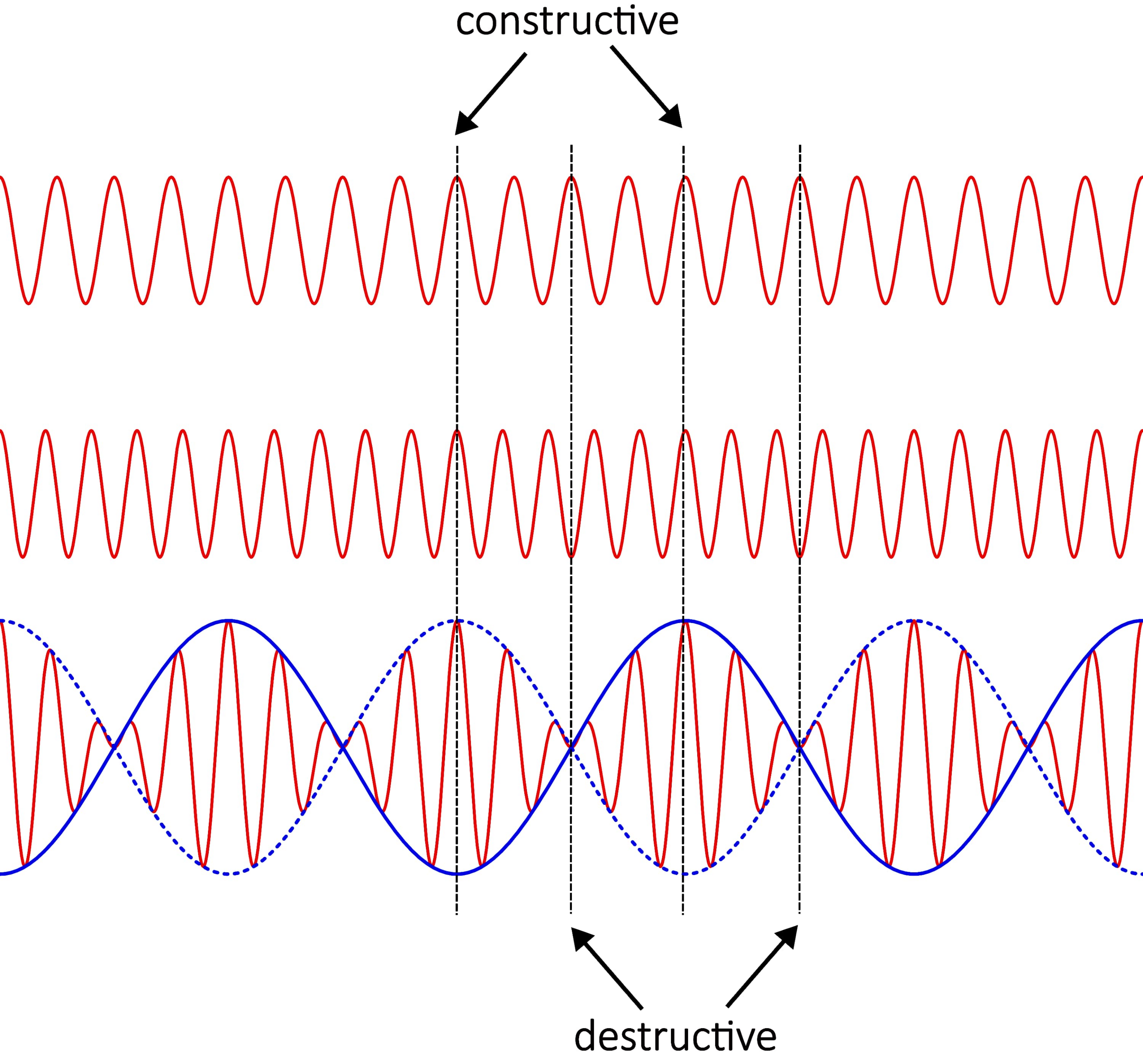
$$S_1 = a \sin \omega_1 t = a \sin 2\pi\nu_1 t$$

$$S_2 = a \sin \omega_2 t = a \sin 2\pi\nu_2 t$$

according superposition principle

$$S = S_1 + S_2 = a \sin 2\pi\nu_1 t + a \sin 2\pi\nu_2 t$$

$$= a(\sin 2\pi\nu_1 t + \sin 2\pi\nu_2 t)$$


constructive
destructive

using $\sin\alpha + \sin\beta = 2\cos\frac{\alpha-\beta}{2}\sin\frac{\alpha+\beta}{2}$

$$s = 2a\cos\pi(\nu_1-\nu_2)t\,\sin\pi(\nu_1+\nu_2)t$$

$$s = A\sin\pi(\nu_1+\nu_2)t$$

Here $\boxed{A = 2a\cos\pi(\nu_1-\nu_2)t}$

A = amplitude of final wave.

The amplitude (A) will be maximum when

$$\cos\pi(\nu_1-\nu_2)t = \max = \pm 1$$

$$\cos\pi(\nu_1-\nu_2) = \cos k\pi$$

$$\therefore\ k\cancel{\pi} = \cancel{\pi}(\nu_1-\nu_2)t$$

$$k = (\nu_1-\nu_2)t \Rightarrow \boxed{t = \frac{k}{\nu_1-\nu_2}}$$

Hence maximum amplitude $(k = 0, 1, 2, 3 \ldots)$

ie $t = 0, \frac{1}{\nu_1-\nu_2}, \frac{2}{\nu_1-\nu_2}, \frac{3}{\nu_1-\nu_2}, \ldots$

Time Interval between two successive maxima of sound

$$= \frac{1}{\nu_1-\nu_2} - 0 = \frac{1}{\nu_1-\nu_2}$$

ie frequency of maxima $= (\nu_1 - \nu_2)$

The amplitude (A) will be minimum

$$\cos \pi (\nu_1 - \nu_2) t = \text{minimum} = 0$$

$$\cos \pi (\nu_1 - \nu_2) t = \cos (2k+1) \frac{\pi}{2}$$

$$\pi (\nu_1 - \nu_2) t = (2k+1) \frac{\pi}{2}$$

$$\boxed{t = \frac{(2k+1)}{2(\nu_1 - \nu_2)}}$$

Hence minimum amplitude $(k = 0, 1, 2, 3 \ldots)$

$$t = \frac{1}{2(\nu_1 - \nu_2)}, \frac{3}{2(\nu_1 - \nu_2)}, \frac{5}{2(\nu_1 - \nu_2)} \ldots$$

Time interval between two successive minima

$$t = \frac{3}{2(\nu_1 - \nu_2)} - \frac{1}{2(\nu_1 - \nu_2)}$$

$$= \frac{3-1}{2(\nu_1 - \nu_2)} = \frac{\cancel{2}}{\cancel{2}(\nu_1 - \nu_2)}$$

$$\boxed{t = \frac{1}{\nu_1 - \nu_2}}$$

frequency $f = \frac{1}{t} = (\nu_1 - \nu_2)$

Formulas :-

1) wave length :- $\boxed{\lambda = \frac{v}{f}}$

2) Angular wave number :- $\boxed{k = \frac{2\pi}{\lambda}}$

3) a) frequency :- $f = \frac{1}{T}$

b) Angular frequency :- $\omega = 2\pi\nu$

4) Speed of transverse wave :

$$\boxed{v = \nu\lambda = \frac{\lambda}{T}}$$

5 Displacement :-

$$y = a\sin(\omega t \pm kx + \phi)$$

6 Constructive — Distructive

Phase diff. $\phi = 2n\pi$ — $\phi = (2n+1)\pi$

Path diff. $\Delta x = n\lambda$ — $\Delta x = (2n+1)\frac{\lambda}{2}$

⑦ Speed of Transverse wave :-

in String: $v = \sqrt{T/m}$

in Solid $\boxed{V = \sqrt{\eta/\rho}}$

⑧ Speed of Longitudinal wave :-

In fluid $v = \sqrt{B/\rho}$ (B = Bulk modulus)

in Solid $v = \sqrt{Y/\rho}$ (Y = Young modulus)

9) Newton's formula (Velocity of sound)

$$\boxed{v = \sqrt{P/\rho}}$$

⑩ Laplace Correction :-

$$\boxed{V = \sqrt{\frac{\gamma P}{\rho}}}$$